Johannes Kepler

Optics

Paralipomena to Witelo & Optical Part of Astronomy
translated by
William H. Donahue
Johannes Kepler

Optics

Paralipomena to Witelo & Optical Part of Astronomy

Translated by
William H. Donahue

Green Lion Press
Santa Fe, New Mexico
Kepler, Johannes

Optics; Paralipomena to Wicel; & Optical Part of Astronomy / translated by William H. Donahue.

Complete English translation of Kepler’s Ad Vitellionem paralipomena, quinque, astronomiae pars optics traducta (Frankfurt, 1604), with introduction, extensive footnotes, bibliography, and two indexes.

ISBN 080009-12-8 (cloth)


QC353.K46 A3 2000

Library of Congress Catalog Card Number 00-104402
Contents

The Green Lion’s Preface ix
Translator’s Introduction xi

Kepler’s Optics
Kepler’s Dedication to the Emperor 5
Epigrams 11

Kepler’s Preface 13

1 On the Nature of Light 17

Appendix to Chapter One, and Airing of Aristotle’s Arguments on Vision in De Anima 43

2 On the Shaping of Light 55

3 The Foundations of Catoptrics and Place of the Image 73

1. Refutation of Euclid. Wield, and alhazen 73
2. True Demonstration 76

4 On the Measure of Refractions 93

1. On the Debate between Tycho and Rothmann upon the matter of refractions 93
2. Refutation of various authors’ various ways of measuring refractions 96
3. Preparation for the true measurement of refractions 104
4. On the sections of a cone 106
5. What kind of quantity measures refractions? 110
6. Causes of the quantity of refractions 123
7. Consideration of those things that Wield advised were necessary for astronomy. 142
8. Whether the refractions are the same in all times and places. 146
9. On the observation of the Dutch in the far North 151
10. Conjectures from antiquity concerning refractions 155

5 On the Means of Vision 171

1. Anatomy of the Eye 171
2. The Means of Vision 179
3. Demonstration of those things that have been said about the crystalline in regard to the means of vision 191
On the varied light of the stars 237
1. On the light of the sun 237
2. On the illumination of the moon 241
3. On the circle of illumination of the moon and the earth 244
4. On the earth’s circle of illumination 246
5. On the phases of the moon 247
6. On knowing the age of the moon from the quantity of the phases 248
7. The paradox that never ever was there a real new moon 251
8. On full and partial illumination of the illuminated bodies, and the earth’s penumbra 252
9. On the lines of the moon’s phases 256
10. On the moon’s spots 259
11. On the mutual illumination of the moon and the earth 263
12. On the first phase or rising of the moon 268
13. On the light of the other heavenly bodies 271
14. On the light of comets 274

On the Shadow of the Earth 279
1. On the form of the shadow 279
2. The paradox that the moon is not obscured by the earth’s shadow 276
3. On the redness of the eclipsed moon 282
4. On the pull of the eclipsed moon 287
5. Problem: To measure the refractions in the regions at the greatest distances by observation of eclipses of the moon 289

On the Shadow of the Moon and Daytime Darkness 297
1. The occasion of this enquiry 297
2. Examples from historical accounts, that the moon’s shadow brought night into the day 299
3. Whether it can happen that in a central conjunction of the luminaries, the sun is still not entirely hidden? 309
4. A number of corollaries on eclipses of the sun 313
5. On the partial occultations of the other heavenly bodies 314
9 On Parallaxes 319
1. On the observable position or place of the heavenly bodies, or the reckoning of it beneath the fixed stars 319
2. On the altitude of the heavenly bodies from the center of the earth and the parallax resulting from the distance of the eyes 321
3. On the diurnal parallax resulting from the distance of the earth’s surface from the center 323
4. A most easy and succinct derivation of the diurnal parallaxes in longitude and latitude, using a new parallactic table 328
5. On the parallax resulting from the distance between the sun and the earth, or the annual parallax 330
6. Short appendix on the curved tail of comets 332

10 Optical foundations of motions of heavenly bodies 335

11 On the observation of the diameters of the sun and moon and eclipses of the two, following the principles of the art 347

Problem 1. To construct an eclipic instrument 347
Problem 2. To measure the sun’s diameter with the instrument 350
Problem 3. To observe the sun’s diameter through a slit 353
Problem 4. To observe the moon’s diameter through the instrument 354
Problem 5. To judge the ratio of the apparent diameters 355
Problem 6. To estimate the quantity of the defect 360
Problem 7. To find the true ratio of the diameters 362
Problem 8. To capture the image of the eclipsed sun on the instrument 363
Problem 9. To extract the true image of the eclipsed sun 364
Problem 10. To extract the image more skilfully 364
Problem 11. To show the true image of the eclipse 365
Problem 12. From the image, to find the ratio of diameters, the visible distance of centers, and the quantity of the eclipse 366
Problem 13. In an eclipse of the sun, to compute the moon’s diameter easily 367
Problem 14. To extract the inclinations of solar eclipses: Maestini’s method 369
Problem 15. To extract the inclinations using the eclipse instrument 370
Problem 16. To mark out the inclinations even on the floor 371
Problem 17. From the marking of the eclipse, to learn the inclination 372
Problem 18. To take lunar inclinations with the instrument 373
Problem 19. To note incursions as is ordinarily done 374
Problem 20. To note the phases as is usually done 374
Problem 21. To compare the moon’s visible place to the fixed stars 376
Problem 22. To note the times expeditiously 381
Problem 23. To elicit the time of the phases from the extension of the eclipses 383
Problem 24. To dig up the visible latitude of the moon from the sun, as well as the longitude 390
Problem 25. To find quickly the visible latitude at another moment 391
Problem 26. To find the moon’s distance from earth 393
Problem 27. To investigate the instant of true conjunction, the true latitude and the difference of meridians 399
Problem 28. To reckon how much the observation of eclipses shall have swerved from the truth 399
Problem 29. To find the elongation of the sun from the meridian 400
Problem 30. To give the angles of visible latitude 409
Problem 31. Whether it is possible for the beginning of some solar eclipse to decline towards the east, and for the end of another to decline towards the west; but for the beginning of an eclipsed moon to be in the west, and at another time for the end to be in the east 413
Problem 32. Whether the visible path of the moon is a straight line 419

Conclusion 432
Kepler’s Index 435
Bibliography 451
Translator’s Index 455
The Green Lion’s Preface

The book you have before you, reader, is not merely the first English translation of the Optics: it is the first complete translation into any language. This is astonishing, in view of the esteem in which Kepler’s optical work is held. This book is the culmination of the perspectivist tradition in optics, which began with the ancient Greeks, was augmented and redirected by Arabic writers such as Alhazen, and taken up by Europeans in the thirteenth century. At the same time, Kepler’s Optics inaugurates the modern approach to optics, with a clear understanding of how the eye works, a physical account of refraction using something very much like a wave front, and a new level of understanding of how lenses form images. Descartes based his optical work largely on Kepler, and every optical theorist since then has been indebted to this book. Green Lion Press is glad to be able to present this epochal treatise in a form accessible to modern readers.

In this edition, as in other Green Lion books, care has been taken to make the book reader-friendly and durable. Kepler’s marginal notes have been kept in the margins, with the translator’s notes at the page bottoms, as footnotes, not the annoying end notes that are often found in scholarly works. Further, Kepler’s end notes have been included as footnotes, or, when too long to fit at the bottom of the page, as appendices directly following the passages to which they apply.

Page numbers of the 1604 edition have been placed in the inner margins, to allow easy reference either to that edition or to the text in A KGEP II, which also includes the 1604 page numbers. The marginal page numbers also facilitate the use of Kepler’s index, in which the original page numbers have been retained.

The 100-odd diagrams have been redrawn, with the exception of two instances where a reproduction of the original woodcut was preferable (these have been reproduced from the 1604 edition). In almost every instance, diagrams, repeated as necessary, are placed adjacent to all the text that refers to them.

The book is constructed in the traditional way that fine books have been made, with sewn signatures. The paper is acid-free and the cover boards and binding were chosen with strength and durability in mind.

Green Lion Press prefers to use typefaces and sizes that are elegant and easy on the eye. However, the length of this book led us to use ten point type size and a font, Times, which, despite being
somewhat inelegant, is very efficient in the use of space. It also occurred to us that this book, arising as it did from practical concerns, and intended by its author to be a compendium of practical observational techniques, might most appropriately be presented in a form that smacked more of the workshop than the study.

Dana Densmore and William H. Donahue
for the Green Lion Press
Translator’s Introduction

In the summer of 1600, Kepler assembled a large wooden instrument (described at the beginning of chapter 11 of the present work), and set it up in the Market Square in Graz to observe the solar eclipse of June 30/July 1. His experience with this instrument drew his attention to certain optical matters that had been troubling astronomers. From a few notes written down at the time of the eclipse, his optical project grew to encompass metaphysical and theological speculations about the nature of light (Chapter 1), a thoroughgoing attempt to account for refraction, involving a novel treatment of conic sections (Chapter 4), a study of the anatomy and physiology of the eye (chapter 5), a historical study of solar eclipses (Chapter 8), and many other matters relating to the light of the heavenly bodies and how it behaves. The result is one of the most important optical works ever written, which, even when it is wrong, is wrong in an interesting and fruitful way.

The problem that Kepler had initially encountered, that initiated his optical studies, involved the way light flows through pinholes. Astronomers had been puzzled about why the moon seemed smaller in a solar eclipse than it did at other times. Kepler figured out why. In a characteristic series of numbered sentences jotted down on a piece of paper at the time, he set out point by point the way a finite luminous object and a small but finite opening interact to form a slightly enlarged image. (An expanded version of these notes constitutes chapter 3 of the present work.)

At first, Kepler had it in mind to publish this important discovery in a separate small work, which he had actually written during the following month. But his exile from Styria, move to Prague, and involvement with the work of Tycho Brahe intervened. Much of his time in the next year was taken up with the orbits of Mars and of the earth.

With the death of Brahe in October, 1601, everything changed. As his successor in the post of Imperial Mathematician, Kepler was expected to produce works that would reflect well on his patron, Rudolph II. In particular, he was to complete the work unfinished by Brahe and to produce the astronomical tables that would bear

---

2 Letters to Michael Maestlin, 9 September and 6/7th December 1600, in JdGW XIV, nos. 175 and 180, pp. 150–1 and 159.
the name of Rudolph. Kepler filed sheets after sheet with computations and ruminations about the orbit of Mars, which, he believed, held the key to the deeper astronomy. But in the spring of 1602, his investigation took a surprising turn: he discovered that the orbit could not be perfectly circular, but had to be squeezed in slightly at the sides. The unexpected complications that he introduced, together with difficulties with the Brahe family regarding use of the Tychoic observations, led him to realize that the Mars book would not be finished as soon as he had hoped, and that he had better find something else to fill in. His thoughts returned to the book on pinholes.

However, the project turned out not to be as simple as it had seemed at first. As Kepler outlined it in his dedication to the Emperor, he thought he should also discuss the other main optical problem in astronomy, the refraction of light in the atmosphere. That, in turn, required an understanding of refraction itself, which demanded a study of the nature of light. And since there was some uncertainty about how the eye itself interacted with astronomical sights, it seemed that he needed to understand how the eye functioned. To fill these needs, he worked his way through Witelo’s *Perspectiva,* itself no insignificant feat, and compared Witelo’s treatment of vision with the anatomical and physiological approaches of Vesalius and Plater. Further, his study of refraction suggested to him that curves other than circles would be required to explain it, and that led to a study of Apollonius’s *Conics,* which he recast in Keplerian style. And, having gone this far, he thought he would round it out with a few chapters on “the light, the place, and the motion of the heavenly bodies,” and the theory of parallax, so important for the moon’s motion. As a result, although he confidently promised the book for Christmas of 1602, he was still hard at work on it in the summer of 1603. In May he wrote, “measuring refractions: here I get stuck. Good God, what a hidden ratio! All the *Conics* of Apollonius had to be devoted first, a job which I have now nearly finished.”4 He at last sent the finished manuscript to the Emperor in January, 1604.

1 Witelo, latinized “Vitellio” or “Vitellius,” was a Polish scholar of the thirteenth century who studied at Padua and Paris. The little that is known of his life is presented in detail by Lindberg in his introduction to the *Theories.* His principal work, titled *Perspectiva et Opticae,* was the foremost European optical treatise in the Middle Ages and Renaissance, its only rival being Alhazen’s treatise. It was published in three editions in the sixteenth century. The edition Kepler used was Riccioli’s *Opticae Theoricae,* which included Alhazen and Witelo in a single huge volume (with two separate pagination).

2 Letter to Hucbaut von Hohenberg, nos. 256 in JACOB XIV p. 396.
The book everywhere shows signs of the haste with which it was written. Syntax is often jumbled, words misspelled, numbers wrongly cited, figures incorrect, and demonstrations vague. Once the book was in press, Kepler had second thoughts about many passages, for which he wrote notes, often extensive, which he advised the reader to consult first. Production was difficult owing to Kepler’s having entrusted the book to a printer in Frankfurt, perhaps hoping that, in this great center of publishing and marketing, production would be quick and efficient and sales brisk. As it turned out, the pace was far from quick, and it was not until the summer of 1604 that the book finally appeared, with a lengthy (and incomplete) errata sheet appended.

The resulting work is historically important in many ways, some of which, at least, have not yet been explored. Attention has been disproportionately directed to those parts of the Optics that are seen as points of departure for later scientific theories, covering, for example, the inverted retinal image and the way lenses work. Keplerian studies are now entering a phase in which scholars are trying to understand Kepler’s thought as a unified whole, in the context in which he lived and wrote. In this regard, there is much in the Optics that is worth studying. In particular, the way in which Kepler’s astronomical works are related to his work in optics has received little attention. Chapter 10, in which Kepler argues, on the basis of their optical works, that the ancient optical writers were heliocentrist, is a notable example of this, as is his stated intention of building his unfinished Hipparchus on the foundation of his Optics.5

As the book’s translator, I can easily sympathize with Kepler’s frustration at the difficulties of the task and the amount of time it took. The range of subjects and materials to be covered was enormous. I have spent considerable time in rare book collections researching Kepler’s sources, with only partial success. Kepler’s labor of gathering and weighing the reports and opinions of various authors, and of trying out various mathematical models of his own, evidently left little time for the actual writing.

My approach to the text has therefore been somewhat different from my treatment of the Astronomia nova in my translation of that work (see JKNA). This is not really a finished work, and to smooth off the rough places would misrepresent it. I have accordingly followed the Latin more closely than I did in the Astronomia nova, at the expense of fluidity in the prose. The only point on which I have chosen to depart from a strict rendering of Kepler’s text has been

5 Cf. p. 453, below.
to break up some of his long sentences, which could not be clearly reproduced in English because of our language’s lack of syntactic markers. The entire translation has been reviewed by classicist Bruce M. Perry of St. John’s College, whose help has made it much better than it otherwise would have been.

In writing the footnotes, I have aimed primarily at clarifying obscurities and providing references to sources which, though perhaps familiar in Kepler’s day, are obscure to us. The editions of Frisch (JKOO) and Hammer (JKGW) have more explanatory material than is present here; readers who are interested may consult those works. Some passages, notably Kepler’s physical explanation of refraction and his resulting refraction law, cry out for study and interpretation, but this is too much to burden a translation with. Such matters deserve separate studies, such as those that have already been done by Stephen Straker, David Lindberg, and others. What readers need is a few notes to get them past the difficulties that the text presents at the level of simple comprehension and that is what I have endeavored to provide.

The title, like many of Kepler’s titles, presents peculiar problems. It appears to be the title of two books, one of which is about Witelo’s optics, and the other about astronomical optics, an impression that is confirmed in the text (for example, pp. 91, 171, and in the running heads themselves, which are different in the two halves of the book). Nevertheless, the book does have a coherent theme: it is about light, and therefore its proper subject is Optics. Kepler tacitly acknowledged this in referring to it later as “mea Optica.” The title he gave it is too modest, suggesting only that it has a few things to say about Witelo, and then will go on to talk about some astronomical matters. So I believe that, in calling it Optics, I am only restoring to it its rightful name.

The other problem with the title is how to translate “paralipomena.” This latinitization of the Greek παραλιπομένον literally means “things that are left on the side,” which can either denote things omitted or neglected, or things that are ancillary or supplementary to some central matter. One might then think that Kepler means to discuss things that Witelo neglected. However, his use of the preposition “ad,” “in,” shows that he had the latter meaning in mind. So the opening lines of the title become “Supplement to Witelo.” I was still unhappy with this because it lost the odd flavor of the original. Then it was suggested to me that this word is an early example of the German fondness for using Greek words in titles, exemplified,
Translator’s Introduction

for example, by Kant’s Prolegomena to Any Future Metaphysics. If so, then perhaps the best approach is simply to carry it over from the Latin, as Kepler had from the Greek.

Acknowledgements

This translation was supported by a translation grant from the National Endowment for the Humanities, whose support is gratefully acknowledged. There were also many individuals who contributed to the project in one way or another.

Special thanks go to Peter Barker and Katherine Webb, who provided food, lodging, and friendship during two extended periods of research in the remarkable History of Science Collection at the University of Oklahoma. Thanks also to Tom and Haydie Callaghan, gracious hosts during a stay in the Boston area for work at the Houghton Library. The staff at the respective libraries were most helpful, especially Mariýn Ogilvie and Steven Wagner of the University of Oklahoma.

The staff of the Meere Library at St. John’s College, Santa Fe, helped out in more ways than I can acknowledge here, especially Inga Waine, Director, and Laura Cooley and Heather MacLean. Lisa Richmond, Director of the Greenfield Library at the Annapolis campus of St. John’s, was energetic and helpful in arranging for a loan of the facsimile of the 1604 edition of the Optics.

Bruce M. Perry of St. John’s College, Santa Fe, reviewed the entire translation, catching many misinterpretations and infelicities of expression. Special thanks to David C. Limberg for lending me a copy of his reprint of the Thesaurus and reading and commenting on parts of the translation. Peter Barker, Eva T. H. Brann, Bernard R. Goldstein, Owen Gingerich, William Kerr, Peter Machamer, Thomas North, Bruce Stephenson, Stephen M. Striker, and James R. Voelkel all helped in various ways with comments and answers to queries, for which I am grateful.

The typesetting and layout were done in \TeX, using the superb ZZ\TeX book layout software written by Paul Anagnostopoulos.

William H. Donahue
Santa Fe, New Mexico
September 19, 2000
Johannes Kepler

Paralipomena to Witelo

whereby

The Optical Part

of Astronomy

is Treated
Paralipomena to Witeio

whence

The Optical Part
of Astronomy
is Treated;

Above all

on the Technically Sound Observation
and Evaluation of the Diameters
and Eclipses of the Sun and Moon.
With Examples of Important Eclipses.

In this Book, Reader, you have, among many other new things,
A Lucid Treatise on the Means of Vision, and the Use
of the Humors of the Eye, against the Opticians and Anatomists,
by the author

Johannes Kepler, H. I. M.'s

Mathematician.
AD VITELLIONEM
Paralipomena.
Quibus
ASTRONOMIAE
Pars Optica
TRADITVR:
Patris
De Artificiosa Observatione
et Aestimatione Diametrorum
dictiorum in Sole & Luna;
CVM Exemplis insignium Eclipsium.
Habet hoc librum Lectorem, inter alia multa noua.
Tractatum luculentum de modo visivis, & humorum oculi
usque ad Ophtalmicos & Anatomicos.
AVTHORE
IOANNE KEPLERO, S.C. M°
Mathematico.

FRANCISVRTI,
Apud Claudium Marnium & Heredes Ioannis Aubrii
Anno M.DCIV.
Cum Privilegio S.C. Maiestae.
Dedication

To Rudolph II
Ever August Emperor of the Romans

P.F.P.P.

King of Hungary, Bohemia, etc.; Archduke of Austria, etc.

Since this one time, most August Caesar, has been forth that extraordinary
astronomer Tycho Brahe, who, in his most meticulous observations and incredible
precision rivalled the very nature of things, and since Germany has, as well as
those ancient lights of the mathematical faculty, Peurbach, Regiomontanus, Apianus,
and those whom the Viennese Academy can enumerate in a lengthy list,
also now this new exemplar, sublime and admirable, of carefulness and perfection.
I therefore think it fitting, as the Professor of this whole art are energetically
following in the footsteps of this preeminent boy, and compare themselves with
his example, and especially since the Germans labor in it, each in his own capacity,
that they may yet continue to retain the prize for this glory, hitherto set
before them, within their nation. For the treasury of the sects of Nature is
exhausted, its richness beyond telling, and anyone who brings forth some new
thing about it into the light does no more than open the path to others for
subsequent investigations. Moreover, for me, who have devoted considerable time
to optical studies, the Tychoan astronomy, in which this Your Majesty's most
brilliant court made me a participant while Tycho was still alive, provided the
occasion, with the result that I considered certain theorems of optics, trifling in
appearance, but bearing the seeds of great things, worth pursuing and unfolding
at greater depth. For I thought it unworthy in optical science, since senses and
instruments are required in astronomy, while in optics, geometrical certainty is
not lacking, that optics be surpassed by astronomy, and that one cannot establish
in the former by demonstrations what in the latter the eyes have comprehended;
it is much more unworthy that when summoned by astronomers to assist them,
optics does not appear, and it is unable of itself to soften the fault of hindering the
precision of astronomy, most Augustus and the Court. And so I have considered it no
small honor, if it might fall to me to undo the knots that have been in the way,
by a good method and the clarity of demonstrations, and to lead the optical scien-
tific through to that degree of subtlety that it might satisfy the astronomer. Which
plan of mine, after Your Imperial Majesty approved it, I began to follow through
with the greatest industry, by the collection and publication in book form of those
things which either I had discovered once in Syria, while drawing a stipend from
the provincial Governors, or had to be added to the discoveries later, which were
in fact the most difficult of all, and required the most work.

And there were two things that Tycho Brahe's accurate carefulness, first
brought out into the light, that pertain to optical science: one, on the refraction
of the light of the heavenly bodies: the other, on the diminution of the moon's
diameter in solar eclipses. Although several other passages also appear through-
out that man's writings in which he sent the astronomers back and exhorted them
to probe the mysteries of optics, these two main points are nonetheless of the
greatest weight. And in fact Brahe did carefully investigate the measure of refractions at all degrees of altitude, but since the cause of the measure had not yet been made evident by optics, there arose involves discussions about the universe and about the elements, and it was not sufficiently clear whether the refractions were the same or different at all times and in all places. From this, it finally follows that sometimes as observed, attributing too much to his own diligence and carefulness that he had applied among the things to be observed, and supposing the place of the celestial body to be known with complete precision, throws the demonstrations into difficulty: the nature of light, heat by the inconsistency of optical causes, does not always allow such precision of instruments. And so I have devoted Chapter 4 to this matter, before which, because of the method of the investigation, Chapter 3 and part of Chapter 1 had to be presented. Moreover, the earth’s shadow is drawn into the discussions, once the refractions have been found, and once it is known that the sun’s rays do not spread out in the same straight lines from the source all the way to the earth, and much less so beyond it. And further, since this matter is in doubt, we make a test of the universal proportion of the measurement of the celestial bodies (since this depend on the earth’s shadow), that is, of the thing, foremost in astronomy, that the public praises and the philosophers too praise and revere. And so, to preserve astronomy’s dignity, and to storm this hostile fortification of doubt, although in Chapters 4 and 5 several steps in the big undertaking were made, and were, so to speak, cut into the extremely hard rock: by the sharpness of intellect, there nevertheless remained so much work as to require Chapter 7 especially for it. Now, in the diameters of the luminaries, which was the other subject, if the sense of sight commits any error, this affects not only the whole theory of eclipses, but also most of all is what I have just spoken of, the measurement of the celestial bodies. And thus, so that the tables of the moments which, broken off by the author’s death, your Majesty proposed for completion in this time, and is providing for with a not insignificant expenditure—so that these tables, I say, may achieve due soundness in their most conspicuous part, the labors of the sun and moon, I have resolved that every stone must be turned and nothing must be overlooked that might leave any hesitation. Therefore in Chapter 2 I have explained the extent to which instruments can discover us in investigating the dimensions of the luminaries, and in Chapter 5 I have begun to speak of the extent to which the sense of sight is itself beset with errors. And since the complete account of vision had to be set out as a whole, which is brought to completion with refractions, and with the images of things seen, and with color, it should not appear strange to anyone if I degrees at some length into the sections of the cone in Chapter 4, which give form to the refractions of the eye, the optical entertainments of Joannes Baptista Porta in Chapter 2, the nature of light and colors in Chapter 1, and other things elsewhere. For although these matters might be not at all concerned with astronomy, they are in themselves worth knowing. Again, to the discoveries in Chapter 11, a matter that is commonly said to be easy, I have added discoveries, and have

1 "Rerum copiae XI inventae, quod volgo littera facie, annoto addid. . . . . The syntax
shown how to measure the diameters of these luminaries without chance of error, while in Chapter 8, on the occasion of eclipses of the sun, I have shown how to compare them with each other. And this much I echo Brahe but provided me with the occasion of writing about.

Further, since the things just enumerated includes the greatest part of what can be said about astronomy from optics, and the book would now almost deserve the title, Optics of Astronomy, I thought I would make a compendium of the work if, whatever little were remaining, I should add it too. And these were about the light, the place, and the motion of the heavenly bodies, which I have gone through in Chapters 6, 9, and 10. For both the sixth and the ninth are ancillary to the eleventh, the former for estimating the moon's diameter, the latter for demonstrating optically the remarkable appearances of eclipses, and the paradoxical changes of direction. Therefore, those things that I have found out in other chapters about these three remaining attributes of the heav-

enly bodies, I have, as much as they allowed, amplified and set forth in a much more polished manner. Especially in Chapter 9, the theory of parallax, which alone of all astronomy is the most difficult and vexing: this, I say, I have aided with the most easy short cuts, and have thus improved it so, that it might be taken as almost new, and I have constructed a new parallactic table, which does indeed take its name from this theory of parallaxes, but which has a very general and absolutely compendious application in the whole remaining the-

ory of the secondary mobile bodies, which I shall set forth, God willing, in its own time.

And so, at this time, I have given this out to the public, however much work it involves, so that, because this most beautiful science lies neglected, I may arouse some people from this lengthy sleep to embrace it, and to navigate this practically new channel usefully, which I was the first to open with risk and perhaps with some expense; that is, to untie more dexterously the knots over which I have sweated, if they shall perhaps find anything lacking, while distinguishing those things which I have proposed as certain, provided only that they not fly into these most pure rites of Apollo, with, as they say, unwashed hands, rashly taking on something greater than their powers and grasp.

But that I should bring this book of mine forward into Your Imperial Majesty's view, and give it in trust under the protection of this august Name, all the highest Offices persuaded me; for in these times Your Majesty, though dis-

tracted by the war against the Turks, the greatest of all and extremely costly, has not passed astronomy over unattended; and has hitherto supported me, devoted to the Tychoic astronomy, with a liberal salary, by whose help I have carried these things out, and I in turn, by whatever means is in my power, wish to show my gratitude—indeed, my truest deference—and pay homage to this regal affection

1. is opaque here: Chevalley writes: “De plus en chapitre XI. j'ai apporté de nouvelles découvertes à ce qui est communément connu. . . . . . .” which makes some sense but doesn't fit the Latin.

2. ruines d'évèc.
for the liberal arts; and, according to what is due, make commendation to posterity; while those present should also understand through me that it is possible for no one to hide how much Your Majesty has done for these arts of peace, or which hopes of Your Majesty's eventual triumph in this long-continued war, and upon the recovery of peace, they ought to cultivate.

If there is anything further in this work which should give me confidence in speaking in Your Majesty's presence, it is undoubtedly this, that I have expended a huge and disagreeable task in the investigating matters that have lain neglected for so many ages, and these are all different things, about which others have put together separate books for each, as with many patrons: nor have I filled up my soul with speculations of abstract geometry, that is, with pictures "both of those things that are and of those things that are not", which are almost the only things on which the most celebrated philosophers of today spend their lives, but I have tracked geometry through the cosmic bodies portrayed through her, following the Creator's inprints with sweet and pleasing. Finally, that both these works and others of this kind which I have in hand even now, I have taken up to do honor to my profession, which I hitherto followed by Your Majesty's order and salary, at the expense, not so much of health, but of other additional studies, by which I was able, and, in the judgment of my friends, was obliged, to create security for my old age, if such be destined, and for my family. If any inconvenience redounds to me because of this situation, this one thought, worthy of a German man, makes it light and pleasant for me: that it is beautiful even to die in honest service of such a Prince, and in this to be overcome in valor by soldiers, but to defend vigorously an assigned task, as if it were some eagle. Should Your Majesty most clemently approve of this most deferent affection of mine, I shall consider myself sufficiently happy: more happy, where he shall also have found in this work that which is adequate to His divine judgment on all the arts, and that which his mind, previously satiated with the most outstanding discoveries, may yet desire from this work, but most happy by far if the same Clemency that I hope thus far experienced should continue to favor me. For it will thus be by no means a cause of fear in me that that most pernicious enemy of arts, Indigence, might cast me down, compared by starvation, from this task of mine, as from a citadel entrusted to my loyalty, nor will there be any doubt but that Your Majesty will have sent me that assistance and those provisions in time, upon the receipt of which I might be able to bear up under siege, and finally, with all difficulties overcome, happily to complete the rest of my studies for the glory of God, and the celebration of Your Majesty, and for the ease of the human race.

May God Most Good and Most Great preserve Your Majesty unharmed for as long as possible, and, with your victorious armies, may you drive forth

---

1 This is an allusion to Plutarch's famous saying: "Max is the measure of all things: of the things that are, that they are; and of the things that are not, that they are not." These words were quoted by Plato in the Theaetetus, 152 A. However, Plato discusses being and nothingness in the context of pictures in the Sophist, beginning about 235 D, so this may be what Kepler was thinking of when he wrote these Greek words.
from Christian necks, and do so to as great a distance as possible, the tyrant of the house of Ottoman, advanced in years, odious in barbaric pride, insufferable in blasphemies. At Prague, 28 August, which day has brought Your Imperial Majesty’s fifty-third birthday, famous for the most honor-bearing embassy of the King of the Persians, in the year of human salvation 1604.

Your Imperial Majesty’s
Most Humble
Mathematician
Johannes Kepler.
Epigrams

On the Optical Books of
Johannes Kepler. His Holy Roman Majesty’s Mathematician

Epigram

of Ioannes Serres the Elder, Secretary to Christian II, Elector of Saxony.

He who desires to observe the fire-bolting eye of the world,
Or who wishes to perceive the eyes with his own eyes:
Let him, flashing with your mind’s eyes, O Kepler,
Hide himself right in the bowels of your book; 
Not should he fear the rays of the radiant, and flying atoms,
And the wondrous phantasms that multifarious color bears;
[Not] the mirror be it a simple line or compound;
Since those things never seen by the eyes, you will give for seeing.

The Author’s Epigram

upon his eyes, and his treatise on the eyes.

Even:
O dear mind, we have lost our sharpness, while the lights of the true
We have sent to your threshold through our glazing.
Without this marriage, you would remain blind: of the work,
Give some return to your partners: give, sweet sister!-

Mind:
What should I do for those in distress? When did an implacable hour
separate me far from your hospitality?

Even:
Snatch us away from darkness, lead us to whatever light you will go to;
And from whate’er fear of death you lack, deliver us also,

Mind:
As far as possible, I shall do it; only let faire favor the speaker,
I shall make you mortals eternal with my writings.
Here also I bring back the losses that you have borne for me;
And bring I shall cast rays upon its brilliance, even with blemishes.

Another with the same sense

The eye speaks.

I devalue life in exchange for fame: for a name, perception
Teach, O soul, how to die more fruitfully, so as not to die
Preface

Astronomy, which deals with the motions of the heavenly bodies, principally has two parts. One consists of the investigation and comprehension of the forms of the motions, and is mainly subservient to philosophical contemplation. The other, arising from it, investigates the position of the heavenly bodies at any given moment, and has a practical orientation, laying the foundations for prognosis. These two parts fly up into the heavens, supported as Plato used to say, by (as it were) a pair of wings, geometry and arithmetic, each of which serves both of the parts referred to, though geometry serves the contemplative part more, and arithmetic the practical part. And thus the highest apex of astronomy culminates in the arithmetical part, which embraces the tables of motions and the ephemerides derived from them.

The geometrical part has many others beneath it, according to the variety of subject matter which an honest evaluation applies to geometrical astronomy. For to trace it from the beginning, in astronomical demonstrations there are two kinds of principles: one, the observations, and the other, the physical or metaphysical axioms. Now around the observations, there arise the three parts of astronomy. The first is the mechanical part, dealing with inanimateness, suitable for observing the celestial motions, and the way of using them, which is known to astronomers, the great Tycho Brahe, published five years ago. The second is the historical part, comprising the observations themselves. The reader should be informed this 24 books of the most meticulous observations of this sort, embracing the past 40 years or so, were left by Tycho Brahe, which I hope will come forth into the light at some opportune time. But because all celestial observation takes place through the mediation of light or shadow, and because the media between the stars and the eye have a variety of modifications, and because those things that we observe in the heavens are either motions (whose kind includes retrogradation), station, and so on), or arcs (that is, angles at the observer), or luminous bodies, and because all these are considered in optical science, hence arises the third, optical, part of astronomy, which I am treating now, through a brief recounting, as if included among the principles of the old things that Wino treated methodically, or the new things that Tycho Brahe treated here and there, on this subject.

The other kind of principles in astronomical demonstrations, the physical or metaphysical principles, together with the subject matter itself of astronomy, which is principally the motions of the heavens, a physical matter, makes up

1 Tycho Brahe, Astronomiae instauratae mechanicae, Wandelburg 1598. In TBOO V, p. 3–162.
2 Although Kepler had hoped to publish these observations, since his comment in Vol. 1 of the Kepler Manuscripts in St. Petersburg, quoted by Frisch in JKG 1, p. 192: he never succeeded in doing so. The two first published by the Iusius Albertus Cunzius in a book dated 1627, on his title page through the colophon the title Historia coelestis. They appear in TBOO sub. 30–13.
the fourth part of astronomy; namely, the physical part, which deals with the efficient causes of the motions, or the movers; the formal causes, or figures that the movers serve for; the material causes or orbs, and the physical interposition or remission of movers, when part, if God should grant me life, I shall encompass by means of Commentaries on the motions of Mars (built out of the Tychoic observations, and up to the adamantine foundation of astronomy of the motion of the sun, re-established) by Tycho Brahe, and of the fixed stars set in order by him, which I think I can call the key to a deeper astronomy.¹

You see, dear reader, the position that is occupied by the Tychoic (that is, the truest and most accurate) astronomy. He collected most ample material for future building in the observation books; he showed the soundness of that material in the Mechanic: he laid two very solid foundations for the house, as I have just said, through the Catalog of Fixed Stars, described very accurately and truly, which has played the role of the best cement, and will serve to glue together the material of the observations; and through the theory of the sun that he established, which has the firmness of a foundation and is the central post, embracing all the arches, right in their tops, the door too, with groined roof belonging to all the chambers.² But he brought the front of the house to completion, I mean the theory of the moon, the primary palace and portfolio, so that the house might be suited to habitation. To this, in the present book for this purpose, windows and stair, are in part added and in part replaced where they were broken out. The armory, or the theory of Mars, was constructed; doors, or publication, will be added in the near future. There remains the office, the oratory, the dining hall with bedroom, the study, above all of which is built a platform, in the place of a watchtower, for catching a view of the ages, the theory of the eighth sphere, and the ages of the ploughs.³ These are indeed mixed with some


² This was published in 1609 under the title Astronomia Nova. It is in Vol. 3 of JKGW, and has been translated into English by the present translator (New Astronomy, Cambridge 1927).

³ Kepler is alluding to the presence of the sun's (that is, the earth's) motion in the apparent motions of all the planets. Here is in irony here, however, largely because of his erroneous estimation of the solar parallax. Bode's solar eccentricity, the celestial parameters of his solar theory, was incorrect. See C. A. Wilton, "The Error in Kepler's Accurate Data (for Mars)," Centaurus 13 (1969), 2–18, reprinted in Wilton's Astronomies from Kepler to Newton (London: Vinton, 1989).
timers of ancient observations, rougher and consumed with rot; nevertheless, these too, as far as possible, will be smoothed and strengthened, so that in the end it will be possible to place on top of it the most or highest pinnacle, the table of the Rudolph. That this may also come to pass at the first possible time, pray with me to God All-great and All-good, if in fact you admire the Tyeconic astronomy, that by means of the arms of Emperor Rudolph he may push back that sworn enemy of the name "Christian", and that he may grant his people victory. For if that should happen, there is no doubt that the Most Wise Emperor will then provide suitable outlays in the greatest abundance, since even in these upheavals of war he does not interrupt them.

Those things that come to be considered optically in astronomy are either of objects themselves set before the sense of sight, where images of things, or rather, light and shadow, are considered; or the medium through which vehicle the light of the images passes, because of which some light comes to us refracted or finally, the visual instrument, or the eye.

Again, the objects themselves are either the celestial bodies (the sun, the moon, and the stars), the motions, or the places of the bodies. We properly begin with the bodies. Now in bodies, in astronomical terms, we consider nothing but their image, which they send down to us by the aid of their light, which they are endowed, and of that image, the shape and quantity, chiefly of the sun and the moon, but, regarding the earth, that of its shadow.

For the most noble and ancient part of astronomy is the eclipse of the sun and the moon, a subject that, as Pliny says, is in the entire study of nature the most wondrous, and must like a porter. Anyone who ponders this carefully will find if he will refuse to have recourse to faith in holy scripture both that there is a God, a founder of all nature, and that in the very mechanics of it he had care for the humans that were to come. For his theater of the world is so ordered that there exist in it suitable signs by which human minds, likenesses of God, are not only invited to study the divine works, from which they may evaluate the founder's goodness, but also are assisted in inquiring more deeply.

For, I implore you, what is the cause, if it is not wise, for nature's playing such games in the sun's and moon's bodies, by which not only humans, as history bears witness, are turned to wonder and stupefaction, as long as they are ignorant of the causes, but even the quadrupeds, by Pliny's testimony, commonly take fright?

Furthermore, the extent to which humans are assisted by eclipses of the luminaries in all of astronomy, all the books of the astronomers bespeak. For, as regards the motions of the sun and moon, and the lengths of years and months, this entire theory first arose solely from the observation of eclipses, nor could it be constructed otherwise. Moreover, it cannot be smoothed and polished further except by considering eclipses of the luminaries more accurately and truly, which is the aim of this book.

Now anyone who will consider how closely all the rest of astronomy is linked to the sun's motion, and how much the moon, participant in day and
night, assists us, when all other means fail us, will rightly believe that all of astronomy is supported by these observations of the luminaries, so much so that these darknesses are the astronomers’ eyes, these defects are a conspectus of theory, these blunders illuminate the minds of mortals with the most precious picture. O excellent theme, praiseworthy among all people, of the praises of shadow!

And thus the quantity of the image which the moon or the sun, whether whole or eclipsed, shows us, and of the shadow which the earth stretches out to the moon, must be carefully investigated by the astronomer. The diameters of the other stars are sought out to the extent that, if neglected, they will render the observations uncertain, and to the extent that we are burying ourselves to know the same things about them as about the bulk of the sun and the moon’s bodies.

Next, these images come down to us thanks to the light in the sun, which is direct and intrinsic, and in the moon, which is reflected and extrinsic.

Even though these images are obvious to everyone’s eyes, all practicing astronomers complain that it is with difficulty that they are measured. This is partly because the bodies have a very narrow apparent size, and partly because they constrain the eyes with their exceptional light, so as to prevent their fulfilling their function in seeing. But even in this place nature has not forsaken those desirous of learning, for she has shown a procedure by which we may accomplish in darkness, without detriment to the eyes, what is completely impossible in clear light, with the eyesight directed towards the sun. Since this method is worthy of admiration, and also was grasped most ingenuously by the practitioners of the art, it is fitting that it not be held as contemptible and neglected among astronomers, but adorned with geometrical demonstrations, and illustrated with instances, which was done by me three years ago, on the occasion of the eclipse of the sun of 1600, in this manner which follows shortly, although I shall preface a few things first. For because many things, not only about the direct ray, but also about the reflected and refracted ray, were overlooked by Wetics, and many things that should have been explained a priori were only brought in extraneously from experience and set up in place of axioms, I thought it a good idea to look a little more deeply into the whole nature of light, and to relate to its principles those things that appear, insofar as possible at present, in case readers should come along whose mental powers for seeking out the arena of light might be able either to be more aroused or even more assisted by these discussions. Although these things are not a little remote from the proposed subject, there are also not a few things that are to be cleared up for astronomers by all the kinds or images of rays.

* An example of Keplerian wit: the solar eclipses of 1600 and 1601 are not considered until the final pages of this lengthy treatise.
Chapter 1
On the Nature of Light

Albeit 1 that since, for the time being, we here verge away from Geometry to a physical consideration, 2 our discussion will accordingly be somewhat freer, and not everywhere assisted by diagrams and letters, though the chains of proofs, but, looser in its conjectures, will pursue a certain freedom in philosophizing—despite this, I shall exert myself, if it can be done, to see that even this part be divided into propositions.

In the place of Common Notions 3 I am giving a preliminary admonition on vocabulary: what the Greeks denote with the single term "bending back" καταρακτήριον 4 of rays, the Latins for the sake of distinction divide into two classes, establishing one class of reflections and the other of refractions. Now if words are to be diversified because of the different natures of things, it is preferable to choose those in which the nature of any particular thing is expressed as nearly as possible to the usage of the common people. Anathema, this does not seem to have been brought happily to pass by the Latin critical writers.

For this much is clear to me, that the proper usage of the Latine word "rectum" or "recta" in Latin, concerns things that are bowed, and which, if the force were to cease, would return to themselves; hence, the stretching of the ligaments and the bowed appearance of the member is also called "bending the knee." 5 Other uses are analogous to these.

The Greek word διανομή 6 however, most properly expresses that

---

1 Gubernator in Latin. This is a very odd word with which to begin. It is the adverbial use of the term of governor (the rest, the remainder), and usually means "moreover" or "apart from this," or something similar indicating a transition from what went before, therefore seems appropriate to use in an archaic English word and keep Kepler's sentence unbroken, to retrieve the odd flavor of the original.

2 According to the prevailing Aristotelian division of the sciences, mathematics and physics were sharply distinguished in that the former treats abstract forms while the latter treats the concrete. The former deals with physical lines, but not physical objects; whereas optics deals with mathematical lines, but not physical mathematical lines. (Aristotle, Physics H 2.194a 10-12.) Kepler carefully preserved this distinction, using geometry as an intermediate between observation and physical theory. See especially New Astronomy, pt. I, trans. R. H. Darnton (Cambridge 1995), p. 162.

3 Kepler lies alludes to Euclid's introduction of what he calls "common notions": καταρακτήριον immediately following the postulates. They are concepts that are common to all of mathematics, as distinguished from the postulates, which are proper to geometry.

4 This is the noun formed from the sub-dimension, from καταρακτήριον or back and κατά κατά back and back.

5 Kepler raises to the intuitive form. The present indicative is καταράω. Kepler's remarks are correct, except that the word does not imply the presence of "opses" in the thing bent.

6 Grown thee.

7 Kepler confuses the matter by giving the intuitive of the middle voice rather than the active. This is unorthodox, for which see note 6.
which is "frangere" (to break in Latin), which presupposes a dissolution of continuity (most especially of word) and a loud noise. Thus *chirurgus* in Greek is a small branch grown from a tree. *Attikàdáos* is used to describe the noise which in Latin is called "fragor" (noise of breaking), from "fragendo" (breaking). If this be compared with the characteristics of rays, the word "flectere", if you look to its proper sense, clearly does not agree with them: for nothing of the sort pertains to rays, whether accidet upon mirrors or by water. On the contrary, both of these are most correctly described by the same word "tractio" (breaking). For both are truly broken, the one by the surface of the mirror, the other by the surface of the water, and the broken parts constitute a rectilinear angle. Thus although in a broader sense something is also said to "bend aside" which, while it should have been single, has gone off into two contiguous straight lines, exactly like the rays: nevertheless, for the sake of avoiding the undesirable implication, the word flectere is clearly to be excluded. For in this signification, *reflexus* (bending back) would pertain to both classes of rays, exactly like the Greek word *katalektia*; but because rays are affected in one way by mirrors and in another by water (for the one rebounds from the mirror towards those parts whence it came, while the other bends down from the surface of the water into the depths, and towards those parts opposite those whence it came), let us therefore follow here the convention of the optical writers, and use *reflexus* names: let the one be denoted by the Virgilian and special word "Perseverrition"; and the other by the glosae name "intraction", so that the propositions themselves may allude to the nature of the thing.

For the demonstrations of characteristics of this kind, however, all philosophers and optical writers establish a certain comparison between physical bodies (and their motions), and light, which comparison we shall explicate somewhat more broadly.  

---

8 There is, pace Kepler, no sense of hearing in this word, which simply means "branch" or "twig". Nor must the branch be separated from the tree.

9 This word *tekhnè* is somehow somewhat like *kírè*, for which see note 3, is actually derived from *kýrè* and means "safe, secure, firm, sound". The two nouns are unrelated, nor does *kírè* have anything to do with *kírè*, as Kepler implies (although the latter is related to *kírè*). See H. Roesch, *Griechisches Etymologisches Wörterbuch* (Heidelberg, 1909), vol. I pp. 663-667.

10 Kepler believed light to be a propagated *specus* (ray, image) and not a moving *compositio* (substance). Had he held the latter opinion, he, like Newton in his Query 20 of the *Opuscula*, would probably have argued that reflection does not occur at a single point.

11 *Reflexionum*. CL. *Iuris VII 23*, where the participle form *repertum* is found.

12 *Infracta*, an action most apparently coined by Kepler from *inficinger* or "break, affect, or disjoin*.

13 The following consequence is a resuming of the theory sketched out in Kepler's letters to his teacher Michael Maestlin in 1595-letters 21, 22, and 23 in *Kepler* vol. 1 (pp. 286-301) and first published in chapter 2 of the *Mysterium Cosmographicum* (Franken,
First it was fitting that the nature of all things imitate God the founder, as the extent possible in accord with the foundation of each thing's own essence. For when the more wise founder strove to make everything as good, as well ordered and as excellent as possible, he found nothing better and more well advized, nothing more excellent, than himself. For that reason, when he took the corporeal world under consideration, he settled upon a form for it as like as possible to himself. Hence arose the entire category of quantities, and within it, the abstractions between the curved and the straight, and the most excellent figure of all the spherical surface. For in forming it, the most wise founder played out the image of his revered trinity. Hence the point of the center is in a way the origin of the spherical solid, the surface the image of the utmost point, and the road to discovering it. The surface is understood as coming to be through an infinite onward movement of the point out of its own self, until it arrives at a certain equality of all outward movements. The point communicates itself into this extension, is such a way that the point and the surface, in a commuted proportion of density with extension, are equal. Hence, between the point and the surface there is everywhere at unevenly absolute equality, a most compact union, a most beautiful compositing, connection, relation, proportion, and communisaurance. And since there are clearly three—the center, the surface, and the interval—they are nonetheless one, inseparable as none of them, even in thought, can be absent without destroying the whole.

This, then, is the aesthetic, this is the most fitting, image of the corporeal world, which anything that aspires to the highest perfection among corporeal created things takes on, either simply or in some respect. The bodies themselves were confined separately within the limits of their surfaces and could not by themselves have multiplied themselves into an orb. For this reason, they were endowed with various powers which, though they derive their tests in the bodies, nevertheless, being somewhat free than the bodies themselves and lacking corporeal matter through they do consist of their own kind of matter which is subject to geometrical dimensions, may proceed form and might try to achieve an orb, as appears chiefly in the magnet, but appears plainly in many other instances. Who wonder, then, if that principle of all adornment in the world, which the divine Moses introduced immediately on the best day uncertainly created matter, in a sort of instrument of the Creator, for giving form and growth to everything—i.e., I say, this principle, the most excellent thing in the whole corporeal world, the matrix of the animate faculties, and the chain linking the corporeal and spiritual world,

[190c] The present account differs not only in being more succinct, but also in the dynamic notion of the point unfolding itself into a sphere.

36 Cf Plato's Timaeus Miv. ad.
37 Ibid. In the Tentium Intermedium (Frankfurt, 1870), Section 126, Kepler writes, "Now as God the creator has played [ergo] all he has also made nature; his image, so to-day, and the frame is just the same as the one he had played [ergo] for her." (KGW 4p. 246.)
38 As the density increases the extensity decreases, so that the quantity commensurable of the two remains constant.
has passed over into the same laws by which the world was to be furnished. The sun is accordingly a particular body, in it is within faculty of communicating itself to all things, which we call light; to which, on this account at least, is due the middle place in the whole world, and the center, so that it might perpetually pour itself forth equably into the whole orb. All other thing: that have a share in light imitate the sun. From this consideration there arise, in a way, certain propositions, which are among the principles in Euclid, Wicles, and others.

**Proposition 1**

To light there belongs an outflowing or projection from its origin towards a distant place. For it was said that light had to be communicated to all bodies. This communication had to come to pass through a conjunction of dimensions, for we said that light falls under geometrical laws, and that it is considered in place as a geometrical body. Therefore, it will be communicated either by the approach of its source to the object, which is absurd, and is no communication; therefore, it remains that it be communicated through a local egress, and an outflowing from its body.

**Proposition 2**

Any point whatever flows out in lines infinite in number. That is, in order to illuminate all of the surrounding orb, which is what we said must happen. But the spherical has infinite lines.

**Proposition 3**

Light, in itself, is fitted for moving forward into infinity. For since it partakes of size and density, by the above, it will be able to vanish into nothing with no size, because size, and thus density also, can vanish through division to infinity. So much concerning the essence. But also, the force projecting it is infinite, because, by the above, light has no matter, weight, or resistance. Therefore the ratio of power to weight is infinite.

**Proposition 4**

The lines of these projections are straight, and are called "rays". For we have said that light strives to attain the configuration of the spherical. However, its true geometrical genesis lies in the equality of the intervening spaces, through which the middle point is spread out into a surface. But these are straight lines. If light were to make use of curves, there would be no equality in the spreading, and therefore nothing similar to the spherical.

The same thing is also proved, or rather declared, in this manner. The ends of motion are various. For nature strives after either the unity of the parts, or their separation, and both take place most easily by a straight motion. And things that are closer together are understood as being more united, and straight lines are the shortest of all lines between the same points. Therefore, the motion that unites a thing, such as the motion of heavy things towards the earth, or of iron to a lodestone, must necessarily occur along a straight line; otherwise, not all parts of the motion would tend to the same end, but, somewhere in mid path, that which
was to unite with the other would turn aside from this striving for union. The same is to be understood of the contrary motion of separation, which is called “violent” in the realm of nature. For to it also there belongs a motion this is the contrary of the motion of uniting; and therefore it is straight, since to the straight only the straight is contrary.

To light belongs, not uniting, but something similar to separating, and a certain most violent projection or flowing out. Therefore it is also a straight motion. Or, if you prefer, call it the union of that light in the solid body with the object to be illuminated. For the same thing will follow.

Nor does a curved line follow from the nature of light. For by Prop. 3 it, in itself, is fitted for being called forth to infinity, whole curved lines, insomuch as they are curved, return to themselves, and are bounded.

Proposition 5

The motion of light is not in time, but in a moment. For as is demonstrated by Aristotle in the books on motion, there is a certain proportionality of time to that ratio that exists between the moving power and the weight or movable bulk, or to the ratio of the weight to the medium. But this moving force has an infinite ratio to the light as it is moved, because light has no matter, and therefore no weight. So the medium does not resist light, because light lacks matter by which resistance could occur. Therefore, the swiftness of light is infinite.

Proposition 6

With departure from the center, light receives some rarefaction along the breadth. For by 2 and 4, light goes forth in infinite straight lines. But these are closer together at the center, because there are the same number in a narrow place as there are in a greater space. But this is the definition of rarity and density. Therefore, it is rarified along the breadth.

Proposition 7

With departure from the center, a ray of light receives no rarefaction along the length; thus, it is not the case that the longer the ray, the more rare or diffuse, at least not because of that length itself. For in the geometrical genesis of the spherical, nothing of the sort can be conceived. In contrast, light strives for that genesis by its communication of itself. Besides, the ratio of the projecting power to that which flows out is infinite since it lacks matter, as was said above; and, by Prop. 5, the motion occurs in a moment; and, by Prop. 3, it goes to infinity. Therefore, the projecting strength is equal out to infinity, namely, that very strength which is also at the origin. So also, the strength of a ray is equal along the length.

19 Aristotle, Physics VI 8, 213 a 22: “For we see that the same weight and body is carried faster through two causes, either by a difference in the through which is through water rather than earth or through air rather than water, or by a difference in what is carried, if the other things are equal, because of an excess of pleasance or lightness.” (Translation by Joe Sachs.) Note Kepler’s change of Aristotle’s speed into time: Kepler’s ratio should, of course, be inverted.
Proposition 8

A ray of light is no part of the light itself flowing forth. For, by Prop. 4, a ray is nothing but the motion itself of light. Exactly as in physical motion, where its motion is a straight line, while the physical movable thing is a body; likewise, in light, the motion itself is a straight line, while the movable thing is a kind of surface. And just as in the former instance the straight line of the motion does not belong to the body, so in the latter the straight line of the motion does not belong to the surface.

Proposition 9

The ratio that holds between spherical surfaces, a larger to a smaller, in which the source of light is at a center, is the same as the ratio of strength or density of the rays of light in the smaller to that in the more spacious spherical surface; that is, inversely. For, by 6 and 7, there is the same amount of light in a smaller spherical surface as there is in a more spread out one, and therefore it is that much more compact and denser in the former than in the latter; if, however, the density of a linear ray were different depending upon position in relation to the center (which is denied in Prop. 7), the situation would be different.

Proposition 10

Light is not impeded by the solids of bodies as such, so as to prevent its passing through them. For whatever is impeded is impeded or repelled by something of the same kind, as a body by a body. Solids, as such, have three dimensions. To light, by 6 and 7, there belong only two dimensions. Therefore, light, or its rays, are unaffected by solids as such, nor do they interact as regards solidity.

Proposition 11

The pellicial is one body whose geometrical constitution, or the positions occupied by the internal parts, is established by means of a certain form. For the geometrical definition of the humus is found in Aristotle; that is called "hydrous" which is not bounded by itself. Therefore, all the least parts are mutually bounded by themselves, and the whole is entirely united by the internal parts, not actually divided by surfaces. But now, by Prop. 10, the solidity, as such, does not impede light in passing through. Therefore, whatever is one, is pellicial. And if

20 This is a correction of the linear inverse relation stated by Kepler in Chapter 20 of the Mysterium cosmographicum. See Kepler, p. 71.

21 In his appendix to this chapter (p. 446), Kepler says, “For something is transparent when it is seen through, and pellicial when shone through by some light.”


23 Here Kepler refers to the following enmote. To p. 18. The logical connection is in the origin of the word “pellicial.” However, things that are black in the highest degree are excluded, for, although they are one, they are nonetheless not pellicial, because of their property of the highest blackness, as
there are things that are partly liquefied, and partly not, such mixed things do not constitute one pellicul body. Further, this definition is so much extended to all natural bodies, an extension which Aristotle clearly approved in the book On Honors, 22 nevertheless, had as well as being things are pellicul. Nor is it an
impediment for them to be colored, for pellicul bodies should be so, as is shown below.

Proposition 12

Light is acted on by the surfaces of whatever bodies it encounters. For those which fall under the same kind are fitted for acting on each other mutually. Light, by Prop. 7, is of the same kind as bodies because of the surfaces by which they are bounded. Therefore, it is suited to being acted on. But it is also actually acted on. For to be bounded is to be acted on, and a line is bounded by its point, and hence is acted on by it. The motion is a straight line, whose boundary is some extent a point on the encountered surface. And the boundaries of the ultimate rays of light are the ultimate points, that is, the surface, which is, as it were, composed of them.

Proposition 13

The surfaces of dense bodies, or bodies of which many parts fill a small solidity, are also in a dense dense; namely, in that respect in which light and bodies mutually act on each other. For though surfaces, as quantities, are assigned no thickness, which belongs to bodies, nevertheless, inasmuch as they are surface of dense bodies on account of this material geometrical characteris-
tic, they too are considered dense, since they are material surfaces. More clearly, density is an affect of matter, which assumes three dimensions of these, two be-
long to the surface. Therefore, they participate in the density of bodies according to their measure. But above, a sort of density of the surface was ascended to light in the same respect.

Proposition 14

Light passes through the surfaces of dense bodies with greater hindrance, to the extent that they are dense. For since, by Prop. 1, motion is a property of light, the properties of rectilinear motion also are properties of light. Therefore, being

follows in Prop. 17. And you should take the word, "colored", with the same limitation, as in line 20 following. Here, 'the, the supremely white color is excluded, not insomuch as it is color, but through a property, namely as it can only be represented by hue of bodies which are no pellicul'. When I paraphrase this opinion to Aristotle's own line 28 and on p. 13 line 42, it is because the passage cited, after he had spoken of water, air, and other bodies thus described, but is, as pellicul, he added, "it is in them, and participates in the other bodies, to a greater or lesser extent." (On Sense and Sensibles, 3, 439 a 22.) He seems to have been saying that aside from the things that are usually called "pellicul", this range is also at the rest of the bodies to greater or lesser extent. Should it later purging instruct you otherwise, strip this opinion of the authority of the Philosopher, and send it back to me taken.

22 Aristotle, On Sense and Sensibles, 3, 439 a 22. See the preceding note.
hindered by a denser medium is also a property of light. However, by Prop. 10, this is not insoluble as the medium is a solid, and therefore, it is hindered insolubly as it is bounded by a dense surface. More clearly, the motion of light occurs naturally with spreading-out, by Prop. 6, since it always goes from a single source to all regions. Therefore, just as the surface, because of its infinite points, resists motion, which is in lines, so a dense surface resists a spreading motion, since density and spreading come under the same gens.

**Proposition 15**

Color is light in potentiality, light insubstantial in a pelletial material. If it now be considered apart from vision. Different degrees in the arrangement of matter, by reason of variety and density, or of pelletiality and darkness, and likewise, different degrees of the spark of light, which is conducted into matter, being about the distinctions of colors. For since the colors seen in the rainbow are of the same kind, whence the colors in things are as well, the origin of the two will be the same. But the former originate from just these causes that have now been mentioned. For when the eye is removed from its position, the color changes. And, indeed, in the common boundary of light and shadow, all the colors seen in the rainbow rebound, so that it is certain that they exist from the redemption of light and from the scattering of a watery material above it. Therefore, colors in things, too, will arise from the same source. And there will be this much difference, that in the rainbow the light is extrinsic, while in colors it is inherent, in the same way that in parts of living animals there are certain lights that are at work. And to the degree that there are differences in the potential heat in gingers from the actual heat in fire, so it appears that light in colored matter differs from light in the sun. For something exists potentially, which does not communicate itself, but is contained within the bounds of its subject, as light, which lies hidden in colors, as long as they are not illuminated by the sun. However, you do not know whether colors too might not scatter their spark of light from the depth of night. But this subject has variously exercised the wits of the most shrewd philosophers, and is

---

21. Here Kepler refers to the following note:

Top. 11, to Prop. 15: Since this definition belongs to colors, according to greater and lesser, it does not hold at all in the perfectly black. For this is the basis of all colors, and is related to colors as a point is related to a line, a period to quinquages, although it is not a quantity. So also the perfect black color is lacking all potential light, and consists in pure materialized darkness. And radiating into a dark chamber, it does not paint the wall black, but gray, nor would it be noticed unless in the image of other things, color about it. It therefore comes pretty much only under the account of something lacking almost all radiation; it is noticed in a picture of the wall. It remains for you to wonder, however, that magics has so great a power that through it light produces its essence, the color black—that is nothing but rays of darkness, painting themselves to some extent on the surface present.

22. Lac. This word appears to have been coined by Kepler. It is the Latin word with a diminutive ending, and evidently means a small bit of light. Since the English word "luminous" is already taken in English for a bright speck on the sun’s surface, a spark of light has been chosen instead.
of such obscurity that it cannot be cleared up in the present account, particularly since it is not particularly relevant. Should you object that darkness is a privation, and can therefore not become something positive, an active principle (namely, one that radiates and colors the walls). I would likewise raise cold in objection, which is a mere privation, and nonetheless also becomes an active quality in matter.

Proposition 16

Light passing through colored bodies is affected everywhere, both at the surface and in the solid, to the extent that it is colored. For those things that fall beneath the same genus are suited to act upon each other mutually. But light and color fall under the same genus, by Prop. 15. However, by the same, color inheres in the matter of the pellicoid body. And matter has three dimensions, so therefore, for light also, the action is upon that color which is in the depth of the medium.

Proposition 17

The opaque is something that is broken up into many surfaces, and something that possesses great density, and something that possesses much color, whether in quantity or in quality. For the opaque is that which does not transmit the rays of light. But, by Prop. 12, many surfaces greatly impede the rays, just as, by Prop. 14, the surface of something dense greatly impedes them. So also, by Prop. 16, the surface of a colored body greatly impedes them, whether the depth of the medium adds quantity to the color, or the color itself recedes greatly from brightness, and participates greatly in darkness.

Further, that nothing is absolutely opaque, Aristotle also allows in the book on perceptibles.27

Corollary on Method

Therefore, since light has two aspects—that of the essence, by virtue of which it is light, and that of the quantities which it acquires—it also possesses two operations,26 the prior that of local motion, and the posterior in the order of nature (which is prior in its goal or form) which is illumination (with which heating is conjunct). Of these, local motion belongs to light because of of quantities, while illumination belongs to it because of its essence, by virtue of which it is light. In conformity with these two operations, it also has two objects—quantities and color—with which matter is conjunct. And because local motion also has two kinds of objects—the medium through which it passes and the thing towards which it is carried—there also exist two aspects of this operation in light; in the

27 Aristotle, On Sense and Sensibles 3, 139a 22.
26 Various. This is a Latinization of the Greek ἐνέργεια, used by Aristotle and Galen to mean something like “operation” or “function”. Cf. Galen, Opera, ed. C. G. Kühn, Leporello 1832–33, VI.21. In Aristotle’s philosophy, it is also the technical term usually (but misleadingly) translated “actuality”, as opposed to ἐνέργεια, “potentiality”. Joe Sachs, who discusses the term in the glossary to his translation of the Metaphysics (Aristotle’s Metaphysics, Green Lion Press 1999, p. 61), translates it “being at work”. Cf. Aristotle’s use of it in Metaphysics 1044a 26.
case of the medium. light penetrates the medium and is refracted by the medium if it is denser; in the case of the thing, light strikes the surface which it encoun-
ters, and is made to rebound by it. These two aspects are distinguished from each other in different ways. For by motion, light is both spread through the medium, and dashed against a surface, and in turn is either picked up by the surface of a pellicac medium, or is made to rebound by the surface of the boundary. Finally, as regards illumination, which is the primary task of light; colors are diluted by light, and light is tinted and dyed by colors. All of these will be treated in order in what follows.

The reader should note here the origin of the fourth species of light, which the other optical theorists treat ineptly. 29

Proposition 18

Light that has fallen upon a surface is made to rebound in the direction opposite to that whence it approached. For motion is an attribute of light, by 1, and therefore so is a particular species of motion, namely impulse [plexus]. For what in physical motion is hardness of the colliding bodies, which consists of the permanence of the surfaces, in light is the bare surface, or the bounding and shaping of bodies. For by the very fact that physical bodies are even themselves bounded, they are understood to be hard. Soft and moist things, on the other hand, are defined by way of their not having a proper boundary. But the cause of rebounding’s occurring in both physical motion and light is in the violence of the motion. So since the motive force cannot all be destroyed by the collision, the motion will accordingly continue beyond the end of its line, which is the surface. But it cannot do this straight forward, for in the former case, body is in the way of body; and in the latter, surface is in the way of surface: in the former the motion would go into the solid, in the latter partly so, as we shall hear. What is left, therefore, is that it move in the opposite direction. These things may perhaps be put more concisely thus. Impulse is an action, and is between contraries, but with action there is a corresponding effect [passio]; therefore, this is also the case in impulse. But the contrary to the impact by which light strikes a surface in one direction is a rebound in the other direction. For both the power that drives a movable towards something, and the power that drives it back from something in the way, are the same, because the collision is considered to occur at a point.

29 Earlier optical theories distinguished three kinds of light: direct, reflected, and re-
fracted. See, for example, Alhacen IV. 1, Deeunae 1 p. 102. The three branches of
optics corresponding to these species are Optics proper, catoptrics, and dioptrics, re-
spectively. To these Kepler adds communicated light; see Prop. 22 below.

30 Aristotle distinguished between natural motion and forced [impulsa] motion (see, for
eample, Physics IV. 8, 215 a 1). The standard Latin translation of fluens was viscosus,
which adds a sense (quite foreign to Aristotle’s thought) that the force involved might
be unnecessarily great or destructive. Although Kepler uses the standard term, his
understanding of it seems closer to Aristotle’s meaning than to the sense of the Latin
word.
Proposition 19

Rebounding occurs at equal angles, and the rebound of that which strikes obliquely is to the other side. This is V. 10 in Witsel.\footnote{Thesaurus II pp. 195-6.} For an equal power also pre-supposes an equal motion. But the power at the point of collision is one and the same; therefore, the motion is also equal. But that which is at equal angles is equal. Therefore, the rebound of moveables will be at equal angles. I say also that it will be to the other side from that whence the movable was brought into collision, if this applies. For if it were always to rebound in the same line, it would always have to have approached directly, by 18 preceding. But most often it approaches obliquely. Accordingly, then, too the direction from which it approaches is set obliquely in the way of the surface, not directly: therefore, the rebound is also in the opposite direction obliquely, not directly. Or, more clearly, when something moves obliquely towards a surface, that motion is compounded of motion perpendicular and motion parallel to the surface. But the surface is only turned to face that part which is perpendicular to it, not that part which is parallel to it. Accordingly, it does not impede that part which is parallel to it, but allows the movable, in rebounding, to head in the other direction, just as it had approached. Let \(CD\) be the surface, \(BD\) the motion of the light. Let \(BD\) be extended to \(E\), intersecting \(CDF\) at \(D\), and let \(CDE\) be equal to \(CD\). Therefore, since the surface \(CD\) is supposed not to impede the motion \(B\) in the direction from \(B\) towards \(AE\), it will not change the angle \(CDE\). For if \(B\) were not to rebound at \(D\), it would make the angle \(CDE\) equal to \(BDF\), because the line \(BDE\) would be straight. But, while not affecting the angle \(CDE\), \(B\) nonetheless rebounds to the side of \(BA\). Therefore, the amount of motion that would have been allowed to \(B\) in the direction from \(B\) towards \(AE\), had it not rebounded, is the same as the amount allowed to \(E\) even though it does rebound: not, however, towards \(E\), because it rebounds, but therefore towards \(A\), so that \(CDE\) and \(CDA\) are equal. But \(BDF\) and \(CDE\) would also have come out equal without the rebound. Therefore, the angles of incidence \(BDF\) and of reflection \(ADC\) are equal.

Proposition 20

Light that has approached the surface of a denser medium obliquely, is refracted towards the perpendicular to the surface. For by Prop 14 and 16, the effects upon light here are opposite to its motion, and similar to the medium. But motion spreads light, and the medium is posted as denser. Therefore, the medium impedes light so as to prevent its spreading. But the evidence (argumentum) of spreading is oblique incidence, because when light...
falls obliquely upon a surface, it also falls directly upon the same surface extended, and therefore between the oblique and direct rays an angle is interposed, and by the angle the rays are spread. Let the oblique ray be \( AB \) falling upon the surface at the point \( B \), let a line touching the surface be drawn (let it be \( BC \)), and from \( A \) let the perpendicular \( AC \) be dropped. Thus the light from \( A \) is spread to one side through the angle \( BAC \), and the amount of light that was within the angle \( BAC \) in parts closer to \( A \) is the same amount as is also on the line \( BC \). Let \( AB \) be extended to \( D \), \( AC \) to \( E \), so that \( DE \) is parallel to \( BC \). If a denser surface does not stand in the way at \( BC \), the light will be spread further, and there will now again be the same amount of light at \( DE \) as there was at \( BC \). But if \( BC \) is the surface of a denser medium, it will check this spreading out. By Prop. 14, and this will occur in greater or less degree. But the light \( BC \) if it moves further without spreading out, will have to occupy as much space on the surface \( DE \) as on \( BC \), and therefore will have to cut \( A \) from \( DE \) some space equal to \( BC \); let this be \( FE \).

Now on \( ACE \) is the limit of all the spreading out of light, because the ray \( AC \) is at a right angle to \( CE \), and consequently \( E \) will be the common limit of the lines \( EF \) and \( ED \). So \( BF \) will be parallel to \( CE \), and therefore will also itself be perpendicular to \( BC \). So light, coming as far as \( ED \) without any spreading out, would occupy the space \( EF \), and the same light descending as far as the same place without any disturbance, would occupy the space \( ED \), spreading and thinning itself out in the same ratio. Therefore, when a denser medium \( BC \) comes in between, hindering the spreading out, it makes the light occupy a space that is intermediate between \( EF \) and \( ED \); let this be \( EG \). Therefore, the ray \( AB \) will be refracted at \( R \) and below the surface of the denser medium will become \( BG \), approaching the perpendicular \( BF \), which was to be demonstrated. The question arises, however, by what faculty could it happen that the influence of a pellicul surface be imposed upon light? I answer: motion belongs to light, by Prop. 1, and therefore also the species of motion, and the rest of the accidents, namely impact upon a denser surface, and the overcoming of it, and a certain amount of resistance by the medium overcome. Moreover, that this also happens necessarily in physical moveables, wherever a globe is spun into the water, provided that it goes beneath the water, is shown thus. For it may be permisible here for me to use the words of the optical writers in a sense contrary to their own opinion, and to carry them over into a better one.

Let \( BC \) be the water, \( AB \) the motion of a small sphere, let \( CR \) be extended to \( B \) and \( FB \) to \( I \). Now since the motion \( AB \) of the small sphere is, from a certain perspective, composed of \( IB \) and \( BH \), it also will happen that the depth \( BF \) resists it as well as the breadthwise thickness \( BH \). The prior impediment makes its descent slower, and blunts it, as long as it descends; the latter, on the other hand, pushes it back from its line; so that the motion that was going to move along \( BD \), is pushed back from \( BH \) and becomes \( BG \). This happens of necessity.
in projectiles, which are impeded by a denser body. In light, that portion of the motion which is in the direction of $B$ is that motion of thinning out, while that which is in the direction of $BF$ or $BE$ would not vary the light's own matter (so to speak), but only transports it. To this transporting, by Prop. 10, the body or thickness $BE$ does not present any hindrance; but only the surface $BC$ does so. And this, indeed, not otherwise than to the extent that it participates in density, by Prop. 14. It does not, however, participate in density in depth, for then it would be a body, not a surface. But it is dense in length and breadth, as in the parts $BH$ (for with respect to this line alone it is most greatly oblique to $AB$). And the light is carried or thinned out not towards the part $BC$ but towards $BH$. Therefore, the surface from the part $BH$ resists this motion, and hence there in a way exists a kind of bending back of $AB$ into $BG$, entirely similar to those which occur in projected natural bodies.

I shall make an attempt at the same comparison of physical motion with the motion of light using other, denser proofs. For what in physical motion is a denser medium that is nonetheless yielding, or a weight that is in itself at rest, but is in motion through the impulse of another body, is in the present consideration a denser surface, but one which nonetheless may transmit light and may to some extent yield to the transmission.

Now as in physical motion a weapon sometimes collides with the object at which we were aiming at it, and sticking to each other, they proceed along the same path with a single motion, so the same thing happens here with light and the denser surface which light penetrates, but without matter or the dimension of solidity.

I see, however, that with regard to physical motion, even up to the present day it has not yet been adequately explained why here too there should occur a refraction, or rather since this word is more appropriate to physical motion, a deflection, from the direct line of motion towards the line that is perpendicular to the surface that is in the way, whenever something that makes an impulse strikes upon it oblitraby. And because this whole matter depends upon the principle [ratio] of the balance, it should be derived from its source. For I don't think there is going to be any other place given me where I can more conveniently go through with this demonstration.²²

In the Mechanics, Aristotle asks what the cause is, why the beam of a balance, which was previously inclined, returns to equilibrium when the support [bracket] is above the balance and the pans are empty; and why, on the other hand, when the support is below, to the extent that the beam is inclined by the weights, it stays there, not returning when the weights are removed.²³ Jordanus

²² For general information on the ancient and medieval traditions regarding the balance, on which Kepler draws in what follows, see David Lindberg, editor, Science in the Middle Ages (Chicago: University of Chicago Press, 1978), Chapter 6, "The Science of Weights," by Joseph E. Brown, pp. 179–205, as well as the sources cited below.

²³ Aristotle, Mechanics, Chapter 2. 850 a 2–29. Present-day scholars agree that this work is not by Aristotle, though it is likely to have been a product of his school. The Pseudo-Aristotle solves the problem by noting the proportions of the beam lying on
did not repeat Aristotle's demonstration, 54 nor did Cardano give his approval in the book De subtilitate, 55 for he would not have substituted another. This is evidently because Aristotle's demonstration is obscure. But Cardano, who was not much clearer, so displeased Guidohaldo 56 that he inveighed against him in an entire book which he wrote on the balance. The reason for the controversy seems to me to be this, that one element of the Aristotelian proposition seems absurd and contrary to sense perception. Jordanus accordingly ignored it; Cardano also rather candidly set out even to demonstrate what was falsely admitted. Guidohaldo, noting this, and undermining it with sense perceptions, appeared (which was remarkable) to deny even that part that is beyond controversy, i.e., laying the blame for Aristotle and others having erred in the attempt upon the slipperiness of the material.

Let the upright support be AB, the beam CD, and let CA. AD weigh equally. I say that anyone who denies that CD is going to return to horizontal equilibrium, 57 or at right angles with BA, declares war not just on antiquity, but on the centre of things, but on the interest of the human race.

On the other hand, let the support be AI, inverted and underneath: let the beam be CD, having its center at A, the end of the support, from the depiction (imagina tio) of Cardano and Guidohaldo, and let CA, AD, weigh equally. I say again that the beam is going to return to equilibrium. If Aristotle affirmed of this arrangement that the beam CD, once inclined, is going to remain so, it will by all means have to be said that this is a lapse in experience, although Guidohaldo


54 Jordanus Nemorarius, De ponderibus propositionibus, XIII, et eandem demonstratorem (Inognito 1533 and other editions). Prop. 2, in Ernest A. Moody and Marshall Clagett, The Medieval Science of Weights (Madison: The University of Wisconsin Press, 1960), pp. 130–131 (Latin text and English translation); English translation of selections in Edward Grant, A Source Book in Medieval Science (Cambridge: Harvard University Press, 1974), pp. 214–15. Jordanus was actually trying to prove a somewhat different proposition: he states, "When a balance has an equal position, with equal weights hung upon it, it does not depart from that position, and if it be separated from equilibrium above the earth, it returns to the position of equality." His proof depends on seven suppositions (Moody and Clagett, pp. 128–9; Grant, pp. 212–13) which allow one to compare downward tendencies of bodies.

55 Girolamo Cardano, De subtilitate rerum (Nuremburg 1550 etc.; ed. Basel 1554, Book 1 p. 24. Cardano notes that Jordanus neither proved nor understood what the pseudo-Aristotle was trying to demonstrate. However, Cardano also seems to argue that there is some power in the angle itself between the support and the vertical that gives power to one side or the other; and incorrectly avers the argument to the author of the Mechanica.

56 Guidohaldo del Monte (c. 1451–1507) was from Monte Barocco (Membregusco in Umbria). I have not found the "Moni Ferrato," mentioned by Kepler in the margin. Guidohaldo wrote: Mechanicae sive (Pescia 1577 etc.). The argument cited by Kepler is in Book 1 Proposition 4.

57 Literally, "equilibrium of the horizon" (Horizons aquilibriums).
took it upon himself to demonstrate the falsity of this in both arrangements. But the experts should see whether the Philosopher’s words might not admit of some interpretation; for I suspect that Aristotle’s contemporaries had weight some easy method in the balance, using a curved beam with the support clapped in the middle of the arc. In this way, whether the balance hang from the support or rest upon it, it can be considered now above and now below, according as the arc curves upward or downward. And then if in this way the center of the beam be above the support, I grant that what Aristotle seeks happens. For the cause is nearly the same as that which makes a cone standing upon its angle not remain there, but turn over, though it meanwhile remains hanging from its angle.

Now the error originated for Guidibaldo from a false principle; he was defining equally weighing things [aequiponderantia], which would stay in place however they might be located, by means of equal lines from the center. But let us come nearer to the subject. Cardano, following Aristotle, says that $CA$ is heavier since it is higher (namely, by the smaller angle $CAB$), while $AD$ is lighter since it is lower (namely, by the greater angle $BAD$), and in demonstrating this he stumbles. The whole matter comes back to the assigning of angles to gravity, a support derived from mechanics. For $CAB$ is acute, and its sides are pulled apart more easily by the weight at $C$ than the sides of the obtuse angle $BAD$ are by the weight at $D$; therefore, the former weight wins out. If this were indeed the true cause, the effect would follow the proportion of the cause, and the ratio of the weights on the inclined balance would be the same as that of the angles. But this is false. And so Cardano shows us another cause, from a distance, but does not work out the demonstration. Although $C$ is heavy, he says, and, being so fitted by nature, is able to be carried towards the center of the cone.

This is not quite what Cardano argues, and not at all what Aristotle said. Cardano writes, “Aristotle says that this happens when the support is above the beam because the angle $FAC$ of the cone is greater than the angle $FAD$... but a greater angle makes the weight greater.” (De substantia, Basel 1554, Book I p. 34: the letters have been changed to match Kepler’s diagram.) Aristotle, however, wrote, “Is it because when the cord is attached above, there is more of the beam on one side of the perpendicular than on the other, the cord being the perpendicular?” (I have used E. S. Forster’s translation, from The Works of Aristotle, Vol. 6 (Oxford: The Clarendon Press, 1915 etc.) The Greek is rather terse, and evidently both Cardano and Kepler misunderstood Aristotle’s sense.
earth very rarely through the extent of $AC$, nevertheless, it does not descend beyond equilibrium, because the equal weight $AD$ would be forced to ascend, contrary to its nature. This begs the question. Indeed, Cardano. I seek to know, not just this, but something further: if in all cases that which is heavier strives to approach closest to the center, why then, when there are unequal weights in the pans, does the heavier one not simply seek the lowest place, while the lighter is lifted to the very top? The circle, then, is this, namely, the same as in the unequal armed balance [statera].\(^{38}\) About center $A$, with radius $AC$, let the circle $AD$ be described, and in it the perpendicular $BAF$. It is evident that neither of the weights at $C$ and $D$ can either descend lower than $F$ or be raised higher than $B$.

And since both are of this nature, that they tend to the bottom, and they mutually compete with each other, they divide up the descent $BF$ between themselves in that ratio in which they themselves are. From $D$ and $C$, let the perpendiculars $DG$, $CH$ be drawn. Now from what has been said, $BH$, the descent of the weight $C$, will be to $BG$, the descent of the weight $D$ as the weight $C$ is to $D$. I say that this is the proportion of the unequal armed balance. For also, because $HAC$, $GAD$ are equal, and $CA$, $AD$ are equal, and $H$, $G$ are right. $AH$, $AG$ will also be equal, and therefore also the remains of the equals $HB$, $GF$. Therefore, as $G$ is to $D$, so is $GB$ to $GF$. From $F$, let a perpendicular be drawn to $CD$, and let this be $FK$. Therefore, since $CAH$, $FAK$ are equal, and $CA$, $AF$ are equal, and $H$, $K$ are right, $CH$, $FK$ will also be equal. Likewise also $AH$, $AK$. Consequently, the remains of the equals $AB$, $AD$, $AF$, that is, $HB$, $GF$, and $KD$, are also equal. And again, the remains $GB$, $KC$ of the equals $FB$, $BC$, are equal. Therefore, as $C$ is to $D$, so is $DK$ to $KC$. And if the beam $CD$, thus loaded, be suspended from the support at $K$, it will be the ratio of the unequal armed balance, and $C$, $D$ will weigh equally, as is demonstrated in mechanics.

\(^{38}\) It is not certain exactly what sort of balance is meant, though Kepler’s argument makes it clear that he has an unequal armed balance in mind. Historically, \(\ddagger\) may be the same as Thabit ibn Qurra’s \textit{Karamun}, or “Roman balance.” See Ernest A. Moody and Marshall Clagett, \textit{The Medieval Science of Weights} (Madison: The University of Wisconsin Press, 1966), pp. 77–117. The derivation of the word \textit{mesar} from the Greek \textit{metra}, which denotes a unit of a standard weight, and its use by Oecumen (De Oratione, 2,159), suggest that it is a fine scale and not a “steelyard” or coarse balance, as is suggested in the Oxford Latin Dictionary (p. 1844). Cf. H. G. Liddell and R. Scott, \textit{A Greek-English Lexicon} (Oxford: The Clarendon Press, 1843 et al., p. 1634.)
The proposition is therefore evident. Therefore, by subsumption, it is evident why the arms of the balance rotate to equilibrium. For since they weigh equally, it is fitting that they also make equal descents on the circle. This is a diminution and stands as a sort of prothorem.\textsuperscript{20} Now to physical motion, which is common to light.

Let $AB$ be a panel,\textsuperscript{21} $C$ the center, $ED$ perpendicular through it; as, if a globe or missile were carried from $E$ into the panel $AB$, it would drive it forward towards $D$; or as if $AB$ were oars, of equal length on both sides, and $ED$ were a river.\textsuperscript{42} For since $ECA$, $ECB$ are right, the arms $AC$, $CB$ are placed in equal balance, and meet the impact of the mobile body, with an equal power. Now let the oblique $FC$ strike at $C$ and let it be extended to $K$. And let the missile or the stream rush in from $F$ to $AB$. Since the angle $ACF$ is less than the angle $FCB$, the parts $AC$, $CB$ will not be impelled with equal force, but the one that resists more will feel the blow more. And the one that faces at an obtuse angle resists more than that facing at an acute angle.\textsuperscript{43} Therefore, the exterior part $CB$ will resist more. For also, if $KC$ should become a support, $CB$ has a stronger turning power\textsuperscript{44} towards $F$ than has $AC$. And there is the same ratio of resistance of equal powers here in violent\textsuperscript{19} motion, as there was of unequal weights above in motion that is in accord with nature: that is, the lines, not the angles, measure it. Therefore, there is

\textsuperscript{20} Prothoronum, in Latin. The word was used by Martianus Capella (5th century AD), *The Marriage of Philology and Muses*, II 138, most likely as a direct translation from the Greek. If so, it would have meant something like "preliminary consideration".

\textsuperscript{21} It is clear from the context, however, that Kepler mean it to denote a preliminary or subordinate proposition (the term is also used in a similar way in the *Astronomia nova*, ch. 59). I have chosen to call them 'prothorons', as I did in the *Astronomia nova*, rendering Kepler's Latin neologism with an English one.

\textsuperscript{42} I have chosen to call them 'prothorons', as I did in the *Astronomia nova*, rendering Kepler's Latin neologism with an English one.

\textsuperscript{43} Kepler may have been thinking here of Jordanus's *Theorem 7 of Part 4*, which states that "the shape of a heavy body changes the power of its weight." (Moody and Clagett, pp. 216-17.)

\textsuperscript{44} Here Kepler uses the Greek word ἐνέργος, defined in H. G. Liddell and R. Scott, *A Greek-English Lexicon* (Oxford, The Clarendon Press, 1843 etc.), p. 1575, as "firm of the scale, full of the scale-pan, weight." It is thus not simply 'weight' or 'force', but something analogous to our notions of torque or moment. In order to avoid the anachronistic use of such terms, 'turning power' is suggested as a translation.
a greater impression of violent motion on \(CB\). So when \(AB\) is moved inoppositely, \(B\) advances more than \(A\). I say that it will happen in such a manner that the center \(C\) is not carried on the line \(FK\), but turns aside towards the perpendicular \(CD\). And that the arc \(AH\) will at length be pushed forth to the shore, if a be artificially held back in this position \(AB\). And that the arrow \(CK\), because of the oblique transversal \(AB\), will go wide of the target to the left. For from \(A\) let a small part of the path of \(A\) be describable; and let it be made parallel to \(CK\), for it will not turn aside farther to the right, since the violent motion itself is not more oblique than \(FK\), and let it be \(AH\). And \(HB\) being connected, let a triangle be constructed above it, with one side \(HI\) being equal to \(AB\) and the other \(BI\) greater than \(AH\). And let \(H\) intersect \(FC\) at \(K\). Finally, let \(HI\) be drawn, equidistant from \(AB\), intersecting \(FC\) at \(I\). Now since \(AH\), \(FK\) are equidistant, they cut off from the equidistant lines \(AB\), \(HI\), the equal parts \(AC\), \(HM\), and \(AC\) is half \(AB\). Therefore, \(BM\) is also half \(AB\). But \(HK\) is greater than \(HM\). Therefore, \(HK\) is greater than half \(AB\), that is, than half \(HI\). Therefore, the center is between \(H\) and \(K\), and is not at \(K\). It has therefore turned aside to the parts \(D\), which is what was to be demonstrated. \(AH\) were so drawn if we were to come together with \(CT\) towards \(F\), so that \(A\) would thus also have turned aside from the path towards the left. \(BM\) will be made greater than \(AC\), and consequently \(HK\) will be made much greater than half \(AB\) or \(HI\). This happens at all inclinations, so that even in the extreme case, where the movable body moves in a line that is close to equidistant to the surface itself, the surface itself, lightly snapped, rebounds almost on its perpendicular, unless an enormous force of motion move it forward considerably. And so this behavior of violent physical motion also flows back to what is analogous in light.

**Proposition 21**

Both the reflected [repercutus] and refracted rays are straight after the place where they are affected. For the nature of light is to move in straight lines, as long as it is not at all affected by the interposition of surfaces. by Prop. 4. Nor are the rays any longer affected by the solidity of the medium after passing through the surface. by Prop. 10. But physical motions are curved into arcs; reflected motion, because its power is finite; bent motion, because the medium contributes something by its solidity as well. These do not apply to light.

**Proposition 22**

Light illuminating colors is reflected in all directions, and the illuminated colors radiate in an orb, as light itself; more strongly, however, on the perpendicular. For all things, even those that are colored, are to a certain extent transparent.

by Aristotle (cf. Phys. vi. viii. 4, 225a 3). The Greek word is \(βίον\). See the note on p. 26 above.
by Prop. 11, and light illuminates colors through the solidity, by Prop. 16. Therefore, it illuminates from all sides. But now, color is a correlative of light for mutual action. But light strikes upon color, and does so from all directions through the solidity. It is therefore reflected as if from surfaces in all directions. But at the same time it is also transmitted, by Prop. 16. Therefore, the reflected ray has the color of the medium or the object, so color radiates, and so on. Hence, those colored bodies that are also in the highest degree smoothest off, nevertheless radiate to all directions, which would not happen if all the light were reflected from the one surface in one direction, and nothing were to penetrate the genuine seat of color, or it would not be roused. For light is in the matter and internal arrangement of a body, not in its mere surface. One may say, if it is more acceptable to him, that the potential lights of colors are brought over and arranged to act, exactly as that heat which is in ginger is stimulated by the approach of moisture, and itself takes fire, and begins to communicate itself, which same thing all seeds do. On the fourth kind [species] of light, the optical writers say little. We call it "communicated light." For both reflected and refracted light is nonetheless the light of that thing from which it has approached, undersignifying that action. But this communicated light now in a certain way becomes the light of that surface which it has illuminated. Moreover, it is in the highest degree necessary for the astronomer to take it into account.

**Proposition 23**

Light descending through the substance of pellucid colored bodies is refracted at whatever point, and the colors of the illuminated body radiate into

---

22 Cf. Propositions 15 and 16, above.

23 Actually, the corollary to 17 is where this word times is used.

24 Here Kepler refers to the following endnote:

To Prop. 22 pp. 21-2. How this fourth kind of light arises—I mean communicated light, where the light of the sun falls from a single direction, and is so taken over by the colored surfaces, whether smooth or rough, that it is not only spread out in an orb while it ought to have been spread only on the side turned away from the sun, if it had remained simple and reflected, but also takes on the color of its surface—this, I have said, originates in two ways, by which I do not know how I can fail to satisfy myself fully. One makes use of reflection, refraction, pellucidity going to some extent through all bodies, penetration of the solar light, or of the day, to some depth, all of which are not yet seen to be fully sufficient. The other does indeed state something, but does not point out the way. Thus an occult account has been adopted, of how light and darkness may be bound by the chains of matter. So it seems that this too still has to be formed out: how they may again be unbound forth from matter by the extraneous light, and ignited, like one torch from another, and whether this can be done through principles either previously assumed or previously demonstrated, or through others in addition. Let the problem be proposed for optical theorists and philosophers. They shall also consider whether the radiances that are constituted in this way can be held to be among the number of those things possessing the highest degree of density. For if things of the highest density should have individual luminous points, each of these would in an orbit in its own right.
the region from the light in an orb, but more strongly in the perpendicular. For some light illuminates colors through the solidity of the substance, by Prop. 16, it therefore illuminates from all sides. And at the same time it passes through, because it is pellucid. But now color is a correlate of light for mutual action. And so, in passing through, in spreading itself out, it gathers up colors from the substance of the colors, which is what "being refracted" is. For here too, as before, color assumes the nature of the surface. But at the same time it is also tinted, by Prop. 16. Thus the refracted ray has the color of the medium. So color radiates past the medium. And because that which is refracted less is stronger, while that is less refracted which is nearer to the perpendicular, therefore those rays which are nearer to the perpendicular are stronger. So in this way light comes to be the property of the colored medium, and is communi-
cated to it.

Corollary

The cause was to have been stated why, when the sun illuminates the air everywhere equally, nevertheless anyone perceives the greatest brightness of that air through which the sun radiates most nearly, so that one and the same region of air is at its brightest to one person, and to another, to whom the sun makes its way differently, it is less bright, for air has its own albedo29, 30 or color. In the cause, has not been clearly enough stated, the reader should think up another, I believe that a basis for halos and twilights is established at the same time.

Proposition 24

Light reflected from the surface of a body; insofar as it is body, is not col-
ored.29 For by 15 color is in bodies through their matter, which has three dimen-
29 This Latin word did not acquire its technical astronomical sense until the nineteenth century; nonetheless, Kepler meaning seems consistent with it.
30 Here Kepler refers to the following enunciation
To Prop. 24-5, p. 23. That light is not colored in reflection a evident from a simple example, if you set out in the sun, in some other, a number of dishes of variously colored liquids (equally pure, however), in such a way that the reflection falls in a shadowy place, and upon a white surface. It thus becomes evident by these examples that the colors are not on the surface, but in the depth, and accordingly shine forth, and therefore inher in matter that is to some extent pellucid. In all cases, colors require body, no less than light (below, Ch. 6). But that reflecting bodies intermingle their own communicated light, spread out in an orb, with the reflected light, is evident in weaker radiations or illuminations. For example certain metallic mirrors represent the face as muddy, which is because they illuminate and tint the retina with the simple ray of their color not much more weakly than the ray of the face, which is doubled or reflected by the mirror. Hence, by Prop. 23, there arises a mixture of the two, with the new color, in the eye.

Accordingly, in Prop. 25, color in a surface is established no differently than is density in Prop. 13 above. And the sense is that oblique light, since it is a kind of surface, is affected by the density of the surface alone, because the density of the corporeal bulk (corpoream; cf. the footnote on p. 97) had nothing in common with it.
sions; a surface has only two. Therefore, there is no color in it. But that something of color is mixed in, this the body to which the surface belongs adds on its own—a small amount indeed which it spreads about, not just towards that place, but also around about.

**Proposition 25**

Light passing through a colored medium is more and more colored, and that which penetrates the medium more deeply leaves with more reddishness. This is evident from Prop. 40, because color, in standing in the way of light, is not only in the surface, but also in the body.

**Proposition 20**

The rays of light neither mutually color each other, nor mutually illuminate each other, nor mutually impede each other in any way. For the rays, by Prop. 4, are nothing other than the motion itself of light and color, nor does light exist in them, by Prop. 8, but now has passed through. This is just like one physical motion's not impeding another.

**Proposition 27**

Light in the same medium is partly reflected, partly refracted, and also partly adheres to the color of the medium, or is propelled back by the color, and thus is divided into many tenacious lights. For to the extent that it strikes upon a surface, it is reflected, by Prop. 18, but to the extent that the surface belongs to a pellicular body, it passes through, by Prop. 10, and it is refracted, by Prop. 20. Therefore, while it is illuminating two objects, one through reflection, the other through refraction, the two together are equal to the one through which it would have illuminated directly; it is accordingly attenuated, since this is the definition of density. The dual nature of the medium itself brings this about. It is assisted by the essence of light, participant in density, which can consequently be divided into more tenacious parts. In physical motion this is not so evident, because physical moveables are all inanimate and hard, while here there is nothing of the sort. We prove this by experiments.

**Proposition 28**

Different lights falling upon the same object are both accumulated and intermingled, among themselves as well as with the color of the object, each in proportion to its strength or density, whence there arises a new color, or rather.

Now, however, all color, through the whole depth of the body, is partly akin to light, because of its potential light, partly contrary, because of the intermingled darkness. Therefore, light is affected by color through the whole corporeal bulk of the pellicular body, not just in passing across the surface, when it is refracted. Here Kepler again refers to the enulose already referred to in Prop. 24 above. See the preceding footnote.
a generated light different from all the others. For because all are participants in quantity, just as they have hitherto been capable of attention, they are now also capable of accumulation. However, this does not occur in a pellicled medium, because the lights are not in it, as such, but have now passed through it. Instead, they are in the object, because it is in this that they are first established and fixed after passing out from their body. They are, moreover, intermingled, because, by Prop. 16, either of them is mutually affected by the color of the object. And there arises a new color, radiating, because, by Prop. 13, colors differ only in degrees of light and darkness, while accumulated lights vary this degree, and therefore vary the color itself.

**Proposition 29**

When the mutual ratio between lights flowing to the same object is excessive, the sense does not discern the weaker light. For to discern is to prepare or distinguish the function of the visual sense. But as the ratio is, so absurds the preparation. Therefore, if all ratios of colors or brightnesses could be discerned, since they are infinite in respect to the increment of magnitude, the faculty of discrimination would have to be infinite, and this the natural philosophers deny. For all sensory inequalities have certain prescribed powers.

**Proposition 30**

Colored lights on those surfaces which have colors that are more nearly akin to light, as on white surfaces, appear brighter than on black ones. For the task of light is illumination, in which (like all agents) it makes what it acts on more like itself, and light has its contrary color or darkness. Therefore, where colors more approach darkness, as black ones, illumination becomes more difficult. And by Props. 28 and 29, blackness wins out in color, which radiates from the illuminated surface. Thus it radiates less from a black than from a white surface, and so is also less seen.

**Proposition 31**

It being postulated that we are able to perceive one particular color out of many equally clearly on black surfaces as on white ones, which can be brought about by the accommodation of a stronger light to a black surface: note the differences of those colors that are radiating mutually side by side will be more rightly noted on the black surface than on the white. For because the white surface has great brightness, the radiating colors closer to the light become violently bright, by Prop. 28. And they will thus efface the colors closer to black, by Prop. 29. This is not so with black surfaces, because they are more a preservation of light. There follows hence a kind of corollary to Props. 30 and 31: that the rays that have flowed to black surfaces are perceived most distinctly, and so white ones most evidently, and if a surface be a mean between black and white, such as blue.

---

53 Here Kepler refers to the following endnote:

To p. 24, Proposition 31 with its corollary is properly carried to Ch. 5 and should be allotted to the cause of color on the retina.
white washed with red, and the like, they will stand about equally in rendering both the individual colors and their differences.

Proposition 32

*Heat is a property of light.* It could be proved that light is hot from the principles we have assumed. For if the life of things is dependent upon heat, and light has been ordained to nurture that life, it therefore ought to give heat. But every agent strives to make what it acts on similar to itself. Therefore light, striving to make matter hot, will itself be hot. That I will establish what I have said by experience. For light alone is always and everywhere accompanied by some heat, according to the measure of its brightness. Concerning the solar light this is most brilliantly obvious, because this is also the most brilliant. Of the light of the heavenly bodies, the proportion of brightness bears witness that the heat of all of them is in a lesser ratio to the heat of the one sun, than to allow it at the same time to be both perceived and evinced by a human, who possesses the sure measure of heat. However, it appears from the effects of the heavenly bodies that there is in the innermost parts of each a perception of their heat. For no outpouring of vapors, whether cold or hot (in comparison with us) can be aroused without some heat to thin them out and raise them up. But all the planets are suited to arousing vapors. Therefore each one has its own effect of heat. Of things containing fire, the matter is again evident. Concerning the extremely diluted light of fireflies, you cannot deny that it is accompanied by heat. For they live and move, and this is not carried out without heating. But the light of burning wood is also not without its heat; for the rotting itself is a kind of gentle fire. Although Aristotle draws a distinction, allowing that it shines ([φωτίζεται]) while denying that it produces light ([βράζεται]), if you understand this most minutely, it is to say nothing other than this, that that light exists in its least intense degree. If in fact it is true that the carbuncle is self-luminous, they do indeed also attribute powers, both to it and to all gems, of which you see it will be fitting that those that are in the carbuncle come forth in some manner through the mediation of heat.

Now it must be proved, on the other hand, that in all other things heat is extrinsic, and depends upon the heat of things being rightly called "passive". It is evident because there is nothing that has heat from itself, since it is material. This is manifest in those things that are made hot by the sun or by fire. For when they are freed from the presence of heat or of fire they again become cold. As for the heat that is in animate things, it is certain that it comes into the body from the arteries. For the bodies immediately become cold when the arteries are blocked. It comes into the arteries from the heart, and I have no apprehension about asserting that there exists in the heart the very thing itself of which Ferrel said there is a likeness, namely, an enduring flame. 53 For to what end are the bags of the lungs, breathing the air; lest life be blocked off for want of air, as happens in fire; the smoke chambers and pulse of the arteries, or the emission of smoke.

53 Jean Ferrel (1497-1558), physician to the French court, wrote a very widely reprinted textbook of medicine "Medicinae" (Paris 1554), later issued in expanded form under the title "Univeria medicinae" (Paris, 1567 etc.)
lest this little fire be smothered by its own waste, the hidden lamp of the heart, the blood drawn across into the heart through a special channel from the very trunk of the vena cava, like oil, whence this flame might live. I do not maintain this, that the air of the lungs and the blood of the arterial vein is nothing more than just food for this flame: nor do I maintain this, that through the arteries there passes nothing more than just the wastes from this little flame, so that all would thus be subservient to the one heart, as its chief. For I grant that the heart itself has been assigned to the service of the entire animal. I grant that these returns, so to speak, are brought into the heart as into a workshop, in order that, being worked out here, they be appropriated in a different way to the body as a whole through the arteries, for the purpose of indispensable usefulness. Meanwhile, the form itself of drawing in and driving out, and the mechanism of nature in the valves, cries out with a raised voice that there is in the heart a creative flame, which would support life from the revenues of the things that enter, and would expel the wastes through the same path with these its works, to the carrying out of which it is ordained—that is, with the vital spirit. It cries out, indeed, that the nature of the body as a whole, and of the heart in it, is so established that the body is most rightly maintained by the waste products of the heart. For if the necessity of conserving this little flame were not to have impelled nature, it might have carried the blood into the heart, and have driven it thence, more gently and calmly, as in the liver, nor was there any need for the motion of sylph and diastole. If, to the contrary, you raise the question, by what means some kind of fire or flame might most directly be preserved in a closed vessel, such as the heart had to be, I answer that this takes place in no other way that by which the heart both is shaped and is opened and closed. I consequently have no doubt that we might also catch this spark of light with the eyes, if it were to happen to us to look into the hiding places of the heart by means of this unimpaired flame, with the light of day excluded. Thus the animal warmth depends upon light. Not to mention that it befits the soul, itself invisible, to have an essence that is akin to light: in which guise it will come, along with light, into the same connection with warmth, to the extent that light is the offspring of the soul.

There is indeed a host of their bodies that is held in and out, but that too comes from a small fire, which, while not so manifest as that in animals, nonetheless exists in plants. This is evident because their body is at length devoured, lying hidden in seeds, and it generates little worms that glow at night, and so ignites the wood, when rot has finally set in, that at night one can even see the spark of light with the eyes.

That there is heat in the earth, everyone knows. That there is an animate faculty in it, akin to light, the guardian of heat, not everyone will admit. I accordingly appeal to the fires of Elma, and to the innumerable hot springs, fetching

---

55 Elma arterialis, putrunary artery.
56 The three "spirits," natural, vital, and animal, had become fundamental to the Galenic system of physiology, though Galen himself did not unequivocally expose them. The vital spirit was related to the heart and to the faculty of locomotion. For the development of the "Galenic" system, see Owsei Temkin, Galenism, esp. pp. 95–108.
up the fiery heat. The same source that produces the heat therefore produces the fire. In oils, sulphur, coal, there is heat as well as fire, but potentially. For when they are made hot in such a way as not to catch fire, they undergo a remission of heat when the fire is removed, but when they are ignited, they show manifestly the source of their heat, when they burn up. Some things create heat through the presence of a humor, and at the same time ignite themselves, such as hay. To sum up: the soul precedes heat; fire or light accompanies it.

Proposition 33

The heat of light is immaterial. For we have made it the companion of light, and light has no matter, as we now consider it.

Proposition 34

The action of light for producing heat is directed toward matter. For all matter is cold, by Prop. 32, light is hot, so they are under the same genus. Thus opposite is acted upon by opposite. This also occurs because of the essential contrariety of the material and the immaterial. In Prop. 10 above, we were concerned with geometrical action and subjection to action by reason of position, where light and the surface were equivalent. Here, now, the action is physical, not reciprocal, for the matter is only acted upon. But the action that appears in turn to carry over to light works through geometrical action: as, for example, a crystal globe exposed to the sun for a long time at last becomes hot, while a ray passing for a long time through the globe is no colder than it was at first.

Proposition 35

Heat in matter is aroused in time. For even if, with its accompanying heat, light is present in a moment, nevertheless, matter, by the fact that it is matter, is subject to time. Therefore, heat, now materialized and passive, can only be generated in time. This is not true of colors, lying hid in the depth of matter. For they were being considered as surfaces, from the nature of light and illumination, and hence their illumination by light also continues to be instantaneous.

Proposition 36

Light destroys and burns things. For by Prop. 34 it acts upon matter. Further, it strives to make the things acted upon similar to itself, in the manner of all agents. Therefore, attacking the matter, in which the essence of all things consists, it destroys it. This happens by dissipation and illumination, so that everything might become light.

Proposition 37

Light bleaches the colors of things in time. For by Prop. 36 it destroys matter. But by Prop. 15 the essence of colors resides in matter, and if this passes away, the color passes away for them. Further, it bleaches them because (by the same proposition) white things are more akin to light. Black things are those having
more of a share in the darkness and density of nature; therefore, there is more of light to take away in black things. But this occurs in time, by Props. 34 and 35.

Proposition 38

Light ignites black things more easily than white things. For by 30 less of the rays is reflected by black things, and thus more is consumed in them. So light places more of its action in black things, and by 36 the cosmos of destroying and igniting. This is the origin of the opinion that rays are concentrated by black things, dissipated by white.

In the place of a conclusion, the sixth proposition of the third book of Wielo should be noted here. Experience testifies that the image of a visible visual perception remains in the vision to some extent even when the bright body, from which the image had come down to the eye, is removed. This happens to such an extent that that surviving image is blended with the colors of other things to the viewing of which the vision, so permuted, is transferred. This is introduced here from experience: we are unable to prove it a priore. For more principles have to be brought in than just those of optics. This alone should be noted against the common manner of speaking, that those images do not come to inhere in the humors of the eye, nor are they images of light or color. For this is inconsistent with the nature of transparent things and of light, and with the principles of optics. That is, an image is always placed with its body, of which it is the image, and when a body is separated off by something opaque, the image is destroyed by the opposing shadow. Thus, by the very fact that humors are transparent, they never receive images, nor transmit them. Further, whatever image there is does not inhere in the opicage coverage of the eye. For again, no color, no opaque surface, is excited and rendered radiant except by a present, uncovered luminous body. The remaining possibility, therefore, is that what inhere in the eye is an image of the action and effect, not of light, but of illumination. In like manner, the sensation of pain from a blow persists, and this is understood as a kind of image of that effect. In like manner also, in visual motion, a kind of image of it carries over into the trajectory, and carries it forth to some place even after the one who had given the motion removed his hand. And since all sensation is carried out through the nerves, and through the spirits that are borne by them, this image of vision will therefore reside in the spirits, not in the humors. It will become necessary below to refer to this relationship.

58 Wielo, Optics III 6. In Theorares II pr. 87.8. The proposition states, “Vision takes place from the action of a visible form on the sense of vision, and from the sense of vision’s being affected by this form.”
The foolish studies of humans have come to such a pitch of vanity that no one's work becomes famous unless he either builds up or burns down the temple of Diana—unless, I say, he either fortifies himself with the authority of Aristotle, or takes a stand in battle against him, seeking to show off. This is indeed the reason why the most true axioms of the optical theorists (simplified upon in this chapter) have hitherto been held in neglect, and, through this paucity of opinions, have undeservedly been regarded as inferior to the Aristotelian darkness, since Aristotle triumphs everywhere, while the optical writers turn a blind eye and privately remain content with their liberties. Therefore, in order to make opposites illuminate opposites by placing them together and to free the Aristotelians at last from the school of the optimists by the arm of either learning or refuting, it seemed right here to discuss explicitly Aristotle's comments on vision. The subject matter is in fact appropriate to the fifth chapter, if one wishes to observe the literal and topical nuances, since the subject matter of the first chapter is light and colors, by nature prior to vision and the eye, which I have postponed to the fifth chapter. But Aristotle's arguments are so constructed that very little must be honed from the fifth chapter, and the rest appear to be most appropriately treated in this place.

First, I shall consider the individual opinions, and those conclusions that follow from them. Next, I shall construct the whole sequence of arguments, and shall respond to them. These, then, are the main opinions.¹

1. Color, properly and in itself, constitutes the subject of vision, and contains in itself the cause for the existence of the visible.²

2. Light is the activity of the transparent so far as it is transparent.³

3. It is in a way a kind of color proper to the transparent itself, which is itself transparent.⁴

4. It is not fire, nor a body, nor anything that flows down from a body; but it is the presence of fire, or of something luminous, and so on, in the transparent.⁵

5. It is the presence in the body of that disposition, or account of which the body is called "transparent."⁶

¹ The following list of “opinions” consists largely of direct quotations from Aristotle in Latin translation. Those that are not numbered 9 and 11 appear to be conclusions drawn by Kepler from these quotations. The source of each quotation is given below.

² De Anima 418b29

³ De Anima 418b9.

⁴ De Anima 418b11.

⁵ De Anima 418b14.

⁶ De Anima 418b20.
6. And these things are to be comprehended thus, that we may know that Aris-
totle is speaking against Empedocles, who had said that light is carried and
spread out in straight lines between us and what surrounds and encloses us
(the heaven), even though we are not aware that this occurs.2
7. The nature of the body that is now light and now darkness, is the same.8
8. And when that body is solely potentially transparent, then darkness persists
there.9

9. And so it is not when it is actually transparent, but when it is potentially so,
that it is then also as a consequence both dark and able to receive colors;
especially because only then is it without color.10

10. The same thing is also affirmed of things that are not seen at all, and of things
this are seen with difficulty: that they are able to receive color.11

11. Further, that which is actually transparent is to be held to be among the
visible, not, however, in itself, but through foreign and extraneous color.12

12. Furthermore, vision (or the movement that is prior to vision in nature, which
I would call “the illumination of the eye”) occurs in this way. Color moves
that which is actually transparent, such as air, while by this being thus
driven, because it is a continuous body, the instrument of vision, or the eye,
is in turn also moved.13

13. And this is one form of vision, namely, when color is seen, i.e., in light, never
by itself, because light is the being-at-work of the transparent.14

14. And so vision (the movement of the instrument which vision follows) occurs
when the sensory instrument receives something from what lies in between.15

15. For the eye is not affected in any way (I mean, the wall) of the eye is not
moved or altered) by the color itself that is seen.16

---

2 De Anima 418b21.
3 De Anima 418b31.
4 De Anima 418b10.
5 This is a quotation from Aristotle, but appears to be an expansion upon no. 8.
6 De Anima 418b27.
7 Not from Aristotle directly, but evidently a consequence of no. 3 above.
8 De Anima 419a13.
9 Enquiry. This is not a Latin word, but a transliteration of Aristotle’s ενεργεία, through
the word Aristotle used in the passage being paraphrased was ενεργόν. I have ac-
cordingly chosen Joe Sachs’s translation of the Greek term as used in the Metaphysics
(Salts Fy: Green Lion Press, 1999). Sachs discusses the meaning of this term on p. 8.
10 De Anima 419a9 and 419a12.
11 De Anima 419a15.
12 Kepler is drawing implicitly on the analogy between the eye and the camera obscura;
in which the image is formed on the back wall of the chamber.
13 De Anima 419a19.
16. And accordingly if it were to happen that the space lying in between were empty of body, nothing might be seen. 19
17. For there is an analogy between vision, hearing, and smell, by reason of the [space] in between. 20
18. There is, furthermore, another form of vision, by which we perceive, not color but other things. Under this category, there is one and the same something in fire and in the sun. 21
19. Nor are all things seen in light: some bring about sense perception or this notion of the instrument [of perception] that precedes it even in darkness. 22
20. For it is also through fire that the transparent [in pores] becomes transparent (in act). 23
21. And of these things: that are perceived at night or in darkness, some, while they do indeed shine, do not give light, λάμπουν μὲν ἀλλ’ οὐ διότι ἐπι- πλενον. 24

From these disputations there appears what I say is the case of the major premise's being evident, that the motion of the instrument [of vision] that is necessary for vision may occur. For the seeing of color, Aristotle requires two motions of the air or of a body of this sort: one by light, to bring its transparent into action (which motion is sufficient for seeing a luminous body); and another by the color of the object seen.

Now, as regards the first opinion, it is indeed true, provided that a proper definition of color also be supplied. For the reason why color has the power of moving the instrument of vision, is because it is of the nature of light. And so it is a primary and inherent property of light to alter the walls (and therefore the eye).

But for this it does not suffice color to be akin to light; it also must have been actually illuminated by light, and must have imbued a certain light, which in this chapter is called "communicated light."

In the second disputations, Aristotle defines light, not, I think, in its nature, but to the extent that it is in characteristic of the process of vision. 25 However, even if it appears impossible to scrutinize deeply the very nature of light itself,
it is nonetheless best to scrutinize some things that more nearly concern the true nature of light, before passing on to how light functions. For it is certain that we know most rightly what anything can do to something else, when we have understood what it is in itself.

The same is also to be said of the transparent; that Aristotle defines this no differently than if he were giving an explanation of the word. For something is transparent when it is seen through, and pelliculic when shone through by some light, but neither happens except in light. And he approaches much nearer to the nature of things who points out the characteristics by whose presence bodies are suited to being shone through, whether light be present or absent. For it is certain that by the approach of light the nature of bodies is not changed; and nonetheless some bodies remain dark in both cases, while others (remain dark) only when light is absent.

Thus in the third opinion it happens that the two have power, both light and the pelliculic. For the denomination of the words is as if light were in mixture primarily for the sake of the pelliculic, which is without. For light exists for the sake of the colors to be illuminated, while the pelliculic exists for the sake of both light and color; that is, in order that the colors might be illuminated by a single sun that is absent.26 And so light is not the activity of the transparent, but rather the activity of colors, just as are they seen, or radiate.

In the fourth, I wish to teach if light (or, if one would draw distinctions with Scaliger, illumination27) is not a downward flow from the luminous body, how it is present in the pelliculic. If the sun is present in the air, and is nonetheless attached to the heavens, it will therefore be present in the air through an outflowing. Unless perchance something of occult philosophy lies hidden in those wondrous words; at which, indeed, one for whom the unknown will be pleasing may marvel, (though) those desirous of learning are not satisfied by empty words. Further, according to what has been demonstrated in this chapter, the ray is not in the transparent (transit as it is transparent), but was there, or was as if there. It is, on the other hand, only in the colors and surfaces of things.

Thus I place the fifth in all respects. For if we have not the true points of the term [transparent], it is indeed not actually transparent unless light in passing through it strikes upon the wall. Light is thus then not the presence of that disposition, but something greater than that presence. For the term does not originate from light itself, but from the motion of light through a body, or of vision through a body. Or if the nature of the transparent is more acceptable [as a criterion], light and the form of the transparent are distinct to such a degree that they cannot be under the embrace of the same physical category. For color—that is, light of any kind—would be desired of the transparent; surface would be desired, which

---

26 That is, the sun is not in the same place as the colors, hence, the pelliculic medium is necessary.

27 Lumen. This is the term used by J. C. Scaliger, Exercitationum exercitatorum adversus. Consular. liber LXI. Paris 1637. Ex. 73. ______. Frankfurt 1592. p. 259.
is not denied of light; and finally density would be denied, which is itself also left to light.

Nor do I think anything different of the sixth, than this: that under the person of Empedocles the truth itself is spoken, provided only that you leave out this, that the light that is thus extended from the sun all the way to the earth is present in the intermediate [space] according to the proportion of the density and opacity of matter; that is, that it is not present in the transparent to the extent that it is transparent; and that this is the reason why it is not perceived by us.

Let the seventh and eighth be judged by the proceeding.

Now in the ninth, it does not therefore follow that the transparent is capable of receiving color, if it lacks color. For the transparent, to the extent that it keeps this form of corporeal nature, does not contain a potency for receiving color; and without such a potency a simple negation of the thing, as is known, does not pull it a disposition. Rather, to the extent that the transparent happens to be more colored (since nothing is absolutely transparent), it will be that much less transparent.

The tenth flatly denies that things lying hidden in darkness have colors.

Again, the complaint is: the same as it was with the transparent; [he argues] so if there are no colors unless they are seen. Common usage, as Aristotle himself admits, is the master of speech; and common usage does not lose colors so narrowly. If you say that Aristotle did not wish here to extend the attribution of color beyond what pertains to vision, I say in turn that the most correct line of enquiry is that first of all the nature of the thing be considered in itself, and then what it could do in something else. For by this procedure [at Aristotle's], things are indeed confused and made obscure. Color, then, is really in the things themselves, even if they are not illuminated, and if they likewise do not radiate and are not seen. Nor does the illusion of the polished make the colors more the polishing, but the illumination of colors makes the medium be traversed, and makes it be truly said to be polished.

The eleventh also includes the transparent [periscopus] among the visible, which are the things seen [conspicuo]. But things that are seen [conspicuo] and things that are seen through [periscopus] are opposites, as the force of the terms imply. As for the matter in question, color is seen in itself, as Aristotle testifies, but color, to the extent that it is actually seen, is a characteristic of the surface, not of the body, to which Aristotle also agrees. Therefore, only surfaces become visible through color. But the polished is a body, not a surface; therefore, by Aristotelian principles, it cannot become visible through color. Unless you should take the word "color" here with another concept, which would give rise to extreme ambiguity.

Now in the twelfth you may indeed find many things lacking. First, if for the motion or alteration of the senses' instrument, no more is required than that whatever is between the color and the instrument be totally transparent, that is, be actual; carried over into the possession of light, then no illuminated color whatever will be seen. But it cannot be proved by any experience, that vision of color occurs in air that is everywhere penetrable by light but not illuminated by
color. An instance is given: \(^{25}\) If the sun alone give light, and it be such at the present cetera surface in which there is color, and let it be in the air of the body after that surface. If it be supposed to be extended. It will therefore not be able to illuminate the surface which is grazed with the outermost ray, but it will illuminate all the air placed in front of it. Nevertheless, vision will not follow, even by an eye set in the space of the illuminated air. Experience, on the other hand, testifies the contrary; to the extent that some color is illuminated more strongly, it is seen that it, rather back more evidently, and that happens continuously. Therefore, where there is no illumination of color, there is also no vision of it, whatever may happen because of the transparent. So also, experience testifies concerning the transparent, that in the event that the light, i.e. it is more noticeable, the vision of colors that are beyond it is that much more hindered, because the presence of light in the transparent destroys the definition of the transparent.

If you should ask Aristotle what kind of motion is, if color moves the actually transparent, and when this moves the eye, he will say, I think, "alteration" (ch. 5). If you should ask, in what quality (i.e. as a charge), he would answer, in color. Therefore, the transparent is carried over from non-color to color, and again from green to reddish black. Therefore, the same transparency, in the same sense of itself, will be pronounced with absolute clear colors, and moreover, will move different senses of vision, and with that confusion of colors that would make all see the same confusion of colors, but, as experience testifies, it will make this one see green, another black, a third red. I ask, moreover, by what procedure will the judgment be able to accomplish this? And here, in a moment, will such a depth of air be altered, and indeed how will it support so many colors in the same part of itself? And so, since this fabricated some is not counteractable with the effect, and since it also may not itself admit various modifications at one and the same moment, according to the variety of views, it is to be held as nothing. Therefore, let us embrace the true opinion described in this chapter, and established by irreputable examples (experimentum): that from the sun, from the colors illuminated by the sun, there flow can [venus] [passive] similar to each other, and that in this flow itself they are diluted, until they strike upon a medium that is in some position opaque and there they represent their source; nor that vision becomes ten will be said at ch. 5 below when the opaque wall of the eye is colored in this manner, the vision being composed when the images of different colors are unconfused, and distant when they are not in transmuted.

\(^{25}\) Here Kepler refers to the following edition.

So p. 32. I am saying that the case is given, not ever in actuality, but on the fact that the sun alone gives light for it never really happens that the sun alone gives light, for all the air and all the earth shines along with it.

\(^{26}\) Kepler appears to mean the 3rd Book II ch. 5. Near the beginning of that chapter (1698634). Butchard says, "Stimulation appears to be some sort of duration in medium of color."
Chapter 1

For unless such an outflowing, and the dilution of the outflowing form, be adopted, the eye will never be made by Aristotelian principles to be affected in one way by the vision of a constant thing, and in another by that of a moving thing, with the quantity seen and the color seen being set at an equal level by way of compensation. On this, see also ch. 3.

Opinion 13 has this right, that colors are only seen in light, but it assigns a false cause, when it brings in light for the sole of the transparent, which, as I have said, owes its effect to the transparent instead. On the contrary, the reason why light is required for seeing colors is because colors do not radiate or emit a form in a hemisphere, unless they are illuminated by the light of the sun or of torches. Thus opinion 14 must be completely reversed. To the extent that the eye is more affected with respect to vision by what lies in between, things will be seen through the medium less rightly and transparently. Accordingly, the vision is most perfect when the eye is completely unaffected by what lies in between.

And 15, plainly the eye is altered by the form of color passing through the petulant body, without the assistance of the petulant.

And 16, if the region between heaven and earth were entirely empty, vision of those things that are now seen in heaven would occur most accurately of all (not, however, the vision of the sun in the heaven, of which that Philosopher spoke, since the weakness of the visual instrument hinders this),

But others should see what is to be thought of the analogy of the senses introduced in opinion 17. For what if the analogy were formed this way, instead: that once a determination about vision is made by us, we now use that as a norm to reason about hearing and about smell? And in fact, concerning smell we would then detest that something of the odor flows out from the actual substance of the object and is received in the nostrils, and that this is perceived all the more strongly to the extent that the medium and the distance is diminished. Therefore, the medium here contributes nothing to informing the sense. In fact, this flow persists in time, when the substance has been removed and the source is driven up. And as for the proportion of the effluvium to the medium (that is, to the odor has the proportion of fire, and is expelled upwards by the thickness of the air. This has been noticed most evidently in the extremely high and completely bare mountains of Carinthia, as well as other places. For indeed, those who are walking through there are met with a most frequent and plainly ambrosial odor from the flower-laden valleys lying below.)

About hearing, the case in which the form of a beat goes forth is certainly more complex than that of odor, but easier than that of color. For the medium of light is in a moment. The form of a beat, however, runs out in time, and therefore accommodates itself more to material explanations. And as for the medium, I again ask what I asked before: if there is a medium for giving a form to hearing, why is sound spread farther in pure air, if not because matter hinders sound, while the more any medium deserves to be called a medium, the more material it is?

30 Aristotle, *On the Soul* 450a7, ascribed to Democritus the opinion that if the intervening space were empty, or air in the heaven would be visible.
Why, again, if the medium gives form to hearing, as we dress to sounds more correctly from nearby than from a distance? For I think it fitting that where the cause is augmented the effect should be augmented. Thus in general, all sentences happen when the instrument is affected. But in order that the senses may be affected by things that are absent, the explanations of effluxion have been brought in, so that what could not be done by the things themselves could be accomplished by the forms. Media, on the other hand, exist only by the necessity of nature to excluding a vacuum. Since these were destined to become a hindrance to effluxion, they were purified and spread out: that is, they were made pellucid (i.e. allowed, stretched out, and developed) to that light, and the things standing in air that give off odors and sounds, might pass through them, so that these forms too might be spread around.

The next four opinions contain a contradiction, which I have to the Aristotelians to discuss. Is it not final nothing at all wanting; fire and the celestial light do have something in common, and opinion 19 was taken from experience. The heavens, fire, nature wood, and things of this sort are perceived at night. And what Aristotle attributes to fire in his way in opinion 20 I also attribute in my way. For him, too, fire gives its form to the medium (for which see above); for me, too, fire illuminates colors. But, for Aristotle, it does so because [light exists within], and [light] is the actualization of the transparent. In my (in the final opinion), resin wood and salted kids do not do this: they only shine (philosophically). Therefore, these things are perceived when the air is not actually transparent, for light is the actuality of the transparent, and this is absent. Here, therefore, the eye is not affected at all by the medium, neither is it affected by the object itself which is at a distance. But it is also not affected by an unflowing; which Aristotle denies. Therefore, vision occurs when the eye receives nothing. The same can be objected to opinion 14. If in the complete absence of an external medium, vision is not carried out, how does it happen that at night one sees a flashing of his own eyes, which is inside the eye? If you say thus the humor of the eye serves as a medium, report that it can also serve as a medium for seeing something external, when that is in contact with the eye. But the error proposed by Aristotle that prevents this from happening is the lack of an external medium.

From this it is clear enough that the optical writings differ from Aristotle on nearly all the opinions. There must secondarily be a flaw in his arguments, which I shall now weigh.

The sequence is this:

1. In every sense, the arrangement of perception is moved by some perceived thing (hominis). 2, 3 For sense is an effect, not one that leads to a seeing to be [noeautai], but one that leads from sense to actuality. Similar to

2 In the first edition and in JGWA (Vol. 2, p. 233.3), this word is given as sentences. This does not appear to be a Latin word, and Frisch (JGWO 2, p. 19) changed it to sentences, which is also not in any dictionary of word list, but is a plausible formation from the verb sense. I have adopted Frisch's correct version.

3 Here, and in the rest of the summary of Aristotle's argument, Kepler includes certain Greek words without translation. I have therefore included the Greek in square brackets after the English translation of that word.
Vision is a sense. Therefore, the instrument of perception is some perception.

H. Color, if it be moved to the eye, is not seen; therefore, a medium is required to come in between in vision. This is corroborated by the analogy of the other senses.

III. Therefore, it is either color or the medium that moves the vision. But it is not color; I understand that this is because, since the medium lies in between, color does not touch the instrument of perception; while there is no motion without contact. It therefore remains that it is the medium that moves the instrument of perception.

IV. That which does not move [something] by itself, moves it because it is previously moved by another. The medium does not move the vision by itself; therefore, it is moved by another; i.e., colored by color; or, what is the same thing, colored (tactually visible) moves the medium.

V. Color moves the medium towards vision, which [medium] we call "not lucid," but color does not move the medium as if it exists in darkness (I understand that this is because in darkness no vision follows). Therefore, a dark medium is not pellucid; or, what is the same thing, color and things of that sort are not seen pellucid in darkness.

VI. If darkness is the privation of something, light is an active state [lightness] of that thing, because their apperitum is opaqueness [lucidity]. The darkness is the privation of the pellucid, as has not been concluded; therefore light is an active state or actuality of the pellucid.

Reply to these arguments:

The first is absurd. I only add, by the way, that Aristotle used not to have so carefully avoided having vision need to be some sort of perceiving away [be present], as if this were unwise to think about something that has been given to living things for their being. For in all instances as has been shown in this chapter, light practices deathly remitt with all matter chiefly with the family of the beasts, which are as it were materialized darkness. The eye, however, both consists of matter and is black. Therefore, it is opaquely determined by light. Hence, the pain from looking at the sun; hence, in part, that of the eyes of the elderly.

To II: On the vision of colors, an induction32 was made. However, the existence is given of seeing flashes of the eyes of vision of no image remaining.

31 On the Soul II 8: "Sensation consists, as has been said, as in being moved and being acted upon, for it is held to be a sort of alteration." (1083a3-4). "Even the term 'being acted upon' is not used in a single sense, but it is either a kind of destruction by something contrary, or, instead, a preservation of that which exists potentially by something that exists in actuality and is similar according to potentiality is related to actuality." (11324-5).

32 This is Aristotle's induction rather than a statistical sampling. For Aristotle, this word meant the presentation of the universal common to particulars through the use of an example. See Proclus, commentary on Aristotle's Physics 1.7.4-7, and Sacks' discussion of the term in Metaphysics of Kant. For Green Law Press, 1993 p. 114-115.
in the spirits after the removal of the visible object; of the vision of impropriety of the humors of the eye after headaches. Therefore the conclusion cannot be drawn for all vision; and so the conclusion is not universal. But there is also more in the conclusion than in the premises. For even if it be granted that there is always some medium between the color and the vision, it nevertheless still does not follow that a medium is absolutely necessary for giving form to seeing or for moving the sense of vision.

Now other causes are given why colors that are moved right up to the eye are less perceived.

1. If color, that is, the surface of the visible object, were to touch the eye, the eye would feel pain. 2. OYK, one eye could be hooded. 3. No more color could be perceived than can fit within the circle of the opening of the pupil. 4. The first point is, that by the contact of the eye and the surface to be seen, all illumination of color, without which vision is impossible, is excluded, except for the amount of the spark of light that the eye of certain animals maintains. 5. That there is another additional underlying cause, alleged either by what has been said hitherto or by Aristotle, that explains why an object touching the eye is not seen, is clear from this: that even when the object is excessively close to the eye, however much of the external medium remains, it is nevertheless perceived confusedly, and is almost imperceptible, but is surrounded by an indeterminate and nearly shadowy edge, the cause of which is explained in chapter five below. And thus mere closeness also does harm, even without consideration of the medium. The eyes of old people also prove this, which see things more correctly from a distance than from nearby.

Once these causes of the given experience present themselves, one cannot restrict the conclusion to one particular cause, much less to one other than these. This is all the more so, in that here the foundations for explaining vision are being laid.

To III. There is an insufficient examination in the major premise. For it is neither the body of the sun or of color, nor the medium, that moves the eye, but the forms or lights, or the rays of the sun and of the colors descending through the medium, and vibrated spherically onto the hemisphere. For the way of all vision is one and the same, whether of the sun, or of color, or of fire, or of things that shine [τὰ χαλάσματα], as far as the alteration of the wall of the eye is concerned. The only difference is that colors must be illuminated first, while the others are bright but themselves. Also, the sun cannot make other things bright. Indeed, Aristotle seems to take it as generally admitted that a form, like a mere property, cannot stir up any motion except insofar as it adheres to the underlying polished site. This, however, is false, as has been shown in this chapter. For light or rays descending from illuminated things is affected in certain ways, insofar as it is light, not insofar as it adheres to the polished site. Among these effects are emission and spreading out, and their contrary reflection and reflection or condensation. Therefore, nothing prevents there also being certain activities of the same light or rays, namely (as shown by experience) illumination and alteration of the walls, by which colors and light are not only poured forth, but are also impressed and their consistencies destroyed.

But that light, in flowing down, undercuts these effects without benefit of a
Chapter 1

host or of the pellucid. I prove as follows. Reflection occurs with respect to place, and without time; but local motion without time does not belong to any material body.

But what if one should say that Aristotle proposed this motion or effect of the pellucid analogically, and not entirely physically? I answer that in that case the Aristotelian meaning coincides with the meaning of the optical writers, though the words differ. But why did he not rather notice the nature of things and set up an analogical motion in light? Now, what is known of others is they frighten form, a mere accident; cannot support any assertion. I reply, from the same philosophy, that a body, cannot be moved in a moment. And so, the motion of light (without time) is such as to square with the quality of its body (without matter); the two being analogical.

In sum, Aristotle seems to distinguish between the form of colors and the light [70. διόικησις], while nonetheless, from every color, when it is seen, light is spread.

So, since the minor premise has been dissolved, the conclusion also does not follow. Thus it is easy to reply to III, wherein minor premise was the conclusion in III, and was found to be false. For the medium does not move the sense of vision either in itself or moved by anything else. Nor is the conclusion in any sense true once the premise is destroyed; unless one should say that being passed through is being acted upon. However, to the extent that the medium is passed through more freely, it is that much less acted upon; now it is acted upon in any part but at their mutual interface, in which respect it cannot be a medium. For in order to carry the form of colors all the way across to contact with the eye, Aristotle interposed a solid body, and wanted all of it to be moved by color. However, the pair of bounding surfaces is not an interpreted body.

Hence V is also refuted, because it depends on the same. But it will also not be good if it be cast according to my principles, in this way: "Light passes through a pellucid body, but does not pass through a dark one; therefore, these things that are in darkness are not pellucid." For it begs the question, since the very thing that makes darkness is the absence of light from the pellucid. Nor does the pellucid take its name from the opacity itself, as if that which is not in actually pellucid were not of that nature. For it would be pellucid if light were present.

Now VI is most. A reply is made to the minor premise by distinction. The essence of the pellucid, though as it is pellucid, is not light, whether present or absent; but the internal dispositions of the body alone. Darkness does not deprive the pellucid of this essence, and consequently light does not bestow it.

But if "pellucid" means the same as "acting pellucid", and likewise "darkness" means "not acting pellucid", illumination accordingly obtains for itself the name "pellucid". But upon this interpretation, illumination is nothing else but a form emanating from colors, nor does it serve Aristotle's purpose, who makes light [70. διόικησις], the actuality of the pellucid, prior to the form of color by nature and in the understanding. For, in his view, since the pellucid is induced and actually by the form-giving light [75. η领导下], and afterwards color moves this actually pellucid, and impresses upon it its own form.

I expect that the academics are going to bring up something against this.
and are going to focus on how to place the honor of their master (who himself never ever sought it) before the truth. For the rest, you, whoever you are, whom it pleases to contend with me, let it be known that you are going to be held unworthy in this ring unless you enter into my chamber [camera] described in Chapter 2 following, which was the only thing that Aristotle lacked. If you ignore this after being warned, the same excuse that saved Aristotle will not save you.

In Ch. 5 Sect. 4 below, when the opinion of J. B. Portia is examined, you will find a summary and abridgement of this disputatio.30

30 There is a pun here: Chapter 2 presents the theory of the pinhole camera, or camera obscura.
31 See below, p. 224
Chapter 2

On the Shaping of Light

That a ray of the sun cast in through any kind of crack, falls in the form of a cleft upon a plane object, is a fact obvious to everyone. 3 This is seen beneath roofs that are split open, in shrines, perforated windowpanes, and beneath any tree. Left on by the wonder of this thing, the ancients had devoted their energies to finding out the causes. Aside from that, no one has yet appeared to me to have carried out a proper demonstration of the problem. Wielo, three hundred years ago, claimed that this happened because of some sort or other of equality of the rays. Thus he demonstrates Proposition 39 of Book II by Prop. 35 of the same book. But he himself does not cover up the flaw of his own demonstration, saying in Prop. 35, "it may be that a property of the rays contributes much to this." 4 In dealing in these ambiguities, he shows that he had not understood the true cause, which is gathered obviously from another part of his demonstration. Following him, Isocrates Pismus, Bishop of Cambrai, 5 rejected the opinions of others, and among them you may be surprised to note the actual true cause, and withdrew with Wielo into the hidden recesses of the arcane nature of light; in Book I Prop. 5. Hartmann, who published Pismus, and set about removing the flaws in the proofs, left this hesitation. Pismus, though, introduces the words of the others, through which the true cause is set out: "Others," he says, "assume as a remote cause the roundness of the sun, and the intersection of the rays as the proximate cause." Nevertheless, the demonstration itself, which he immediately appends, without doubt following the opinion of those same authors, again hides the truth, and attributes to these very words a sense that they cannot have. It is credible that nearly the same thing happened to Aristotle. For when, in Sect. 15 Ch. 10 of the Problems, 6 he has raised the question, "Why, if the rays of the sun during an eclipse should pass through the leaves of a plane tree, or through the

---

1 Here Kepler refers to the following censure:

"The ideas of Copernicus, as the Keplerian cosmology, was Archbishop of Canterbury. However, as Horace's editions of the Persian Wisdom were published in 1542, and other editions, which Kepler evidently used (see below), erroneously described Picham as "Episcopus Cameracenae." The passage cited is in the demonstration of Book I Prop. 2.

2 The Problems, or Problems, is not a work by Aristotle himself, though it may incorporate some of his thought. The passage cited is in Book 15 Ch. 11 (vol 7 of modern editions). Similarly, the passage cited below is in Ch. 1 on Ch. 5. See W. D. Ross ed., Works of Aristotle (Oxford: Clarendon Press, 1927 etc., Vol. 7.)
fingers of both hands placed crosswise to each other, crescent shapes would be made upon the pavement. He in fact adduces the true cause when he says: "That two cones come together at their vertices in the narrow space of the opening; of which the base of one is in the sun, and that of the other on the pavement; and that it is consequently necessary that when the sun has its upper part removed in the shape of the crescent moon, the same thing happens to the ray below on the opposite side." But in Ch. 5, when he explicitly asked what he had assumed as a principle in Ch. 10, (namely,) "Why a ray that has entered through four cornered figures does not make four cornered shapes, but circular ones?", he ascribes the cause in part to the weakness of vision "that one who is unable to catch those rays that withdraw into the corners of the opening, in view of the brightness of those that pass through the middle of the opening," and in part he appeals to the actual form of vision, "because vision may arise through a cone of rays going forth from the eye, whose base is a circle." By this introduction of an irrelevant cause, and one which is not accepted in optics, he again brings darkness upon those things that he deduces from these in ch. 10, to such an extent that neither Wiele, nor Psamnius, nor any subsequent person that I know of, understood Aristotle.

Several years ago, some light shone forth upon me out of the darkness of Psamnius. Since I was unable to understand the very obscure sense of the words from a diagram drawn in a plane, I had recourse to seeing with my own eyes in space. I set a book in a high place, which way to stand for a luminous body. Between this and the pavement a tablet with a polygonal hole was set up. Next, a thread was sent down from one corner of the book through the hole to the pavement, falling upon the pavement in such a way as to graze the edges of the hole, the image of which I traced with chalk. In this way a figure was created upon the pavement similar to the hole. The same thing occurred when an additional thread was added from the second, third, and fourth corner of the book, as well as from the infinite points of the edges. In this way, a narrow row of infinite figures of the hole outlined the large quadrangular figure of the book on the pavement. It was thus obvious that this was in agreement with the demonstration of the problem, that the round shape is not that of the visual ray but of the sun itself, not because this is the most perfect shape but because this is generally the shape of a luminous body. This is the first success in this work.

In addition, when both Aristotle and the man I have called "Psamnius" to the end of untangling the subject, introduced the most beautiful example of the ray of the eclipsed sun being similarly eclipsed when it is taken through a small hole, they afforded Reinhold, Gemma, and my teacher Mueldin, the opportunity to

1 Aristotle. Problematik XV. 6. 91b 3-25. For the most part, Kepler paraphrases Aristotle rather than quoting him.
2 Kepler uses the Greek word στειρόμαχον here.
3 David Lindberg has suggested that Kepler drew his inspiration for this book and thread procedure from Albrecht Dürer's Unterweisung der Messung (Nürnberg, 1525). See Dürer's treatise on Vision, p. 188.
4 Experimentum.
apply the theorem to a use that is no less noble. For these authors I have named had taught astronomers how to use a compass to measure the magnitudes of solar eclipses, the ratios of the diameters of the sun and moon, and the inclinations to the vertical of the circle drawn through the centers of the luminaries, avoiding the inadequacy of the eyes, and avoiding the error which generally occurs in a bare estimation. And so, from that time, however many solar eclipses were documented by eminent mathematicians, it is likely that they were observed in the way just now described. For aside from this, no other sure procedure can be established for measuring something that happens in the sky.

It is indeed well worth while here to see how much detriment would result from the ignorance of the proof of this theorem. For since it escaped a number of authors, the result was that in believing in the theorem without restriction they fell into a large error. For however many eclipses were observed in this way, they all had come out much greater in the sky than it appeared in the ray; all showed a much greater lunar diameter in the sky than in the ray. Hence it is that Phoebus of astronomers, Tycho Brahe, in his wonder, was driven to such straits as to pronounce that the lunar diameter is always a fifth part smaller in conjunctions than it appears to be in oppositions, even though it is in the same distance from us in both instances. I would nonetheless not deny that there are other underlying causes for the lunar diameter’s in fact appearing somewhat smaller at conjunctions, for which see below.

It is my hope in these pages to remove these considerable difficulties, which wall off our entry to the prediction of eclipses and to an exact reconstruction of the moon’s motion, by a straightforward demonstration of the theorem, and by laying bare the sources of the errors which displayed themselves for me to examine through a careful consideration of the solar eclipse that occurred in 1600.

Definiton

All points from which straight lines can be drawn to some single distant point, making a pyramid, are said to belong to one luminous surface, whether they in fact belong to the same or to different surfaces, because that pyramid is understood as being cut by a single perpendicular plane.

Postulate

A pair of lines of illumination arising from the same luminous point, even though they actually come together at this their origin, are to be regarded as equivalent to equal distant lines within the limits of some perception provided that they have a very large ratio to the base by which the two are conceived.

planarium (Wittenberg 1555, et al); fol. 198 tthis is the edition Kepler had: abo Rainer Gemma Frisius, De rebus astronomica et geometrica (Antwerp and Louvain, 1545), ch. 18. For Mersenne’s contribution to the observation of solar eclipses, with a camera obscura, see ch. 11 below.


91 See below, Ch. 11 Problem 22, p. 423.
Proposition I

If there were a single point within the limits of sense perception from which light is radiated, the ray upon a wall interspersed perpendicularly would have a shape similar to the window through which it had entered in a perpendicular incidence, and the ratio of the dimensions of the illuminated wall and the window would be the same as that of the depatures of each from the luminous point. This is Wituco 199 and 100.12

Let there be the wall or the plane ABCD, and the luminous point E on high; and let there be a window of any shape whatever, which shall now be FGH, interspersed between the two. Now let a perpendicular descend through the transparent body, such as the air, from E to both the center O of the window FGH and the plane ABCD; and let this be EI. I say that the part of the surface ABCD illuminated by the luminous point E will have a shape similar to the window FGH, that is, also a triangle, in the present case.

Demonstration

Now since the whole region of the window FGH and all its points lie exposed to the luminous point E without anything opaque lying in between, the rays from E will fall upon all these points, by 1 2. Therefore, the radiating straight lines will likewise be led through the infinite points of the edges of the window. Let the window FGH have any number of bounding straight lines. Let one of them be FG; whose ends are thus F and G; let which let two straight lines be joined from E; thus, by Euclid XI 2, FGE are is the same surface. And by Euclid XI 1, all the points of the straight lines FG, EM, EK, and however many descend from E to FG, are in one surface. Now the ray EOI is by opposition perpendicular to both of the surfaces FGH and MKL. Therefore, by Euclid XI 14, they will be parallel planes. Consequently, by Prop. 16 of the same book, the two lines of intersection FG and MK of the same plane MEK with the two parallel planes FGH and AC will be parallel. The same will be demonstrated in the same manner for GH and KL, for HF, and LM, as well as for an infinity of sides, if there were any. Therefore, by Euclid

12 Theorems pp. 37-8. Wituco’s propositions are purely mathematical, and concern only cones and cylinders. Kepler extends them considerably here, and applies them to a concrete situation.
XI 10, the angle $FHG$ of the intersecting lines $FG$ and $HG$ will be equal to the angle $MKL$ of lines parallel to the former and intersecting. In the same way all the angles of the one will be equal to all of the other. But the sides are also proportional to the sides. For by Euclid XI 17, the planes $AC$ and $FHG$ cut the intersecting straight lines $EL$, $EM$, $EK$, and however many others, in the same ratios. Thus as $ME$ is to $EF$, so is $KL$ to $EG$, and alternately, as $ME$ is to $EK$, so is $FE$ to $EG$, while the angle at $E$ is common. Therefore, by Euclid VI 8, $FEG$ and $MEK$ are equiangular. Consequently, by Euclid VI 4, $FG$ and $MK$ are homologous, and so are all the sides of the one to all the sides of the other. Therefore, by Euclid VI Def. 1, the figures are similar. But beyond the limits $MKL$ of the lines descending through the edges of the window, there falls no ray, but a shadow, by 4 of the first chapter. Because the parts surrounding the window $FGH$ are by supposition opaque. Therefore, the portion of the illuminated wall $MKL$ has a figure similar to the window $FGH$, which is what was to be demonstrated. Now let the centers $O, I$ be joined with any angles whatever, such as with $FG$, $M$, and $K, L$, here, I say that $EO$ is to $OF$ as $EI$ to $IM$. But since $OF$ and $IM$, or any such others, have an equal ratio to those diameters, because the figures are similar, and since $EO$ is the distance of the window from the luminous point, while $EI$ is the distance of the wall from the same point, the other part of the proposition is clear as well.

**Corollary**

It follows hence that from any point of a luminous surface a perpendicular ray is cast upon an intercepted wall, the base of which ray is similar to the shape of the window. Thus the ray descending from the whole luminous surface to the illuminated wall consists of shapes that are potentially infinite, similar to the window, mutually overlapping, and falling upon approximately the same region of the wall. These shapes individually would nonetheless have their own proper boundaries, if they were separated.

**Proposition 2**

If there shines a single point, remote from a wall and a window (which are near each other) by an incommensurably great distance, the light upon the perpendicularly intercepted wall will resemble not just in shape but also the quantity of the window through which it passed in a perpendicular path. In the former diagram, let the luminous point be $E$, the window $FGH$, the illuminated wall $MKL$, and let the ratio $EO$ to $OF$ be incommensurably great, such as it would be if $EO$ were to measure the enormous distance between the window and the sun or moon, while $OF$ measures the tiny diameter of the window. I say that $FGH$ and $MKL$ are equal to each other within the limits of sense perception. For since $FO$ or any line whatever from the center of the window $FGH$ has a perceptible ratio to $OH$ (because the surfaces are supposed near each other), but $OF$ has an imperceptible ratio to $OE$, $OE$ will also have an imperceptible ratio to $OE$. 


Consequently, by the postulate, the rays $EF$ and $EO$, joined by the base $FO$, having a magnitude imperceptible in comparison to them, are equivalent within the limits of sense perception. But the angles $EOF$, $EIM$ are equal, and are in the same plane of the triangle $EIM$, so by Euclid I.27, $OF$ and $IM$ are equidistant, and by 33 of the same book, are equal within the limits of sense perception. Further, the same can be demonstrated concerning any lines similarly drawn from the centers $O$ and $I$. Therefore, the whole $FGH$ is equal within the limits of sense perception to the whole $MKL$, but it is also similar by the first proposition of this chapter. Therefore, "If there shines $u$ single point..." which was to be demonstrated. \[15\]

**Proposition 3**

If an window could be a mathematical point, the illumination of the squarely inscribed wall would precisely assume the shape of the illuminating surface, but inverted; and the ratio of the diameters of the luminous surface and the illuminated wall would come out the same as that of the distances of each from the point of the window. Let $FGH$ be the surface to be illuminated, $NQP$ the luminous surface equidistant from it, and let the window be at point $O$. In accord with what was previously demonstrated, let straight lines be drawn from $N$, $Q$, $P$, and any other points, to $O$ and beyond it to the surface $FGH$, which lines represent the rays of the luminous surface, and let them be $QF$, $PH$, $NG$. Now, since all meet at $O$, they will thus cut each other when produced, and the ones on the right will become those on the left, and vice versa. Further, because two straight lines $QF$ and $PH$ intersect each other, they are consequently in one plane, by Euclid XI.2, and now by Prop. 16 of the same book, because two equidistant planes $NQP$ and $GHF$ are cut by the plane $PQOH$, the common sections $PQ$ and $HQ$ will be equidistant. In the same way it can be proved that $NQ$ and $PH$ are equidistant, and vice versa $NQ$ and $GF$. Thus by Prop. 10 of the same book, the angle $EHG$ of the meeting lines $EH$, $HG$, is equal to the angle $QPN$ of the meeting lines $QF$, $PH$, equidistant from the former ones, and $GEH$ is equal to angle $NQP$, and $FGH$ to angle $QNP$, to their opposites, respectively. Further, because the plane $NQP$ and $GHF$ are parallel, they will cut $POH$ and $QOF$ in the same rates. Accordingly, as $PO$ is to $OH$, so is $QO$ to $OF$, while the angles $POQ$ and $HOF$ are equal, since they are vertical angles. Consequently, by Euclid VI.6, these triangles are equiangular [with each other].

\[44\]
and $PQ$ and $FH$ are homologous.\footnote{Here Kepler used the Greek word ἀνάλογος.} And thus also are all sides of the one to all of the other. Therefore, the whole figure $FGH$ is similar to the whole $QNP$, by Euclid VI Def. 1. Further, let the centers $I$ and $E$ be joined to $P$ and $Q$ or to any point whatsoever of opposite extremities. Accordingly, $IF$ and $FQ$ will also be equidistant by Euclid XI 16. And because $IF$ and $QF$ cut each other at $O$, the angles $1OF$, $EOQ$ will be equal by Euclid I 15. But $FIO$, $QEO$ are also equal being by supposition right angles, and consequently the remaining angles $1FO$, $QEO$ are also equal, by Euclid I 32. Therefore, the sides are proportional, and as the distance of the wall $OF$ is to the line on the illuminated surface $IF$, or any other line whatever, so is the distance of the luminous surface $OE$ to the corresponding line $E$. Which was to be demonstrated.

**Corollary**

It follows hence that through the individual points of any window, which are infinite, individual (and thus infinite) inverted images of the luminous surface are transmitted to the illuminated surface, following each other in the same order in which the points of the window themselves have.

**Proposition 4**

The quantity of every illumination on a wall is greater than the space of the window through which the illumination is sent in. For if, on the one hand, we pretend that it is a single point that shines, the rays transmitted through the boundaries of the window, since they meet at their origin, are proportionally farther apart as they go forth, and thus take up more space on a more remote wall than they do at the closer window, by 1 of this chapter. If, on the other hand, it be a surface that shines (as it always is), the proposition is all the more true.

Let $PNQ$ be the luminous surface, whose center is $E$, and let $FGKO$ be the window. Thun by the corollary to 1, the center $E$ of the luminous surface will create the figure $FKLMO$ on the wall similar to the window $FGKO$, and greater, by 1 of this chapter, or equal within the limits of sense perception, by 2. Now by the corollary to 3, through the individual points of the edges of the window, individual inverted images of the luminous surface are transmitted, such as you see at $M$ and $L$, transmitted through the points $O$ and $H$. And since $FOM$ is the ray from the center of the luminous body, and the middle of all those
that intersect each other at the point $G$, the remaining ones are therefore either beyond or this side of, and the one that descends from the point $E$, which is on the inside with respect to the window, is now made to be on the outside by the intersection that takes place at $G$. The same description can be applied to all the points. In this manner, a perimeter will be created that is greater than $IKLM$. But previously, this figure $IKLMI$ was greater than the window $FGHO$. Therefore, this new expanded figure is much greater than the window $FGHO$, which was to be demonstrated.

**Proposition 5**

The shape of a ray on the wall is a mixture of the inverted shape of the luminous surface and the upright shape of the window, and it corresponds to them in position in this way. For by the corollary to 3, the inverted shape of the luminous surface, as if attached to the luminous surface, is drawn around following the boundaries of the window, with the result that by the individual points on the wall it describes lines corresponding to the sides of the window, and doing this on the same side. On the other hand, by the corollary to 1, the upright shape of the window, as if attached to the window, is drawn around on the wall, in an order contrary to the boundaries, of the luminous surface by the argument of 3), with the result that by the individual points on the wall it describes lines corresponding to the opposite sides of the luminous surface, on the opposite side. So the rays form the boundary of the figure that is made, where it has angles, and these rays are sent down from the extremities of the luminous body through the extreme angles of the window. Moreover, a geometrical motion is now assigned to the form of each of the two, because of the infinity of the multiplied forms. Therefore, it is apparent that the boundaries of the luminous surface have a share in each of the shapes.

Let the above diagram serve as the example. The angle $Q$ of the luminous surface, together with the whole triangular image, having descended past $O$ to $X$, describes the line $XV$, corresponding to the line $OH$, when the point of intersection $O$ is carried over to $H$. Similarly, the angle $N$, having descended past $O$ to $Y$ describes the line $JR$ when $O$ is carried over to $F$, and angle $P$, having descended past $H$ to $T$, describes the line $TS$ when the point of intersection is carried over from $H$ to $G$. But $PA$, because it is raised up equally above $FG$, describes $RS$ on the wall, after descending and being taken through $FG$. Thus, the shape of the window is expressed approximately.

Again, suppose that the vertex $E$ of the pyramid $EKLMI$ moves from the
center to \( N \) while the pyramid is attached to \( FGHO \). As a consequence, the angle \( FIL \) of the image of the window will belong at \( Y \). When the vertex is now moved from \( N \) to \( Q \), the angle \( M \) will describe the line \( YX \), corresponding to the line \( NQ \) on the opposite side. And if the vertex is at \( Q \) the angle \( MIL \) will be at \( Y \). When the vertex is transferred from \( Q \) to \( P \), \( L \) will describe the line \( VT \) opposite the line \( PQ \). And then \( IK \) will belong at \( RS \), and \( K \) at \( S \). Finally, when the vertex is moved from \( P \) to \( N \), \( K \) will be separated from \( S \) until \( L \) belongs at \( R \). The equidistance of the opposite sides \( PN \) and \( FG \) brings this about. Thus, if \( BY \), \( XV \), and \( FS \) be extended to their common intersection,\(^{13}\) the image of the shape of \( FGHO \) will be completed. If, on the other hand, \( YX \), \( VT \), and \( SR \) meet when extended, this shape will be precisely similar to the luminous surface \( PQU \), but inverted. Since neither of these things happens, it results that the two are to some degree mingled.

**Corollary 1**

It follows hence that if the sides of a pair of figures that are similar and on opposite sides shall be equidistant, the shape of the ray will perfectly imitate the common shape of the two, but in position it will imitate the window.

**Corollary 2**

But if the angle of one of the similar figures be placed opposite the side of the other a shape with twice the number of sides will be created; e.g., a hexagon in place of two triangles, and an octagon in place of two squares.

**Corollary 3**

Therefore, when the luminous surface and the window assume the shapes of a circle, the ray describes a perfect circle on the wall. Nor is it sense a circle has an infinity of both sides and angles, and thus the two circles can be considered to be equidistant, as in Corollary 1, and to have sides opposite sides and angles opposite angles, as in the second corollary. Therefore, whether you give the created ray a number of sides that is one times infinity or twice infinity, the result is the same.

**Lemma 1.** For the following Proposition

Triangles whose sides cut off the same or equal positions of two equidistant lines are bounded by vertices on a third equidistant line. Let there be the straight line \( NEP \), divided into equal parts at \( E \), and let \( FOG \) be equidistant from it.

\(^{13}\) Kepler uses the singular here, although the three lines obviously do not have a common intersection. He must have meant the common intersections of each of the nonparallel pairs.
likewise divided into equal parts at $O$. Let the straight lines $PO$, $EF$ be drawn until they meet at $K$. In the same way, let $EO$ and $NF$ or $PG$ be drawn until they meet at $L$, and let $K$ and $L$ be joined. I say that $KL$ is equidistant from the bases $NEF$ and $FOG$. For by the triangles $MEL$, $FOI$, the angles $NEL$, $FOI$ are equal by Euclid I. 29. The sine is true of $ENL$, $OFL$, and the angle at $L$ is common. Therefore, the triangles have equal angles, and by Euclid VI. 4 the sides are proportional. Therefore, as $NE$ is to $FO$, so is $EL$ to $OJ$. In the same way it is proved in $UPK$ and $FGK$ that as $EP$ is to $FO$, so is $PK$ to $OK$. But $EP$, $FK$ are equal. Therefore, as $NE$ is to $FO$, so is $PK$ to $OK$. But previously $EL$ was also to $OI$ in the same ratio. As a result, as $PK$ is to $OK$, so is $EL$ to $OJ$. And by Euclid V. 5 as $?O$ is to $OK$, so is $EO$ to $OJ$, and alternately as $PO$ is to $OE$ so is $KO$ to $OI$. Further, $EP$ is equal to its vertical angle $OJ$. Therefore, by Euclid VI. 6, the triangles $EOP$, $KOK$ have equal angles, and $OK$ on $FK$ is equal to $OJ$ or $KPE$. As a result, by Euclid I. 25, $EP$ and $KL$ are equidistant. The same is also true of $EFP$ and $EPL$, which in contrast have a common base but cut off equal equidistant portions of $FG$. The proposition is therefore evident.

**Lemma 2: Problem**

To find the point that is the same number of diameters of the luminous surface from the luminous surface as it is in diameters of the window from the window. Let $NFE$ be diameter of the luminous surface, and $FOG$ be the diameter of the window, equidistant from it and lying perpendicularly below it, and let $FO$ be the perpendicular to the sun. From $N$ and $P$ let straight lines be drawn through the ends $F$ and $G$, and they meet the point of meeting being $I$. Say that the is the required point. For since in triangle $NEI$, $FOG$ is equidistant from the side $NE$, $IE$ will be to $EN$ as $IO$ is to $OF$, with the result that as $IO$ (the distance of $I$ from $O$) is to $FO$ (twice $OF$, which is the diameter of the window), so is $IE$ (the distance of $I$ from the luminous surface $F$) to $NP$ (twice $NE$ and the diameter of the luminous surface), which was to be accomplished. It is moreover clear from this that the diameter of the window must be less than that of the luminous surface.

**Proposition 6**

When a window is distant from the wall by the same number of its diameters as the luminous surface is in its diameters, the manner of the shadows is most evident, and the shape of the eyes portrays equally of both shapes. But when the window is distant from the wall by fewer of its diameters, the shape of the ray and its position appears less near the shape of the window. And when on the other hand the luminous surface is distant from the wall by fewer of its

---

80 Literally, "of the luminous". However, since Kepler uses "luminous surface" superficies luminis in Proposition 6, it seemed justified to supply "surface" here.
diameters, the shape of the ray more nearly imitates the shape of the luminous surface, in an inverted position, the more so as the former is true. Let QRSPN be the luminous surface with center Z and diameter NF, and let the window be FOG with diameter FG, and let A be equidistant from the entire figure of the luminous surface, lying perpendicularly below it. From C let a perpendicular be drawn through O, and let it be SL. And on line E let there be found, by Lemma 2, a point that is the same number of diameters of the window from the window as it is in diameters of the luminous surface from the luminous surface above, let EF and PO be joined, and likewise FG and NO, and let them be extended until they meet at K and L.

I say that if by this position of the window and the luminous surface a ray be created at A, it will be equally relayed both to the shape of the luminous surface E and to the shape of the window O. However, if it be created and it fall close than L towards O, the nearer it is to the center O the closer it will approach the shape of O, the shape of E being effaced. If on the contrary it fall beyond L, the farther it is from O the more precisely will it represent the shape of the luminous surface E, the shape of O being gradually effaced. For this to hold, note first, that the meeting points K, L, by Lemma 1 above, are in the same line, which is equidistant from the luminous surface NE. In addition, all three are on the surface to be illuminated. Further, the rays FK and GL sent down from the center E of the luminous surface past the edges of the window FG carry the shape of the window down to the plane I, by the Corollary to 1 of this chapter, and carry thus very midpoint of it, for all about it there stand as many other shapes as there are points lying about the center E. In the same way, the rays FK, NL sent down from the edges of the luminous surface X, P through the center O of the window, carry down the inverted shape of the luminous surface, by 3 of this chapter and its corollary, and likewise that very midpoint. For about this too there stand as many other shapes, similar and approximately equal, as there are points lying about the center of the window, since such an intersection takes place at each of them. Therefore, since these four rays intersect each other at the points K, L, of the surface I, KL will be the common measure of both shapes, that of the luminous surface inverted, and that of the window upright. Now let the point be chosen, in accord with the preceding problem, that is distant from O by fewer diameters FG and it is from E in diameter XP. And let it be X, through which let a line equidistant from the said diameters, which will intersect the rays at B, M, T, V. Therefore, the part MT will represent quantitatively the diameter of the luminous shape inverted, as it falls above the position X. On the other hand, HV is the diameter of the shape that the window has. Thus, the latter will be greater than the former. Consequently, more of the latter than of
the former boundary enters into the eyes. Contrariwise, in accord with the same problem, let the point be chosen that is distant from $O$ by a greater number of diameters of the window than it is from $E$ in diameters of the luminous surface. Let this be $Y$, and through this, as before, let a line be drawn equidistant from the others, which will intersect the rays at $A$, $B$, $C$, $D$. But because the intersection of the rays already occurred at $K$ and $L$, the ones which were on the outside are now on the outside, and $AD$, representing the inserted shape of the luminous surface, will be greater than $BC$, the diameter of the shape that the window has. As a result, the shape of the luminous surface enters into the eyes more here. 

In order that these three cases be more rightly understood, first let the shape of the window be described, just as it is produced upon the wall by any one point of the luminous surface, and let it be $OY\beta$, and let the inserted shape of the luminous surface be $O\zeta\eta$, of such size that it can be enclosed in the same circle [as which encloses the shape of the window]. Now let the center of the shape $O\zeta\eta$ be moved along all the edges of $OY\beta$. Its position about $a$ will be the triangle that, about $b$, $\beta\alpha\gamma$, and thus with the angle $c$ it has described the line $ax$, equidistant from $O\beta$. About $y$, its turn, will be $\gamma\beta\gamma$, and with the angle $\mu$ it has described the line $\gamma\mu\zeta$. Finally, about $b$ it will be $\epsilon\sigma\rho$, and with the angle $a$ it makes the line $\alpha\beta\sigma$, while with the side $\beta\gamma$ it makes the line $\beta\zeta\eta$. Thus $O\zeta\eta$ has described 4 lines, bounding the figure $\lambda\mu\nu\xi\zeta\eta$, parallel to the four lines of the windows.

In the same way, let the center of the figure $OY\beta$ be moved along all the edges of $O\zeta\eta$. Its position about $\varepsilon$ will be $\theta\varepsilon\zeta\eta$; about $\eta$, $\mu\sigma\zeta\eta$; about $\zeta$, $\alpha\lambda\tau\zeta$; and with the line $\delta\gamma$ it has described the line $\epsilon\zeta\xi\eta$, with the angle $\mu$ the line $\mu\zeta$, and with the angle $\theta$ the line $\theta\eta$. Here you see both the diagonal sides of the triangle and the orthogonal sides of the square presented, on the perimeter of the entire figure, approximately equal, that is, of such a size that both are in the same circle. For those of the triangles are greater because they are fewer, and hence it can be said that the rectangular sides are no more shortened than are the triangular ones.

Now let the diameter of the figure of the window be greater, and let it be $OY\beta$, but let the inserted figure from the luminous surface be $O\zeta\eta$, and let both be situated upon the same center, as before. Now let the center of the triangle be carried around on the edges of $OY\beta$. As positions will be $B, K, \alpha\zeta, \gamma\mu\zeta, \epsilon\sigma\rho$, and the intermediate points. You see that with the three angles it describes the
three lines off, $\overline{AL}$, $\overline{ML}$, while with the side it describes the line $\overline{ON}$. Let the center of the window be likewise carried around on the edges of $\overline{ON}$; its positions will be three, about $\ell$, $\zeta$, and $\eta$, the points being three: before, middle, and after you see the with two angles it describes the two lines $\overline{AO}$, $\overline{AI}$, with the remaining two and the side between them, the common line $\overline{ON}$. Thus this figure is lacking some lost part, preventing it from being wholly similar to the window. The traces of the figure of the luminous surface, however, are very slight.

Contrariwise, let the diameter of the figure of the window be smaller, and let it be $\overline{OP}$, and about the same center let there be the inverted figure of the luminous surface $\overline{ON}$, and, keeping the same two-fold circuit of translation, let the lines just mentioned be drawn, only the proportion being changed. Thus some least part is wanting from the whole figure, preventing it from representing its original, namely the luminous surface, in an inverted position.

Proposition 7. Problem

In a closed chamber, and upon an appointed wall, to represent whatever either is or is done outside opposite the chamber; that is, that enters into the eyes. This art was, as far as I know, first presented by J. Baptista Porta, and constitutes not the least part of the Magia naturalis. But, being content with its view, he did not add a demonstration. Yet it was in fact just this one experiment alone by which astronomers would have been able to make a determination concerning their picture of the solar eclipse.

So, let all the cracks of the chamber be closed, so that not the least particle of light can enter, while opposite the window, which has a view of what is to be represented, let there be a white wall, the other walls being black. Let an extremely narrow hole be made in the window, just as much as suffices the power of vision, in such a manner, however, that if the wall or the window should be thicker than the opening, the opaque parts surrounding the opening should be cut away, until your view through the opening may be unobstructed to all things located outside that you wish to represent. And let the eyes of the viewer be shielded from the light of day for quarter to half an hour, until the images impressed by the spirits in the clear light of day might vanish, in accord with what is said in the conclusion to the last chapter. And let the things to be represented be located in the bright light, either of the sun, or of day, or of torches. I say that all things that are seen both to stand stationary and to happen outside, are going

---

17 Camera clausa. The term camera obscura had evidently not yet become standard. Porta called it a curriculum obscurum (Magia naturalis, Naples 1559, XII, 7, p. 267).
18 Magia naturalis XVI, 6. This technique was also described, though obscurely, by several other sixteenth-century authors.
to be represented inside on the opposite white wall, but in an inverted orientation. For by Chapter 1 22. things outside illuminated by any light whatever tint the communicated light, and scatter it in an oblique manner; by 6 of this chapter the things outside will illuminate the opposite wall within, so that the shape of the illuminated wall is indeed a mingling of the shape of the window, and that of the things that are outside; but nonetheless, since the window is presupposed to be very narrow with respect to the distance of the window from the wall, a minimal part of the shape of the window will be mixed in with the shape of the things standing outside. Tie only thing that will be lacking in this picture is, first, that by the same Prop. 6 the things will appear inside in inverted orientation, and second, that anything outside that falls within the reach of a cone having its vertex at the wall and shaped or limited by the hole, cannot be portrayed upon the wall with its parts artculated, but whatever exceeds in breadth the boundaries of this its cone, will be depicted within along with its parts. For the same reason also, those things that shine directly through the hole shine more brightly and are more confused in the image, because the hole is spread wider from the direct than from the oblique. But the colors will also not be lacking in this picture; for by Chapter 1 25 the colored lights of cones coming together at the hole do not disturb or hinder each other. And because the surface upon which these colors of exterior things radiate is white, hence, by Chapter 1 29, it will receive these rays all the more strongly, and will make them appear brighter. And since the chamber is closed, and it is not the rays of the sun or of the whole sky, or rather of the glowing air, that shine upon the individual points on the wall, but just wherever little part you please that shines on the point opposite it, therefore, by the converse of Chapter 1 27 and 28, the senses distinguish between the individual lights, because they are not at all tinted by a stronger light. This is especially so because we made the other walls black, so that the light not illuminated by the white wall that was originally illuminated, and become bright themselves (by 1 29), and in turn illuminate the representational wall, and thus confuse the colors coming from outside (by 1 27), if these walls too had been white. Because if the chamber were not so tightly closed, although the colored rays on any wall you please are infinite, even without the representation of the hole, nevertheless, by Chapter 1 28 they could not be perceived because of the brightness of day. And because when the configuration of the luminous hemisphere is changed, its image upon the wall is itself also changed, while the things that are done, that is, motions, suddenly change the appearance of the hemisphere, therefore, the image within will also change, and thus the motions of exterior things are to be seen within.

However, in the representation of colors, there is lacking what was proved by 29 and the converse of 30 of Chapter 1: that the distinctions of colors are not well grasped by the eyes, because of the brilliance of the whites radiating upon a white surface, and indeed, the radiations of the dark colors are apprehended no differently than under the proportion of shadows. If you wish to change the color of the surface, the radiation of colors will indeed be made distinct, by Ch. 1 30, but with the polluted common color of that surface, and so weak that it can hardly be grasped by the eye.

The following caution must also be added. First, if the hole be exceedingly small, things are indeed depicted distinctly and in detail, but just as very small
writing is hard to be read by a weak sense of vision, so here too the eyes, imbued with images seen in the brightest light of day, will have to be held in place for a very long time indeed before they may grasp so detailed a picture illuminated by such a bad light. If on the contrary you open up the hole; the picture will indeed be proportionately brighter and more apparent, but will also be proportionately coarser and more confused. Thus the hole should be of an intermediate size. Next, if the wall is at a great distance from the hole, the picture will indeed be more precisely detailed, as the ratio of the hole to the distance vanishes. But there is in turn this consequent disadvantage, that the luminous colors are more dilated, and, being thinned out, move the sense of vision more weakly; and that the air that is in the chamber, mingled with small dust particles, becomes luminous for a longer distance within, and effaces the picture on the wall with brightness. Thus colors come through hardly to a wall that is too fat, and from a nearby hole strike upon and color a whole paper far more clearly. The same is to be held concerning the air and the distance outside. For even when one beholds remote objects directly, their color, diluted, moves the sense of vision more weakly, and is tinted by the blue color of the large quantity of intervening air, and is obscured: and it comes through to the chamber in the same way.

It is also helpful to place some sort of wall, like a brown, opposite the hole on the outside, to keep the sky or the air from ensuing the part of the wall opposite with too much brightness, and to keep weak lights placed next to strong ones from becoming invisible, or to keep the air inside from becoming too bright and washing out the colors on the wall. The picture will be at its brightest if the sun should illuminate the things to be represented directly, when it is near the horizon.

You will avoid the inconvenience of a conical opening in a thicker wall, by making holes on both surfaces of the wall; thus the surfaces of the wall will remain nearly undamaged.

The reader may expect me to follow G. B. Porta here through the rest of the devices that he has in Book 17 of the Magic, but this does not adduce of method, nor does it further my plan.

Proposition 8

The shape of a ray of the sun or the full moon that passes in through an window with an angular shape, with the condition that the center of the window to its distance from the wall is less than the sun's radius to its distance, approaches gradually more and more to roundness as it proceeds further from the window.

For since the shape for the luminaries, by which they pass into the eyes, is circular, the proposition is therefore evident through 6. And thus it is not true simply, that an angular ray that has passed through angular windows becomes perfectly round as it progresses. For the deficiency impinges on the sense of

\[\text{54}\]

Instead of having one large conical opening in the wall, with the window and its hole on either the outer or the inner surface, one could make two smaller conical openings with their vertices meeting in the middle, and place the window in the middle. One could thus have the same field of view while removing only about one eighth as much material from the wall.
vision, if you look into it more carefully. This false persuasion completely racked the brains of the ancients. Further, it is clear from the same principles that if a window be circular, the ray will be circular; if it be a good sized quadrilateral, the ray will not be a simple quadrilateral, but one of obtuse angles drawn back into a circle.

Proposition 9

When the sun is in eclipse, the image on the illuminated wall will also be in eclipse, regardless of the sort of opening, when the image comes in high enough that the ratio of the window to its distance be less than that of the sun to its distance. For the eclipse of the sun consists of the motion of the moon beneath the sun's body, by which motion one and another image of the sun (but always borne or lanate) is constantly shown from beginning to end. The result will therefore be what we learned in Proposition 7, the demonstration of which, carried over to this case, is also valid. In the same way, eclipses of the moon, and its phases as seen by night, are depicted upon the wall, but less vividly.

Proposition 10

In the image or rays of the eclipsed sun seen in through a round window in the proper way, the horns appear not sharp (as they are in the sky), but drawn back and blunted by the little circle of the window. Let $D A C E$ be the true inverted image of the eclipsed sun, in shape and size such as passes in through the center of the window, by the corollary to 3, and on its edges $D, E, C, A$, let circles $D H, E G, C F, A B$, be drawn equal to the window, by 2.

By 5, the pointed shape of the luminous surface is necessarily mixed with the round shape of the window, while by Corollary 3 of that same proposition, this confusion with the circle, insofar as it is a circle, does not at all detract from the likeness of the shape, but only moves the boundaries of the window $D, E, C, A$, out, so as to make the shape $I G K, F B H$. Therefore, only the pointed horns $C, D$, remain in question, for when they are drawn around following the circumference of the shape of the window, they themselves describe such a circumference $H I, K F$, just as was evinced by 6.

Proposition 11

The diameter of the moon in a ray of this kind appears less than it does outside in the sky. Keeping the previous diagram, let the sector $D A C$ be extended, and let the center of this circle $L$, be joined with $C, A, D$. Let the sector $D E C$ also be extended through $N$, and let the center $A$ of this circle be joined with the points $C, E, D$, and produced to $K, G, I$. Since by 3 Corollary, $D E C A$ is the
exact shape of the remaining part of the eclipsed sun, as it really appears in the sky, therefore \( I A : AE \) or \( LD : DA \) or \( LC : CA \) is the true ratio of the diameters of the sun and moon.

But now, since \( F B \) is everywhere augmented by the semidiameter of the window \( E G \) or \( CK \), two factors therefore cooperate to the same end. First, the semidiameters of the sun \( AD, AE, AC \) are increased and become \( AI, AG, AK \). As a result, by Euclid V.8, the ratio of \( AI \) to \( AL \) is greater than \( AD \) to \( AL \). Second, the semidiameters of the moon \( LD, IA, LC \), are decreased and become \( LH, LB, LF \). Likewise equal because equals are subtracted. As a result, by the same proposition, the ratio of \( AD \) to \( BL \) is greater than \( AL \) to \( BL \). But \( AI \) to \( AL \) was also greater than \( AD \) to \( AL \). Therefore, the ratio of \( AI \), the semidiameter of the sun in the ray, to \( BL \), the semidiameter of the moon in the ray, is much greater than that of \( AD \), the semidiameter of the sun in the sky, to \( AL \), the semidiameter of the moon in the sky. This inequality is quite perceptible, since a large enough window is usually chosen.

Proposition 12

The digits of eclipses appear fewer in the ray than in the sky. For in the previous diagram let sector \( 10K \) be extended through \( M \). Now since \( EN \) to \( NA \) is the ratio of the sun’s diameter to that of the eclipsed part, while \( GM \) is greater than \( EN \) by two semidiameters \( EG \) and \( NM \) of the window, the ratio of \( NA \) to \( MG \) is therefore less than that of \( NA \) to \( AE \). But \( NA \) is equal to \( MB \), because \( AB \) and \( MP \) are equal. Therefore, the ratio of \( MB \), the eclipsed part of the ray, to \( MG \), the diameter of the ray, is less than the ratio of \( NA \), the eclipsed part of the sun, to \( NE \), the diameter of the sun. The ratio is often less by a fourth or a third part, according to the breadth of the window.
Chapter 3
On the Foundations of Catoptrics
and the Place of the Image

1. Refutation of Euclid, Witelo, and Alhazen

At the very foundation of catoptrics, the demonstrations of the optical writers are still clouded, in that they require from sense perception the very thing that was to be demonstrated. Further, it is not true that no error arises from this procedure. In the present Optics, we examine refractions with more care, with a view to eclipses and stellar observation, but to get to refraction we had to pass over this gap. So this place, too, must be filled, the clouds must be driven off, so that the sun of truth may shine more clearly.

In theorems 16, 17, and 18 of the Catoptrics, in order to prove any one of those things that are visible through the perpendicular of a mirror’s surface, to appear upon the surface, Euclid makes a false assumption. Let CD be the mirror, B the observer, A the visible object, AC the perpendicular. That the place of the image of the object A is on AC, namely, at E, he proves thus: “For,” he says, “when the position C of the mirror is taken, upon which the perpendicular falls, the visible object A is no longer seen.” If by “taken” you understand “occupied” (that is, that the position C is covered), the axiom is false, even though Euclid brought this forward to the beginning of the book, placing it among his postulates (or, to put it differently, what he calls διώκειν ἐν τοῦθα in this book) that are borrowed from experience. For even if C be covered, or taken away entirely, provided that D remain, A is nevertheless seen at E by an observer B.

Furthermore, this axiom seems to smack of a false belief about the true and real ascent of the image on the line CE, which matters are indeed considerably at odds with the opinions of certain of the ancients concerning the emission of visual rays from the eye. For it is to this end that Euclid in the first postulate defines vision (βλέπειν, in Greek) to be a straight line, and uses it, taken thus, everywhere in [the proofs about] mirrors. Euclid thus appears to speak with

---

Paralipomena to Wielo

a certain wiser force; and assumes things which, though they are not easily granted, yet once admitted lead to no internal contradiction in what follows. And indeed, in this style of philosophizing, truth suffers violence: there exist false beliefs, which even seem to have stuck to Aristotle, when he speaks of his "visions," 3 from this Euclidean school, 4 which he did not well understand. At length, ignorance set up her tyranny under the guise of Art. "Let us now grant that Euclid's axiom is to be understood differently, so as to state that if the observer were situated at A and C were covered, then A would not be seen. Then the axiom is perfectly true, but the conclusion does not follow from it, except for perpendicular viewing. The argument does not carry over from a perpendicular to an oblique observer.

Alhazen and Wielo seem to have had a sense of this. For when they carefully built forth what they had found in Euclid, they in fact omitted this, on the grounds that it was absurd. However, they say something similar in refractions.

Wielo tries to demonstrate the same thing in Book 5 prop. 36, 5 but this most careful author was hampered by the burden of his times, and by his close relationship with the Arabs, which makes his hard to understand today. Nevertheless, the obscurity of the subject also brought the delusion upon him.

First, I say that he does not do well to argue from the location of the thing seen to the location of the image, that is, out of fear that the image might cease to exist if the image should not correspond to the object in position. And indeed, in this way he would easily overturn all of catoptrics. For many things of this sort are different in the image than in the object. Next, for my part, I do not understand the postulate which he repeats from the beginning of the book, except for the amount of light that the Arab Alhazen reveals here in Book V no. 9-10, from which Wielo transferred his own.

Alhazen first gives a lengthy empirical proof that the position of the image is always on the perpendicular from the object above the surface of the mirror. Then, in no. 9-10, he tries to give the causes of this. However that may be, I ever here do not understand anything any better, except this final statement: "The physical condition of natural things," he says, "looks to the size of their principles.

2 The word Kepler uses here is λήθες, etc.; Greek words were often scattered about in Renaissance Latin works in much the same way that French words and phrases are now introduced into English prose.

3 Although Kepler uses the plural, Greek here, Aristotle seems always to have used the singular in De anima.

4 Kepler has the chronology of these two authors wrong: Aristotle died in 322 B.C.; while Euclid flourished around the beginning of the third century B.C.

5 Artificial: This word corresponds to the Greek πρακτικός, and has a somewhat similar meaning. Here evidently mean: "The rules or theory of an art, a prescribed method or system." (Oxford Latin Dictionary, Oxford 1982, p. 177, third definition.)

6 Wielo V. 56 (Theurake II p. 207) and Alhazen V. 9-10 (Theurake I pp. 130-131) say that the image falls upon the perpendicular for both flat and curved mirrors.

7 Theurake I pp. 130-131, II p. 207.
and the principles of mental things are hidden." 9 By these words he says two things. First, he repeats the very thing that was proposed to prove (for they say nothing different), and second, he says by way of appending the cause, that it is hidden. But this is not demonstrative.

He nevertheless seems to be implying that this location of the image on the perpendicular was long ago thus established by God the Creator because it would be best so, and no more fitting place could be given to the image, which he proves by the sameness of the position, or from the contrary variation. Witelo, who also follows him, i.e., Alhazen concerning the soul, which preserves over vision, comes to suspect that the soul might assign the ratios of mirrors, by some particular scheme of its own. Book 5 Prop. 18. 10

And in fact all these affects are consequences of vision, by material necessity, where considerations of purpose or beauty have no place.

Furthermore, in the plane mirror, Alhazen does indeed abide more nearly in the truth, 11 not in such a manner that a certain instance can immediately refute him. He says that when in image is perceived on the perpendicular, it has the proper magnitude belonging to the thing itself. 12 But I object that it is not necessary than it have the proper magnitude; this is evident in curved mirrors, where the quantity is always changing. I therefore ask for the reason why it should have its true quantity on the plane mirror, rather than in the curved one.

But this fact, that they do not give the same cause of this manner in reflection 13 as in reflection, further strongly confutes the Optical writers.

In Book 19 Prop. 13, where Witelo is about to demonstrate that the image created by refraction likewise falls upon the perpendicular drawn from the object to the surface of the denser body, he faithfully repeats the words of Alhazen. 14 This he did, I believe, because he did not consider it prudent either to touch this sore, or to change rashly things that were not fully understood. They argue from the composition of motions on the diagonal, that it arises from motions parallel and perpendicular to the surface of the denser body. It is hard to see the connection, and even if you admit it, a mathematical deduction of what was proposed to be proved will not be forthcoming. And if it he established that the image plays about the perpendicular, it is not as a result immediately clear that it also falls upon the perpendicular itself.

---

8 No. 14, in Thesaurus I p. 131 line 7-8. This is not, however, "what" in the sense of being at or near the end of the section.
9 Theorema II p. 198.
10 These words italicized by Kepler are all adjacent in the Linn, though it was not possible to keep them so in English.
11 V 4, Thesaurus, p. 130. What he actually said was, "in this position, the truth, both at the point seen and of the image, will appear."
12 Recapitulation.
To Alhazen's opinion, Witeloappendstheviewthatwehadnotedaboveasar
irrelevant and false in Euclid. He says, "If on the surface of a transparent body
a point upon which there falls a perpendicular from the seen object, happens to be
hidden by the interposition of something opaque between the seen object and the
point, the object will not be seen." 13 I say that this is false. For provided that the
point be free, from which the ray from the seen object to the eye is refracted,
the image of the refracting object in the depth will perform seen. And indeed
the first and more obvious image of an opaque object, created by reflection, will
become visible, in that we have just said that for its own part it is submerged
and set in the midst of the water, (a matter which perhaps deceived Witelo); while the
second and less apparent image, that of the object radiating in the depth, which
is established by refraction, will appear in the same place, commingled with
the former image of the opaque object, or a little past it. 15

This passage also provides a refutation of what we have just said that Al-
that used for the consideration of a plate mirror, namely that from the quantity
of the image its position also follows. Thus through the subject of refraction a
great instance arises. It is true that in convex mirrors the image appears to be
both smaller and nearer, in plane mirrors it appears equal to the object itself in
both quantity and remoteness, and in concave mirrors it appears both greater and
more remote. Thus the remoteness approximately follows the quantity here. But
in refraction, where the image is equal to or greater than its object, it gets closer,
and when the quantity of the image is diminished, the image itself departs further,
as may be seen in lenses 16 diminishing the images of objects if you look through
them from a distance.

2. True Demonstration

_Next, in order to make evident the true cause of the place of the image,_
_ignorance of which is a disgraceful stain in a most beautiful science. I should
like to have placed before the eyes from the beginning the chief view of the
demonstration that in plane mirrors the images under which objects are seen

---

14 See above p. 73.
15 The notation is from the end of the demonstration of Witelo X 13, _Declarans H II_
p. 215.
In the adjacent diagram, an opaque ob-
ject at H will block the rays between C and
E, and, according to Witelo, an observer at
R will therefore not see the object at E.
16 Point C in this diagram.
17 We might say, "in a plane".
18 Kepler is comparing the areae image of an object at A with the refracted image of an
object at E. The latter will appear along the line BG. In the diagram is the preceding
footnote, "a little past" the image of A, which appears on BD.
19 Perspicuus.
are not changed by reflection, while they are entirely changed in convex and concave mirrors and in denser media. For when a convex surface receives rays that come together in a wide angle, it turns them back into a narrower angle, while a concave surface reflects into a shorter pyramid those that are hurrying towards a meeting with a moderate inclination. Those rays that are about to go into a denser medium, after being refracted, the surface alters almost not at all in breadth but makes them come together slightly more acutely. In depth, however, the surface acts in exactly the opposite way, bringing them together into a large angle. These general statements, I say, are set up before the eyes in the following demonstrations. Now I make ready the demonstration itself.

Definition I

First, I place at the entrance the definition of the Image, taken from catoptrics, into which we are entering. The Optical writers say it is an image, when the object itself is indeed perceived along with its colors and the parts of its figure, but in a position not its own, and occasionally endowed with quantities not its own, and with an inappropriate ratio of parts of its figure. Briefly, an image is the vision of some object conjunct with an error of the faculties contributing to the sense of vision. Thus, the image is practically nothing in itself, and should rather be called imagination. The object is composed of the real form of color or light and of intensional quantities. So, since the image is the work of the sense of vision, some preliminaries accordingly need to be said about vision. Now the image contains chiefly these four things: color, position or direction, distance, and quantity. For each of these it must be explained by what support of the visory apparatus, and by which reinforcements, they are comprehended, even though Willard has explained this same thing in Book 3 and 4. Nevertheless, we have to row up a little closer to the demonstration that has been established.

Proposition I

So, since seeing is a swelling, and receiving occurs through contact, therefore none of the things mentioned will be comprehended without contact, or something in some way of the nature of contact. Contact is understood here as being between the surfaces of the eye and the image or rays which, by the foregoing, flow down from the objects.

---

20 species.
21 Intensional quantities are quantitative properties of things that can acquire different degrees of intensity. The theory of "intensional quantities," as it was called, was developed in the fourteenth century, most notably by Nicole Oresme (ca. 1325-1382), and was familiar to Galileo, Kepler, and other prominent scientific thinkers of the seventeenth century. Cf. Oresme, Configurations of Qualities, in A Source Book in Medieval Science, Edward Grant, editor (Cambridge: Harvard University Press, 1974), pp. 243-255. Kepler, Astronomia nova ch. 53 and 56, Dondaine trans. pp. 376 and 394.
22 Plutarch.
Proposition 2

Thus, when something happens to the form of light and color in the middle of its path, such as either reflection or refraction by either polished surfaces or those of denser media, the light, on the ground that it is entirely and in every respect removed from contact with the eye, is not grasped by the eye or by the sensory faculties standing by to assist the eye.

Proposition 3

Now let this be taken from the senses as generally admitted: that genuine vision occurs when the folding down of the pupil of the eye is exposed most closely to the arriving ray of light. Thence it follows that vision from the direction whence the light approaches, is rendered more certain by this direction of the eye and of the entire face which is like a support.

Proposition 4

And now this is immediately clear as well, that it is necessary that the sense of vision be in error in making judgements regarding the direction in the world or the position of an object, when the forms or rays proceeding from forms, and falling upon a polished surface, are reflected to the sense of vision in the opposite direction. For when, in the above diagram, the eye B is turned in the direction BD, it cannot become fully conscious of the reflection of the ray ADB. that happens at D by 2 of this chapter. Therefore, it imagines for itself the position of the object at A in the direction BDA, namely, at E: thus the image is, as regards place, torn away from its object. The same judgement shall apply for refraction. For although here the directions are not exchanged for their opposites, they nonetheless differ perceptibly.

Proposition 5

Again, since vision is a receiving, and receiving is between contrary which are of the same kind, it is fitting that whatever dispositions of visible things are to the eye common by virtue of their being of the same kind, all those are to a certain extent perceived by the eye with the assistance of the visual faculties and with the mediation of that disposition.

Proposition 6

First, the eye consists of transparent humors, and so in this respect it is capable of receiving light and colors. See Viere I. 39.

---

25 The word Kepler uses here, "vulgar," was used in classical Latin to denote "A double or front-door (esp. in a temple, palace, or sim.)" (Oxford Latin Dictionary, p. 2089), although by Kepler's time it was commonly used for the "vulgar" of the heart.
Proposition 7

Next, its shape is round, and it varies in the different coverings and horns, within and without. But this visible world is itself concise and round, and whatever we behold of the hemisphere or greater with a single fixed gate, is a part of this roundness. It is therefore fitting that the ratio of individual objects to the whole hemisphere be estimated by the sense of vision, in the ratio of the entering form to the hemisphere of the eye. And this is what is commonly called the visual angle, or the vertex of the visual pyramid within the eye, whose base is in the object itself. For in any single gate, the eye becomes the center of the visible hemisphere. Therefore, it is by means of this surface of its instrument (whether it be internal or external) will be stated below in the consideration of the eye) that the strain of vision measures the angles of viewing. For although all the solid angles are in a point, which is the center either of the eye or of a particular one of its coverings, these nonetheless cannot be distinguished in a point; thus a surface is required, by which the eye may measure the solid angle, as is evident from the geometrical writers. This round shape of the eye is therefore itself sufficient that among art principles we might know, by rough estimation, that the eye has a sense of the angles set up about it.

Proposition 8

Thirly, since to each animal a pair of eyes is given by nature, with a certain distance between them, by this support the sense of vision is most rightly used to judge the distances of Visible, provided that that distance have a perceptible ratio to the distance of the eyes. For if the excess be enormous, that distance is apprehended no differently than any other you please, an unlimited number of times greater. For here it is simply the geometry of the triangle, as is more amply discussed below concerning parallaxes.

For, given two angles of a triangle, with the side between them, the remaining sides are given. In vision, the sensus communis²⁵ grasps the distance of its eyes through becoming accustomed to it, while it takes note of the angles at that distance from the perception of the turning of the eyes towards each other. Now when an object is so far removed that the distance of the two eyes vanishes in comparison with it, the axes of the eyes have a nearly parallel direction. But to the extent that the object is closer, the eyes will be more tuned towards each other.

Now in optics we are determining the principles of the place of the image, not only when we employ both eyes, but also when we are using only one. Nonetheless, this faculty of distinguishing distances, which first arises from this companionship of the two eyes, is later derived for single eyes also, through other dispositions of the eye.

²⁴ "Crassa Minerva," literally, "thick Minerva."
²⁵ The power of perception common to all the senses. Aristotle writes, "There is also a common faculty associated with [all the senses], whereby one is conscious that one sees and hears... and this is closely connected with the sense of touch." On Sleep and Waking, Ch. 2, 455a 16.
That is, it is like what I said before in my rough and everyday imagery, which I shall refer to below in ch. 5, that the nature of the object to the hemisphere is grasped by the eye through the solid pyramid whose vertex is at a point of the eye, and whose base is in the seen object. Similarly, now, when we are estimating the distance of the object the triangle becomes nearly isosceles its point or vertex is (in contrast) in some point of the seen object, and its base is in the distance of the eyes. For the ratio applies to all dimensions of the surface viewed, while the distance is only considered in a straight line. Hence in the former instance a solid pyramid is required, and in the latter plane triangle.

Note and Appraisal

The following propositions 9, 10, 11, 12, 13, 14, are a bonus. The main demonstration could stand without them. Experience bears witness that the image is on the perpendicular, even when we use one eye; with the necessary result that even with one eye the distances of points would be grasped. But although this ability does not lie in a single eye, for the following reasons, still it is consistent that it be there solely because of the motion of the head, by which motion a single eye stands in for two that are far apart.

Proposition 9

That distance measuring triangle can also be considered in one eye: the vertex is in a point of the seen object, the base is in the breadth of the pupil, and in that diameter of the pupil which coincides with the line joining the points of the two pupils. Thus the one eye will learn this procedure for setting up the triangle from the two eyes, by becoming accustomed to it, but in this ratio, that if, because of the distance of the eyes, the sense of vision discards the distances of those things that are a hundred paces away, by the breadth of the pupil it would approach to ten paces, or smaller distances. Although there are also other reasons why a single eye may measure distances, it does so through that diameter of breadth rather than through another distance or depth. For although on the inside the eye is everywhere circualt, on the outside the folding doors of the eyelids are divided along the line of distance of the two eyes, parallel to the horizon. For Nature wished both the distance itself of the eyes, and also the opening of the folding doors as well, to be equidistant from the horizon. She did not at all wish one eye to be higher than the other, or one angle of the eyelids to be higher than the other (in animals of the most perfect vision, that is). This is because the plane of the horizon (coming into the eyes by means of its color) makes the greatest contribution to the imagining of the measuring triangle, that is, of its place, to see whether the one is parallel to the other at its base. For this reason, when we gave upon things with our head inverted or at an angle, we use the discriminating faculty in a more muddled way.

Proposition 10

For that reason also, in cases narrower, and in the flat surface of denser media, we strive, with nature's aid, to make the twice from both eyes strike upon the surface with equal angles (or vice versa, for geometrically it is the same thing). For this faculty of setting up the measuring triangle is common to the
two eyes together and to each eye separately. There is a certain coordination of motion, of such a kind that with the turning together of the axes of the two eyes there arises simultaneously some new disposition in the individual eyes (for the sake of the argument, and in a rather general sense, let it be granted for now that the \textit{axis tunicæ} \textsuperscript{26} in the individual eyes is either wrinkled up or stretched out). Therefore, it happens that when the two eyes are directed at the same surface, whether reflective or refractive, and one of them is higher, the wrinkling of the axes must necessarily differ, both from the turning together of the axes of the two eyes, and from the wrinkling of the other axes, because of the difference of refraction, as will be said below. \textsuperscript{27} However, if one tries this, first, he obtains it without difficulty, for he again and again perceives two images in place of one; and second, he does great injury to his eyesight and induces headaches.

\textbf{Proposition 11}

However that may be, the external configuration of the folding doors of the eyelids is a sign rather than a cause of this facility in the individual eye: one must reflect on the internal and authentic cause. Moreover, I have just given one (the wrinkling up of the axes and the contrary dilatation), which would be sufficient, but for the assertion of the Physicists, that this motion in this tunic exists because of the abundance or weakness of light, for which seech 5 below. \textsuperscript{28}

And so, insight \textsuperscript{29} it should be considered that in the humors of the eye there is a certain ratio of density and tension, well known to the eye itself on its own. Thus, since both air, by whose mediation vision takes place and light, which is seen, have established their ratio of density above, it is fitting that there be in the eye the power of measuring either the density or ratio of both the air and the light.

\textbf{Proposition 12}

But we shall steer into the light a number of arguments gathered from the air. For the eye seems to be nearly cut off from the function of seeing when the transparent tunic of the eye, as when the eye is submerged beneath water. Thus it is made completely transparent is the kind of animals capable of breathing, for perceptions objects in air, not in water. For that reason, fish are endowed with a thicker and harder humor, almost bone rather than humor, in order to see through the waters. It is therefore reasonable to believe.

\textsuperscript{26} The “grape-like tunic.” However, what Kepler means here is not the tunic as a whole, but the posterior part of the iris which is sometimes named tunic, from its resemblance in color to grape grape” (Grey’s Anatomy (Philadelphia: Rummey Press, 1924, p. 831)). Kepler’s description of the iris, derived from the anatomical studies of Felix Platter, is in Chapter 5 below, p. 177.

\textsuperscript{27} The change in dilatation of the pupil with convergence of the eyes, first noted by Kepler, is known as the “convergence reaction.” It is described in standard physiology texts, e.g., Barbara R. Laudan, \textit{Essential Human Anatomy and Physiology}, p. 249.

\textsuperscript{28} See p. 187.

\textsuperscript{29} Cf. propositions 6–8.
that animals of the highest rank have an albinous humor in the eye for the very reason that there is a difference in the transparent media of the air and the eye. For had she not kept an eye on this, Nature would have been able to send light into the open hole in the eyes. The same thing, however, may also be brought to light from the shape of the eye. For since it is round, many things happen when the luminous rays enter refracted from the air into a denser round body, which could not occur when the eye is flooded with waters, where, when the albinous humor matches the surrounding medium the round surface of the albinous humor is not filled. At least, therefore, there is in the eye a perception of this airy thinness, in that way in which we have a deeply intird perception of all the objects to which we have become accustomed. See Wiela book 3 pop. 3 and 4. Above, however, in prop. 6 of the first chapter, we attributed to both light and the surface of the air the same condition of density; therefore, the eye too will perceive the density of light. And this happens all the more because, as was made plain in prop. 20 of the first chapter, in relation to this density, light receives something from all media that differ among themselves in density or transparency, and thus also receives something from the humors of the eye: why therefore should the eye or sense of vision in general not also receive something contrary from the density of light, likewise in relation to its own density?

Proposition 15

Fifth, if (by the preceding) the eye perceives the density of light, there must also be a modification and condensation of it, so that it must sense the attenuation that in fact is going on inside the eye and in a way, happens to it, since the humors in addition are expanded in depth, through which, by Ch. 1 Prop. 6, this rarefaction is carried out. For while light passes through this depth, it is spread out in a certain proportion (other things being equal, and ignoring the compaction that occurs through the configuration of the denser humors), and comes out more attenuated in the depth (as it was when it first entered. Nevertheless, the eye might also be able to determine the rarefaction of light that occurs within the eye by using the evidence of the broader illuminated surface in the depth, since by Prop. 7 above we attributed to it a perception of the quantity of the surface touched by light.

Proposition 14

With these things thus proved by the four preceding propositions, I would now like to persuade you, with a geometrical demonstration of what was established only probably, that by a single eye, with the assistance of the higher faculties, and through the senses upon the eye’s humors by the density of light, as by an agent, the distances of points of a seen object, to which distance the diameter of the eye is perceptible, are perceived and computed. For, first, any

---

8. Wiela 3.3: "The organ of the visual power must necessarily be spherical." 5. "The eye is the spherical organ of the visual power, composed of three humors and four layers, arising from the substance of the corneum, intersecting each other spherically."

12. The antecedent of this pronoun is obscure in the Latin.
point whatever radiates in an orbit by 3 of the first chapter, and thus it also radiates in the breadth of the eye, which latitude, be it of the whole eye or of the opening of the eye, is by hypothesis not imperceptible. Next, the sense of vision perceives both the density and tenacity of the images or the light that have arrived from the point through the rays, by 12 of this chapter, and the quantity of the attenuation of these same things within the depth of the eye, by 13 of this chapter. Further, it grasps by habituation the depth of its own humor. Let $c1$ be the diameter of the pupil; $b1$ parallel to it, as far away as the depth of the eye allows. Next, from the boundaries $a1$ and $b1$, let $e1$ and $f1$ be spread with a lesser inclination. $c1$ and $f1$ with a greater one; and therefore, $g1$ and $h1$ will meet at a more distant source $h$, while $e1$ and $g1$ will meet at a nearer one $k$. For since the interior angles of the greater ones are, $g1k1$, when by Euclid I, 32, they are subtracted from two right angles, the remainder $g1k1$ of the former will be less, while $g1k1$ of the latter, will be greater. Thus, by Euclid I, 21, $g1k1$, $h1$ will meet at a more distant point $k$, and $e1$ or $h1$ will be greater than $g1k1$ or $k$. Therefore, since the eye grasps the diameter $c1$ and the depth $e1$, and since it is sufficient to observe the ratio of $h1$ to $g1$ and each of each to $c1$, either by the attenuation of light, or by the very small illuminated part of the inner surface, it will consequently observe $c1$ or $h1$, not indeed, by measuring, but by comparing the distances of the object through this habit, as it were, with the powers of its body, and the extension of hands and of places.

These things are sufficient to establish that three things, we are now just about to demonstrate concerning the partnership of the two eyes, be understood as demonstrated also for the diameter of breadth of the single eye, which is what we had proposed above.

Proposition 15

And the comprehension of color, direction, and distance has been discussed; what remains is the legitimate comprehension of quantity, which I would account for in one word. For it follows upon the comprehensiveness of the angle and of the distance, where the sense of vision, from the sides that come together in the eye and the angle set up between them, makes a judgment about the base of the pyramid, which is the quantity of the object seen. Now let us make a nearer approach to the subject of the place of the image.

Definition 2

The surface of reflection of reflection, the optical centers define for the surface determined by three planes (or it is postulated that a unique plane surface is always applied to three points not placed in the same straight line: the center
of vision, a point of the seen object, and the point of reflection (or of refraction).

In the above diagram in No. 1, it is determined by the points D, A, B.

Proposition 16

This kind of surface, they demonstrate, is necessarily erected perpendicularly above the reflecting or refracting surface. Witsel Bk. 5 prop. 25, Alhani Bk. 4 no. 14. The former carries out the demonstration using prop. 20, and the latter using no. 10, of the same books, namely, that the angles of incidence and reflection are equal, and they at length derive the cause from nature acting through shorter lines, which they had abstracted from Euclid and Ptolomy. And indeed, these operations are not of a form that acts deliberately or keeps a goal in mind, but of matter bound to its geometrical necessities.

Furthermore, we also perceive the images of stars in pans by the same laws of reflections, where the comparison of lines completely attests. Finally, if the demonstration were genuine, it would also follow in the consideration of refractions. However, Witsel, Bk. 10 prop. 1 and 2, and Alhani Bk. 7 no. 9, change the form of the demonstration here, for it is ineffectual here to argue thus from least lines. For we it is enough to establish that reflection takes place in the opposite direction, and refraction towards the perpendicular drawn from the radiating point to the refracting surface. The former is from Ch. 1 Prop. 19, and the latter from Prop. 20 of the first chapter. Each of these we deduced from its own proper cause, from which also followed at the same time the equality of the angles of incidence and reflection. Here we shall introduce in a general manner the argument of which Witsel makes use for refractions in particular, insofar as it can be gathered from that darkness.

Now, since directions are arranged in an orb, let there pass through the point of reflection A the pair of straight lines BC, DE in the reflecting surface, indicating the four directions. Now let the surface of reflection BFGC pass through one of these lines BC. If this is inclined on the reflecting surface DCEF, it will as a result be inclined to one part of the second line DE. Let it be inclined towards D. Therefore, both the incident FA and the reflected AG will veer towards one direction D of the second line DE, which.

33 Themaios II p. 202 and I p. 111, respectively.
34 Kepler did not know of the Optics of Ptolomy. It was cited by Ambrosius Rhodius in 1611, and then lost until it was rediscovered by Laplace in Paris. Cf. Hammer’s note in JGDR II p. 440.
35 That is, the star is so far away that a change in the angles of incidence and reflection will make no difference in the distance traveled by the light.
36 Themaios II pp. 405-6, and pp. 242-3.
37 Note that BC and DE define the surface of the reflecting medium thus, BC should be imagined as coming out of the plane of the page perpendicular to DE.
as we said above, should not happen. For it is required that \( FAD \) and \( GAE \) be equal, no less than \( FAB \) and \( GAC \). Therefore, it is necessary that the surface \( BGFC \) be on the perpendicular to \( DCEB \), so that the angle of incidence \( FAD \) veers to one direction of the second line \( DE \) to the same extent that the angle of reflection \( GAE \), equal to it, veers towards the opposite part \( E \) of the second line \( DE \).

In refractions, the same thing is evident more simply. For since refraction only occurs towards the perpendicular, the line \( CR \), in the diagram of proposition 20 of the first chapter, which is drawn in the surface of the denser medium \( HC \) from the point of incidence \( C \), perpendicular to \( AC \), to the point of refraction \( B \), will extend parallel to that which, coming forth at right angles from the extended perpendicular \( AC \) at \( E \), at any depth whatever in the denser medium, intersects the refracted line \( BG \). Unless this were so, the refracted ray \( BG \) would not approach the perpendicular \( BF \) or \( AE \) directly, but would at the same time veer in other directions. Therefore, by Euclid XI 7, \( AC \), perpendicular to the refracting surface \( HC \), is in the same plane in which the refracted line \( BG \) also is. And the plane surface in which the refracted line is, is called the "plane of refraction." Therefore, the plane of refraction, since it contains \( AC \), perpendicular to the refracting plane \( HC \), will be perpendicular to the same refracting plane, by Euclid XI 18.

Proposition 17

There was a further difficulty in establishing principles. The demonstration itself is straightforward. First, the sense of vision err in direction, as was said in prop. 14 of its third chapter; it imagines for itself an object in the same direction whereon the refracted or reflected ray approached. Next, the sense of vision also err in the angle; for it imagines for itself that the inclination by which the refracted or reflected rays proceed all the way to the centers of the two eyes, is also the same as the inclination or angle by which proceed those rays which approach from the radiant point to the points of the reflections or refractions, corresponding to the eye, by 2 and 7 of this third chapter. And the genuine place of the image is that point in which the visual rays, from the two eyes meet, extended through their respective points of refraction or reflection, by 8 of this third chapter. Now the visual ray of either eye (the luminous line drawn out by the imagination from the eye continuously through the point of refraction or reflection in the same surface with the surface of refraction or reflection, by def. 2, and either eye has its own surface of refraction or reflection, by the same. Therefore, where the visual rays of the two eyes meet, the surfaces of the refractions or reflections also meet, each one passing through its respective eye. Therefore, since the place of the image is in the meeting of the visual rays, by the first definition, it will be in the meeting of the surfaces of refraction or reflection of the two eyes. But those surfaces meet at a visible point of the
object, by definition 2. And since these two surfaces of refraction or reflection are perpendicular to the refracting or reflecting surface, by 16 of this third chapter, and mutually intersect each other, their common intersection will first of all be a straight line, by Euclid X 3, and consequently the object seen and all its images will be in the same straight line, since all are on the common intersection. Furthermore, by Euclid XI 19, this common intersection will be perpendicular to the same refracting or reflecting surface. Therefore, again, all the images of the seen object will be on the perpendicular from the object to the surface, whether refracting or reflecting; and this will happen to such an extent that the distance of the points of the seen object is grasped in the manner described, whether by the two eyes, or by the diameter of the breadth of one eye. And since (in particular) the surface cutting a sphere perpendicularly passes through its center, the common section of two such, that is, the line of the images and of the visible object, will therefore also pass through the center. Therefore, it is not the occult nature of light, not the mind of universal nature, but the breadth of the sense of vision alone, that harmonizes with the causes for the sense of vision's placing the image on the perpendicular.

Proposition 18

If you were to apply the sense of vision otherwise, so that (contrary to nature) the refractive or reflective surface were the same for the two eyes, an entirely different thing would happen. For in convex mirrors and in denser media the images will depart from the perpendicular and will approach the observer. Which, as it is readily to be explored by anyone, and has also been proven to me through experience, I thus shall now demonstrate it in the cause of light.

First, let $EG$ be a convex spherical surface with center $L$, a radiating point at $D$; the vision on $CH$, collinear with $L$, so that the surface of refraction through the two eyes $C$ and $H$ is the same. Let the points of reflection be $E$ and $G$. Let $DE$, $EC$, $DG$, $GH$ be joined, and let $CE$, $HG$ be extended until they meet. Let the point of meeting be $S$. Also, let $E$ be drawn touching the circle at $G$ and $E$, and let them be $AO$, $OK$; and let the points $G$ and $E$ be joined with a straight line, which shall be extended in both directions to $M$ and $N$. Now, since $DEK$ is equal to angle $CEO$, while $DEN$ is less than angle $DEK$, therefore $DEN$ is also less than angle $CEO$, and $DEN$ will be doubly less than the greater angle $CEM$. And since $DGO$ is equal to angle $HGA$ (by the law of reflection), while $DGE$ is greater than $DGO$, therefore $DGE$ is greater than $HGA$, and doubly greater than the lesser angle $HGM$. But $EDG$ is the difference between the greater reduced angle $VED$, and $EDG$, the lesser but made greater. On the other hand, $EDG$ is the difference between the greater $CEG$, made greater by the same measure, and

39 Refractive vs. Reflective. The terminology is becoming confusing, if not confused. This surface is clearly the same as the “surface of reflection or refraction” of Definition 2, as distinguished from the “refracting or reflecting surface” (Proposition 16, enunciations).

39 That is, $NED$ is less than the angle of incidence $BEK$ (and hence is “reduced”), while $EGB$ is greater than the angle of incidence $DGO$ and hence is “made greater”.

70

71
the lesser HGM, reduced again by the same quantity. Therefore, ESG is much greater than EDG. And ESG is the same angle in CSH in which the reflected rays CE, HE meet when extended, and S is the place of the image of point D, by S of this third chapter. But EDG is the angle with which the rays depart from the point D. Therefore, the angle at the image is greater than the one at the radiating point. Therefore, the image will appear to be nearer than the radiating point. By Euclid II.1 I say that S is between DL, the perpendicular from the object to the center, and the perpendiculars through E and G, the points of reflection.

Let EJ be extended until it cuts DL; let it cut at T; and let the points E and G be joined and V the place of the image be at T. Further, let GO be extended toward it cuts DL at V. Also, from the center L let straight lines be projected through the points E and G indefinitely out, and afterwards let a parallel to TE be drawn from D to LE, cutting it at X; Likewise, in a parallel to TG be draw from D cutting LG at Y. Thereupon, if T is the place of the image, c will follow not only that the angle EFT is cut in two by FK, the tangent to the circle at E, but also that the same thing is done to DGY by GV, the tangent to the circle at G, because DEK and KEF are each equal to angle CEG. Likewise, DGV and VGT are each equal to the angle at G, because GHA. Consequently, since EK and KEF are each equal, when equals are subtracted, the remaining XEP and TEF will be equal. But EFA and DXE are equal, because DX and TE are parallel. Therefore, DEK is equidistant between DEK and DXE, DKE and DX and DL opposite the equals. Therefore, in triangles DXY and TFE the sides are proportioned, by Euclid VI.4. Consequently, DL is to LY as DX (that is, DEK) is to ET. But by Euclid VI.3, DF is to LT always DEK is to ET. Therefore, DK is to KT as DL is to LT. But by the same consideration it will be demonstrated that YDG is bisected, and DL is to LT as DV (that is, DG) is to GT. And because, by hypothesis, DGV and VGT are each equal, DV will be in VT as DG is to GT. Therefore, DV is to VT as DL is to LT. Perpendicularly, however, DK is to KT as DT to LT. And, alternately, as the greater, DK is to the less, DV, so is the less, KT, to the greater, VT, which is obviously absurd. And so it
Paralipomena to Vitelo

is false that the vertex $S$ of the visual pyramid $CSH$, whose base at the observer is $CH$, is on the perpendicular $DL$, or that it goes all the way to $T$, as we had previously supposed, and from which this absurdity followed. And thus this is true of all inclinations that are between the perpendicular and the horizontal. But the perpendicular radiation itself, reflected to the sense of vision, moves it, think that the image is on the perpendicular, while the horizontal radiation moves the image away from the perpendicular $DE$ more and more towards the points $E$ and $G$, to such an extent that there is given a position where the line $HG$ from the lower eye, extended through the point of reflection, $G$, cuts the circumference $GE$ again, before it meets the line $CE$ drawn from the other eye to the other point of reflection.

From these things it thus appears not to be universally true that the place of the image is on the perpendicular, unless this restriction also be added, that the sense of vision be so located with respect to the mirror as nature shows.

In concave mirrors the same is illustrated in a contrary manner. For judgement concerning contraries is the same, other things being equal.

There is one further thing I should add. Whatever of this that can be gathered by a single eye, is either imperceptible, when the arc of the circle of reflections, varying the visual angle, is always less than the actual pupil of the eye, and thus the difference between the two reflections is extremely slight; or, if the two eyes were to come so near the mirror, and were to be so greatly inclined, that some large arc lie between the points of reflection, the sense of sight is hindered, as was said before, and impaired, because the proportion between the axes of the two eyes and the narrowing of the opening in the aea has been disturbed.

Proposition 19

It must now be demonstrated that the same thing also occurs in refractions which experience most plainly declares: that when both eyes are in the same surface of refraction, and are observing from a very oblique angle, the image departs from the perpendicular and approaches the eyes, provided that the space intervening between the eyes be moved near enough so that a perceptible difference is interposed between the refractions at the two eyes.

Let the circle $EDAB$ cut the straight line $AB$ at the points $A$ and $B$, and join these to $D$ and $E$, which are any points of the circumference lying on one side. These connections will be the lines $DA$, $DB$, $EA$, and $EB$, which shall be extended indefinitely in the directions of $A$ and $B$, to $F$, $G$, $K$, and $J$. Thus $DB$ and $AE$ will cut each other; let the intersection be $H$. Therefore, by Euclid III.31, $ADB$ and $AEB$ will be equal. But $DHA$ and $EBH$ are also equal. Therefore, the remaining angle $DAH$ will be equal to the remaining angle $HBE$. But $DAH$ and $FAK$ are equal, as well as $HBE$ and $GBI$. Therefore, $FAK$ and $GBI$ are also equal.

Now, with these things being supposed, let the sense of vision be at $F$ and $G$, and let all these points by supposition be on the same surface. Let the denser medium be the straight line $AB$; the point emitting the rays be at $F$: $FAE$ and

---

40 Reading "reflexions" for "reflexions".


Chapter 3

99

GRE be the refracted rays. Thus if the refractions at A and B were equal, as KAF and JBG are here made equal by constriction, it is clear from the above that the image of the point E is going to be at D. That is, at the meeting of FA and GB. And since the points of the circumference towards A are always nearer to the perpendicular from A above AB (provided only that they are between A and the point of the circumference where the perpendicular to AB touches the circle), D will always be nearer the perpendicular than E. Therefore, on the supposition that the refractions at A and B are made equal, the image is the denser medium approaches the eye from the perpendicular, drawn from the observed point E to the surface BA, doing so near the horizontal, since the angle FAB is small.

For the rest, because the obliquity of the incidence EAO to AB is the cause of refraction, by 20 of the first chapter, when the cause increases, the effect will increase, and vice versa. Accordingly, the refraction will be less at A, because EA is less inclined away from the perpendicular, and at B the refraction will be greater, because EA is more inclined. Therefore, since KAF is less than JBG, DAE is also less than HBE. Therefore, ADH is greater than HEB. Therefore, ADH and the place of the image B is the same inside the circumference, and approaches A nearer than the point D of the circumference, by Euclid 12. Before, however, D itself approached nearer to the perpendicular from A than the visible E. Therefore, the image of E approaches much nearer the perpendicular at A that the visible E itself, receding from the point E. Which is what was to be proved.

I: at points of the circumference more remote from A than is the point where the perpendicular to BA touches the circle, is does indeed happen that on the supposition of equal angles of refraction, the perpendicular from E becomes closer to A than the perpendicular from D. But because the angles of refraction are unequal, D, where the angle is greater, again approaches and forsakes the circumference.

And in general, at this great a proximity to the perpendicular, the whole angles of refraction are slight, and thus the differences of these angles are much less perceptible. Accordingly, there is no departure of the image from the perpendicular that can be grasped by the senses, where the sense of vision perceives the object nearly perpendicularly beneath the surface of the denser medium. For other exceptions concerning the place of the image, see Chapter 5 below.

Proposition 20

Hitherto, then, it has been shown that the theorem of the optical writers is not completely general. Thus at the beginning it was more accurately determined, so that the predication might be matched to the subject in breadth [of applicability];
and the instances were shown that there are exceptions to the universal rule of the optical writers. But so that you may see that there is nowhere that errors do not sport out through ignorance, behold now also what sort of difference there is between my conclusions and those of the optical writers constructed upon this theorem. They thought that the image put up a struggle to make its way by the direct path to the surface. Accordingly, even in conical mirrors they give guidance by the example of Alhazen Book 3, Wells Book 7) of drawing the perpendicular from the object to the surface of the conical mirror, quite otherwise than what holds true. For it makes no difference to the place of the image, what sort of mirror surface is placed opposite the object, since the proportions of image formation are all taken from that part of the mirror upon which are the two points of reflection of light on the two eyes. So it is at this part of the mirror, not at the actual perpendicular from the object, that the case of the image's place being on that perpendicular lies. And so one should understand mentally the continuation of the pointers of curvature that had created the reflection on the whole circumference, and above this imaginary sphere one should also draw a perpendicular from the object for defining the place of the image. To make it apparent how great a difference there is in the outcomes of the two opinions, take the following example.

Let there be a parabolic section, the common section of a conical mirror and the plane surface of reflection (by the definition of a parabola in Apollonius), or even [the common section of a rectangular conical mirror and the surface of reflection] (by Archimedes, On Conoids, prop. 12). Let the resulting section be $QY$, and let the point $B$ be the place where each eye (although in a pair of such sections) receives the refracted rays. Accordingly, let the section then be touched at $B$ by the straight line $QD$. And let the sense of vision be at $Q$, and the visible at $Q$, so that $QD$ and $QF$ are equal. If you seek here the place of the image from the optical writers, they direct you to seek the point of the section upon which falls the perpendicular from $Q$, let this be $\gamma$. After $Q\theta$ is drawn touching the section at $\gamma$ and the points $\gamma, \eta$ are joined, they direct you to extend $\eta \gamma$ until it meets $Q\gamma$ let it meet at $\lambda$. They will then say that $\lambda$ is the place of the image of the point $\eta$. But a true reason directs you to find the circle which

---

41 *Passivum*.  
42 *Natio*.
contains the pattern of curvature that the section has at the point of reflection $\beta$. Mixed lines have, moreover, one and another of this sort of circle. Let its quantity be $\xi \beta$, and a perpendicular to $\xi \beta$ be drawn from $\beta$, call it $\xi \beta \alpha$; the center of the circle will be placed upon the line $\xi \beta, \eta \beta$ will be joined, and the place of the image will be where $\xi \beta$ extended cuts $\eta \xi$, namely, at $\mu$. Here you easily see that even if $\beta$ remain the place of reflection, and $\xi \beta \alpha$ also remain the pattern of curvature, and $\xi \alpha$ the eye, while the visible, $\xi \alpha \eta \xi$ moves farther away upon line $\eta \xi$, the perpendicular from $\eta \xi$ is always going to fall upon a point farther away from $\eta$, and the place of the image will be correspondingly farther from $\xi$, towards the outside. And indeed, this difference finally runs off into infinity. Now it is very important for the account of mirrors not to be kept suspended in doubt here.

But enough on the place of the image, which consideration was to be sure entirely necessary for what follows. Be indulgent, O reader, if we have anywhere been more concerned with the Paradromes to Wieldo than with astronomy, I know that the natural method requires that in the beginning the nature of light and its modifications, reflection and refraction, be treated; second, the eye, which depends upon refraction; and finally, catoptries or the image, which is the companion of the visual faculties. Had I followed this method, I would have been able to forgo in this chapter many things that are unessential and that are taken as known by anticipation; and even the coherence itself of the propositions would have been established with tighter bonds, and a more geometrical form. But it seemed to me preferable for gaining the reader’s trust to follow the actual series of my investigations at the same time to place these catoptrical paradromes in subjection to the explaining of refractions, and so to drag these as well as to some extent into astronomical servitude. Otherwise, contrary to the book’s purpose, they would be set up on a royal throne, and would appear to embrace the full scope of the whole work, which is what would have happened had I followed the natural method.

25 Latinism.
26 Lines may be categorized into straight, curved, and “mixed,” which contain something of both of the other kinds, they are curved, but their curvature is not constant (cf. Kepler, Mysterium cosmographicum ch. 2 and Kostolac, De Carlo 12). Hence, the circle of curvature, a concept which Kepler appears to have invented here, is different at different places.
27 Reading visible as visible.
28 Restates specularis.
29 This is similar to Kepler’s professed method in the New Astronomy, in which he was working at the time of writing the Optics. There he wrote, “The scope of this work is not chiefly to explain the celestial motions, for this is done in the books on Spheres and on the theories of the planets. Yet it is to teach the reader to lead him from self-evident beginnings to conclusions, as Ptolemy did as much as he could. There is a third way, . . . that is, an historical presentation of my discoveries. Here it is a question not only of leading the reader to an understanding of the subject matter in the easiest way, but above all, of the elements, understandings, or even chance occurrences by which the author first came upon that understanding.” (Donahue trans., p. 78).
Chapter 4

On the Measure of Refractions

It has been important to astronomy, for the attainment of certainty, to establish the angles by which the rays of stars are refracted from the straight path. It is now also important for the attainment of beauty, to know the causes of this increment of angles. Nor, indeed, is it without use to see whether there is something of certainty underlying this matter, so that we may dare to make pronouncements all the more confidently about whether the refractions are the same at all places. Now that this is attained, very important observations of the ancients must now be treated far differently than if no consideration were given to refractions.

1. On the Debate between Tycho and Rothmann upon the matter of refractions

A long time ago Alhazen the Arabian, and Wietel, following him, set out to explain the matter of refractions more carefully than the ancients had been accustomed to do. And since all our ideas derive originally from experience, they began by investigating with instruments the quantities of those angles with which the rays entering water from air are refracted, then both those from air to glass, and those from water to glass. And since the matter of the heavens, from the opinion of the ancients, was thought to be nearly glassy, that is, crystalline, while air was akin to water, the authors, carried away with boldness, began with the aid of refractions to seek into the heavens' intimate secrets. Experience aided their attempts: a certain ratio of refraction was discovered even in the stars, and it was such that from it, by those experiments that were now confirmed in water and glass, it appeared to be possible to declare that aether is not denser than air, but much rarer than it. This carelessness was long neglected, but after several centuries Tycho Brahe arrived on the scene, who set out to measure with exceedingly accurate instruments the angles of refraction in air, a matter that Wietel had neglected. Contention about this as well as about many other discoveries arose between him whom I have mentioned (Tycho) and Rothman, the Mathematician to the Landgrave of Hesse. There is much controversy on

---

1 Kepler is evidently thinking here of the author of the book whose Latin title is Liber de circumpunctis, which was written, not by Alhazen, but by Abu ‘Abd Allah Muhammad ibn Ma沙rid. Cf. Lalande, Theories of Vision, pp. 209 and 281, and the sources cited therein.

2 This may be the earliest published statement that the ancients believed the matter of the heavens to be crystalline. Although the Latin word 'crystallinum' was used in earlier astronomical texts, it denoted specifically the sixth sphere, the 'circular crystallinth,' which represented the biblical 'waters above the heavens.' when, following Ezechiel 1,22, were thought to be contained in a 'terrible crystal.' Aristotel, whom most medieval writers followed, believed the heavens to be made of a substance that had nothing in common with any ordinary material; cf. De caelo 1,2-3 and Grant, Planets, Stars, and Orbs, pp. 324-370.
refractions in the *Epitome astronomiae* vol. 1, which Tycho published in 1597, and anyone who wants may find it there. For the present, I shall add a summary. Tycho had previously given a notice about being aware of refractions when taking the sun's altitudes. In passing, he assigned the cause to the differences between air and aether. as he had read in Wietelo. Rothmann seized the opportunity, and denied that there was any difference between air and aether by the very fact that no refraction occurred that could truly be assignable to this cause. For he said that, spheres intersected in a circle, even supposing they are different; refractions, as is in accord with this, do not occur throughout the entire circumference of the circle, but only near the horizon. Refractions therefore, do not come from this difference of spheres, but from a lower cause, which does not exceed an altitude of twenty degrees from the horizon. The procedure of enquiring into twilight (which accompanies the sun up to a depth of 18 to 24 degrees) was helpful. He concluded, as a result, that the rays of the stars were bent in the matter that provides the occasion for twilight, which natural philosophers, with the geometrical art, prove not to exceed an altitude of twelve German miles.

Tycho was not neglectful of this case. First, he stipulated that the cause of the refractions appeared to him also to be trifling; the first cause arises from the difference between the denser medium of air and the rarer medium of aether, and the second, which drives refractions near the horizon so greatly downwards, does indeed reside in the vapors about the horizon. And he said that it is not sufficient, for blurring the distinction between air and aether, to demonstrate that no refractions of rays occur near the zenith, at least more that can enter into sense perception; for there can be other ones that are imperceptible, which would be rather small even at the horizon, were they not assisted by some other cause. But it grows suddenly when the stars approach the horizon, because of the vapors, under whose control the guiding principle of this cause is. Or indeed, from the very thing of which Rothmann made note, it appears that the transit for rays of the setting stars through the air 12 miles high, is six times as far as what it is for stars at an altitude of 30 degrees. So he thus far granted that

---

1 The *Epitome astronomiae* was published in Uebrich in 1596 and 1597, as Kepler states. It is reprinted in 1600 vol. VI. Kepler's summary does not exactly follow the sequence of the published text; however, references on this edition for some of the more important points are provided below.

2 Cypriani. Of this, Kepler writes in the *Epitome astronomiae Copernicae* vol. I (1601), Part 3 p. 72. JUERG VIII p. 669. For astronomers, this is that entire time that comes between the first lightness of the air noticeable by the sense and the actual sunrise; or, in turn, between sunrise and the last trace of daylight in the air. The part of Rothmann's letter summarized here is in 1600 VI pp. 111-2.

3 Physic. To translate this as 'Physicus' would be misleading, since Kepler is referring to those who, in the Aristotelian tradition, studied nature or nature.

4 The German mile was reckoned vs five Roman miles, and hence as about 7.4 km. Tycho took the figure for the height of the atmosphere from the *Libri de corpore*, Prop. 6 (for this book, see the footnote on p. 93 above).

for measuring the angles of refractions, the depth and the redoubled vapors, set obliquely to the stars, combine. This he repeated on pp. 92 and 95 of the _Prognosticorum_, with, however, no added correction. Here he also opines that some very small refractions art removed from sight by the exceedingly great distance of the heavens: a view which is somewhat foreign to the way refractions work. Rothmann responded that this is not at all the way that vapors become the cause of refractions. For he said that on the perpendicular they are 12 miles in depth; on the horizon, from the side, twelve times deeper; at an altitude of 30 degrees, six times. It will therefore happen that at an altitude of thirty degrees the angle of refraction will still be one sixth of the horizontal refraction (which even Tycho himself would condemn, since he had measured these angles otherwise from experience), and furthermore, even right near the zenith some refraction will occur. And thus, following his own opinion he presented another way that established vapors act as the cause for refractions: there is a determinate space for the rays of the stars such that they can pass through the vapors unaffected, and so if they arrive at some place of the terrestrial surface through the vapors by a path that is shorter than is this space, they are minimally refracted. But so far as the passage from the side exceeds the established space, the rays are fully refracted. He hoped that in this way he had brought the matter to a satisfactory conclusion. Much was said pro and contra, and the dispute is so involved that I barely could disentangle myself. The opinion in which Tycho remained after this discussion with Rothmann, you have in the _Prognosticorum_. Vol. 1 p. 92. For the rest, as usually happens among the principles of things to be established, the channel is blocked for each. For if they had applied the genuine measure of refractions, it would not have been necessary for Tycho to introduce a twofold cause of refractions, by which I mean the two bodies, the one of air and the other of vapor; nor would Rothmann have denied that light is refracted by some imperceptible amount, even near the zenith. Finally, it would have been apparent that the

---

5 _TBOO_ II pp. 77–8.
6 Letter of 19 September 1588, _TBOO_ VI 1 p. 151
7 _TBOO_ II pp. 77–5.
8 Here Kepler has the following note. To p. 79 and p. 83, and to prop. 9 from p. 136 to 120: Vapor as a whole is of two kinds. One, when it arises from the bowels of the earth, conveyed upward by a certain terrestrial heat; for then it is the nature of ascending, bursing forth from mountainous lands like a fountain, and from the height of the birching forth it is poured down in a circle upon the downward slopes as soon as it is elevated by the mountain cold, and most of the time generates winds in this way. And thus in this state it is immaterial, like water. For that reason, the lights of the stars then appear large and twinkle a lot: by this sign, winds, and the relaxing of cold in winter, are known to be in land. The other sort is of a cohearing, cooled vapor, which stays inert in one place, mixed with thicker smokes, through which the lights of the stars appear dark, reddish, and (as Virgil says [Georgiis 4.93]) 'somber colored', which is by no means transparent. The former kind is rare and ephemeral, the latter common and permanent. The former
2. Refutation of various authors’ various ways of measuring refractions

The means and measure of refractions, even by itself, is established at a high price, and thus, reader, you may not be admitted without adverse consequences: not without first being led through the same briar path of enquiry that I myself have crept through, on the grounds that since you are going to partake of the common fruit, you should pour out your labor as a first libation. This, however, turns out to be for your benefit, that, since there is not yet reposing left over that you might desire in the cause of refractions, you might nevertheless know that no other measure remains, since all crannies have been thoroughly gone over; and also, that you might have the method of seeking before your eyes, cognizance of which alone serves as the greatest argument that this way of measuring has not been assumed a haggardly. For it is the sort of thing that, when striven for, cannot appear to be put forward from the nature of things, unless you were aware of it.

First, this in general is easily established from experience alone; that density is a cause of refractions. This we also demonstrated a priori above, in prop. 14 of the first chapter. Next, this too is certain: that if light strikes perpendicularly upon a surface, it is not refracted, and that it is more evidently refracted as it strikes more obliquely. Therefore, the angle of incidence contributes to the cause, as I tried to deduce a priori above in prop. 20 of the first chapter.

It is therefore evident that the two causes are intertwined, so that density can accomplish nothing if you mentally remove the angle of incidence; for a type of incidence, namely, that which occurs at right angles, evidently strips the dense

most of the time sits upon the mountains, while the latter covers the entire face of the earth indiscriminately.

Thus when I deny that vapors have a part in forming refractions, I mean this last kind, since this used to be adduced as the permanent cause by the authors of this opinion, even when the heavens are very calm. The former kind of vapors, however.

12 Cf. the "thorns" in New Astronomy, introductory remarks to the "Summaries of the individual chapters" (JAXA p. 79).
body of the refraction. A clear argument of this, which I deduced a priori above in prop. 10 and 14 of the first chapter, is that light is not affected by the body of the dense medium, but only by the surface. For incidence is the bounding of a motion, and motion is in a straight line; and the bounds of a straight line are points, and the bound of infinite contiguous lines is a surface (which has infinite points continuous in space), not corporeal bulk.

Accordingly, the cause of refraction consists, not in the corporeal bulk of the dense body, but in the surface. Let these be for us the sure principles of judging the measures.

Thus, it is not a just measure of refractions, which follows from either of these causes alone.

First let us here be those people's opinion that seeks the measure in the bare length of passage through the denser medium. This is a multiple error. Let the surface of the dense body be \( AB \), its bottom \( GC \), the radiating light \( E \), the rays \( EC, ED \). This opinion states that the angles of refraction are to each other as are the parts of the direct rays beneath the dense [medium] from \( AC \) to \( BD \). First, it will be declared that some refraction occurs in direct incidence, which is repugnant to sense and to prop. 20 of the first chapter. Next, the horizontal refraction in a dense body with a straight line surface will increase to infinity. For \( BD \) is to \( AC \) as the secant of no angle, that is, the radius, to the secant of \( AEB \), the complement of the angle of incidence. It is, however, well known that the secant of 90 degrees is an infinite line. But it is known by experience that there is a certain specific limit to the horizontal refraction in water. For the circle, the measure of angles, is bounded by itself, and embraces four right angles. And since refraction is a striving towards the perpendicular, the angle with the perpendicular of the refraction that is parallel to the horizon is not greater than a right angle. Therefore, even the greatest refraction in the densest medium cannot be greater than a right angle.

Third, the cause of refraction will become the corporeal bulk, which has been refuted above in prop. 10 of the first chapter.

Fourth, when the depth of the medium is increased, the refraction will increase. For let \( EC \) be extended to \( G \). Thus the ratio of \( AG \) to \( BD \), which stays

---

13 "Corputernia." This is defined in the OLD as "the putting on of flesh, obesity." In later usage, according to de Cange's Glossarium, it can denote "the matter from which a body is composed." However, Tycho Brahe's use of the word shows it simply to mean "volume." cf. BHOOIP, p. 416.

14 The secant is the magnitude of the line from the center of a circle to a point of intersection with a tangent, in relation to the radius of the circle. At an angle of zero degrees, this point of intersection is the point of tangency itself, and hence the secant is equal to the radius.
the same, will become greater than that of \( AC \) to the same. Therefore, the refraction at \( A \) will be greater, while that at \( B \) stays the same. From this will follow the thing that Rothmann’s arguments appear everywhere to smack of, that \( ACG \) is not a straight line, but an arc, which we denied in prop. 22 of the first chapter, and which is easily refuted by experience. Set up three points in air at the same straight line,\(^\text{15}\) fill the basin with water, line up your sight so that you perceive the end points together, and the middle point will again clearly coincide with the end points, as before in air. And don’t let the illusion of the red bent in water, straight in air, sway you. For this is irrelevant to the experiment. For it is seen, not by the same, but by different refracted rays. But that what I have said follows, is shown thus. From \( A \) let the refracted ray \( AH \) descend to the bottom \( H \). Now let the depth of the bottom be increased from that side, because the angle is increased at the same time, let a more refracted ray \( AJ \) descend from \( A \). It is obviously evident that the points \( A, H, J \), are not going to be in a straight line. And even if, fifth, this opinion seems to connect denser with the angle of incidence, it nevertheless separates them in fact, because it considers the angle of incidence in no other way than to the extent that it increases the length of the passage, which is also increased through another cause, as just now by the lowering of the bottom.

On the contrary, he will be held liable for the same fault who would say that the difference of the sun’s rays of incidence upon the surface of the air and of the earth is the measure of refraction. This is because it is certain that the refractions that astronomers consider are brought about by the air poured around the earth. So let the surface of the earth be \( CB \), let the straight line \( CF \) touch it at the point \( C \), let the air’s surface from the same center \( A \) be \( DE \), and let \( CF \) cut this at \( D \). Next, let some line touch the surface \( DE \) at the point of intersection \( D \), and let this be \( GD \). Thus if \( CF \) be made of a ray of light, it will coincide with a tangent to the earth’s surface, and will be set up at a certain angle to the tangent \( DG \); the difference of the incident rays is \( GDC \). This is diminished gradually, so that at the zenith it is nothing, because the ray from the zenith to either tangent’s perpendicular, and descends at right angles. Therefore, if one should say that refractions are in proportion to these angles \( GDC \), this procedure for measuring refractions will appear elegant, for the reason that for any particular altitude of the sphere of air there follows, its specific maximum horizontal refraction, so that it seems to satisfy experience. In fact, this too is false, because, as stated, it separates the ratio of density from the figure or angle of incidence, which were to have been conjoined. For indeed, by this procedure, the refractions, or at least the increments of the refractions, from zero to the highest, would come out no different, whether the transparent sphere consisted of thin air or glass. But it is clear from experiments that different patterns of increments are also

\(^{15}\) This is done in a basin which will be filled with water.
arrayed according to the variety of dense media. Besides, if the refractions were the result of shape alone, no refraction would occur when the surfaces of the transparent body are planes. Thus, the same straight line has the same angles of incidence in water as it does at the bottom of the water, parallel to the surface. Here the difference in the angles of incidence is zero, while the refraction is not zero.

Let us proceed to those ways in which the two causes of density and angle of incidence are linked.

First to present itself is the one that was thrown up against Tycho by Rothmann, at the beginning. Let the refraction at the horizon be only 3 minutes, he says, and it will have a refraction of a minute and a half at 45 degrees. The thought behind this will turn out to be that the maximum horizontal refraction is established by the density of the medium, and it is gradually spread out through all the degrees of the angles of incidence, so that the extent that the illumination is raised on the arc of a circle, the remaining refractions decrease proportionally from the maximum, until they become nothing at the zenith. Later, both Tycho and Rothmann acknowledged that this way is in disagreement with experience. For in the table of Witelo, Book 10 Prop. 8,17 the refractions near the horizon are also drawn down, not do the ones at the zenith (or nearby) correspond to the horizontal ones, in this proportion of arcs of incidence. The refractions that take place in air drop off much more near the horizon as is to be seen in the tables in Tycho’s Postamentum pp. 79, 124, and 280. For the rest, neither Tycho nor Rothmann considered this sudden increment near the horizon to exist as a result of the actual form of the measure: it seemed to both likely either that the refractions are in proportion to the angles of inclination, or that a new cause—that is, a new body of vapors—intercedes from elsewhere. I, however, say that a measure must be established from when these phenomena follow of necessity, without anything intervening. For the analogy from water and air makes us sufficiently assured to state that the surface of the air alone bears the full responsibility. In water, which is fairly dense, refractions perceptibly depart near the horizon, and at the horizon the maximum refraction is about 37 degrees, and nothing comes to interfere with the surface of water. If this is so, much more so will the refractions decline towards the horizon where the density of the medium is small, and the horizontal refraction is insignificant and slight.

Moreover, by what has been said so far, that final opinion of Tycho Brahe (pp. 95-6 of the Postamentum) has also been refuted, although it was the closet of all so the truth. It is approximately as follows. The standards of measurement of the refractions are established by the length of the passage, and proportional parts of these are taken according to the quantity of the angles of inclination. That of water would become infinite at the horizon; it would arise on

16 [BIBL VI p. 111.]
17 [Table II p. 412.]
18 Here Kepler inserts an asterisk that refers to the note included above on p. 95.
19 [BIBL I p. 88-1.]
of the corporeal bulk, it would be increased with the depth of the medium; and finally, the refraction in air is not in agreement. For in that place Tycho supposes that the horizontal thickness of the air is 142 miles, and 14 miles at an altitude of 60°. Therefore, 1/10 part of the horizontal refraction, namely, 3°, would be due to the altitude of 60°, of which I myself would be due to the inclination of 30° (because the altitude is given as 60°), which Rule says does not happen.

Nor did I leave this remaining case unstated: whether, since the horizontal parallax is established from the density of the medium, the rest might correspond to the sums of the distances from the zephyt. But computation did not give approval to this, nor indeed was there any need to make the enquiry. For refractions would increase in the same pattern in all media, which does not agree with experience.

On this score, the cause of refraction introduced by Alhazen and Witelo is censured. They say that light seeks compensation for injury received from the oblique collision. For to the extent that it was weakened by the encounter with a denser medium, it gathers itself together again by approaching the perpendicular, so that it might strike the bottom of the denser medium with a more direct blow. For of influx, those which are direct are the strongest. And they add a subtle [know not what] the motion of light striking obliquely is composed of a motion perpendicular and a motion parallel to the surface of the dense medium: and this motion, thus composed, is not unalloyed by the encounter with the denser transparent medium, but only hindered. Therefore, the whole motion, as it was composed, fortifies itself again (that is, in the motion that has now been changed through the dense surface there resides the traces of its original composition, so that it becomes neither entirely perpendicular nor entirely parallel. However, it turns aside towards the perpendicular more than towards the parallel, because the perpendicular motion is stronger. They did not explain the matter much better than Macrobius, Saturnalia book 7, who attributed to the sense of vision a hesitation, and a retreat into itself from the encounter. It is exactly as if the form of light were endowed with mind, by which it might reckno both the density of the medium and its own injury, and, using its own judgment and not an extrinsic force, acting and not being acted upon, might of itself perform its own refraction. If this account were true, the measurement of refraction would be easy. For the refractions would increase with the sums of the distances from the zenith, because the impacts are diminished in the same ratio from the obliquity. Thus

20 The horizontal parallax is 3°, one tenth of which would be 3'. However, because the incident rays are inclined to the surface of the sphere at an angle of 30°, the refraction must be reduced by a factor of 1/3, making it about 1°.
22 Macrobius, Saturnalia VII 14.2, p. 448. "Water is denser than husband air, and therefore the sense of vision penetrates it more hectorily, and, being back by the encounter with the water, the eyesight is broken and retreats into itself. But when it goes back after being broken, it does not do so by direct impact, but slowly, in on all sides towards the outlines of the waters, and it thus happens that the image appears larger than its archetype."
if you should ask how much more strongly the sun falls upon the earth from an altitude of 30 degrees than from an altitude of 45 degrees, the answer will rightly be given, that it is as much stronger as the side of a square is longer than the side of a hexagon. For near the horizon the force of the sun is suddenly accumulated, while about the zenith it changes little; and this also happens to the sines.

I devised another procedure for measuring, to combine both the density of the medium and the angle of incidence. For since the denser medium becomes the cause of refractions, it therefore seems to be exactly as if one were to extend the depth of that medium, in which the rays are refracted, to a size that the same amount of matter, in the form of the rarer medium, occupies. For then the rays entered in to the empty space of water towards the visible object without being turned back. When the object, through this kind of image-making, is sunk deeper on the perpendicular, the rays themselves, too, are understood to be sunk and thus lengthened at their other end, where they touch the visible object. Let A be the light, BC the surface of the denser medium, DE the bottom. Let the oblique rays AF, AG, AB, descend, and let AB be extended to D, AG to F, AF to H, where they would have fallen had the medium been uniform. But because it is denser imagine accordingly that the bottom DE is pushed down far enough that there is the same amount of mater in the depth CK under the form of the rarer medium as there is in the depth CE under the form of the denser. Therefore, the entire bottom DE being sunk to LX, the points D, J, H, E, will descend perpendicularly to L, M, N, K. So let the points B, G, F, be joined to L, M, N. The bottom DE will be cut in the points O, P, Q, and the refracted rays ABO, AGP, AFQ will be made.

And it should be noted in this way of measuring, that if CK, FN, GM, BL be extended, they meet again in the same point. For let CA and NF be extended until they meet, and let the intersection be the point R, and let G and B be connected with R. I say that RGM is in turn a single straight line, and likewise RBL. For the triangles EAF, NHF, are between the parallel lines RA, RN, and therefore the angles RAF and FHN are equal, as are ARF and HNF, and the angle at F is common. Therefore, the whole triangles are similar. But side RA is equal to triangle RAF and triangle RAG, and angle RAF is greater than angle RAG by the quantity of angle FAG. And points F and G stand in a single straight

23 Reading "visible" for "visile".
line perpendicular to \( RAK \). In this way, the three points of the two triangles are determined, so that they cannot be other than these particular ones. Triangles \( FHN \) and \( GIM \), in turn, also have an equal side \( IM \) and \( IN \), because these are parallels between the parallel lines \( DF \) and \( LK \). And angle \( FHN \) is greater than the angle \( GIM \), again by the quantity of angle \( FAG \). And this occurs in similar triangles. For angles \( RAE \) and \( FHN \) are equal, as are \( RAG \) and \( GIM \) also. And, thirdly, angles \( G \) and \( A \) stand upon the perpendicular above \( MI \) or the equal side \( NH \), and if \( FN \) corresponds to angle \( RFA \), as which the same thing had occurred before. Therefore, to triangle \( RAG \) is to triangle \( RAE \), in lines and angles, so also will \( MGH \) be to \( NHF \). But \( NHF \) and \( RAE \) are similar. Therefore, \( RAG \) and \( MGH \) are also similar. But they have the sides \( RA \), \( FM \) parallel, and \( IG \), \( GA \) in a single straight line, and the common vertex \( G \). Therefore \( BGA \) and \( MGF \) are equal angles, and consequently \( MGR \) will be a single straight line. In the same way it will be proved that \( RAB \) and \( LDB \) are also similar, and that \( LBR \) is a straight line. What we had given notice of is therefore evident.

This way of measuring is refuted by experience. For the angles of refraction towards the perpendicular \( AC \), such as \( HFP \), are made too great with respect to the horizontal refractions. And if you examine the angles of refraction from workbook and Tycho, the directing point \( R \) of the refracted rays \( BO \), \( GP \), \( FQ \), is not one, but near the horizon \( T \) is highest above \( A \), coinciding with \( A \) at the zenith. Whoever has the leisure may enquire into this either by computation or by a compass. Add that reason itself stumbles, and when it groups in dimensions, it scarcely grasps or perceives itself.

I pass on to other ways of measuring. Since density is obviously a cause of refraction, and refraction itself appears to be a kind of compression of light (i.e., towards the perpendicular), it comes to mind to ask whether the ratio of the media in the case of densities is the same as the ratio of the bottom of the spaces that light has entered into and strikes, first in an empty vessel, and then one filled with water.

This measure is multiple. For, in one alternative, it is conceived in straight lines, as if one should say that the line \( EQ \) (in the above diagram), illuminated refractively, is to \( EH \), illuminated directly, as the density of one medium to the other. Or as if one should say that the refracted line \( EQ \) is to the part \( FH \) of \( AF \) extended, as the density of one medium to the other. In another alternative, it is conceived in plate surfaces as if, the power of \( EQ \) be to the power of \( EH \) or the circle, or any other figure on \( EQ \) be to a similar figure on \( EH \); in the ratio of dense media. Thus the ratio of \( EH \) to \( EI \) would be the duplicate ratio of \( EQ \) to \( EP \). In another alternative, this measure is conceived in the solidity of the prismatic pyramids \( FHEC \), \( EQC \), so that, as medium is to medium in density, so are these pyramids, an empty one to one full of fluid. Finally, because the

24 That is, the "second power", or the square.
25 "When three magnitudes are proportional, the first is said to have to the third the duplicate ratio of that which it has to the second." - Euclid. Elements, v definition 9.
Chapter 4

proportion of the media has a threefold aspect, since they are subject to density in length, width, and depth, I also proceeded to seeking the cubic ratio between the lines \(EQ\) and \(EH\). Indeed, I also considered other lines. From one of the points of refraction, such as \(G\), let a perpendicular be dropped to the bottom \(GY\).

The question will be, whether the triangle \(GTY\)—that is, the base \(IT\)—would be divided by the refracted ray \(GP\) in the ratio of the density of the media. I linked all these measures together, because for all of them the procedure of enquiry is the same.

For whatever the procedure is by which the line, plane, or pyramid might keep the proportion \(EI\) to \(EP\) (or, shorthand, \(YJ\) to \(YP\)) the same everywhere—that is, in the ratio of the media—it is certain that \(EJ\), the tangent to the distance of the point \(A\) from the surface—will become an infinite distance at the horizon, and therefore will make \(LP\) or \(YP\) infinite as well. Wherein the angle of refraction \(IGP\) will become zero, and approaching the horizon will come out to be gradually smaller and smaller, which is refined by experience. For it is greatest at the horizon. Nor do those pyramids brought into the scenario fit in well with prop. 7 and 8 of the first chapter, because the form of light box, the dimensions of its surface, not its body. Finally, this form is refracted right at the surface of the denser medium, and after this effect it is borne with a straight line motion, whatever sort of Sotton encounters it. Therefore, the bottom is the lines \(EJ, EP\), and these continued pyramids are clearly extraneous to the matter of refraction.

At last, therefore, one must go to the actual image of the object, whose place, by our Optics in ch. 3 above, we defined as the meeting of the visual ray with the perpendicular of the incidence. In the previous diagram, let the bottom now be \(IK\), and the depth of the dense medium \(JK\). Let the visible object be at \(M\); erect a perpendicular from \(M\) to the surface of the water, that is, \(MF\) is perpendicular; let \(G\) be the point of refraction. The straight line \(AG\) will be the visual ray, because the sense of vision does not perceive that the ray is refracted at \(G\)—indeed, it reckons that the entire ray comes from the place to which the vision, lying in the way, is directed by the viewer. Thus \(AG\) meets with the perpendicular \(MF\) at the point \(I\). Therefore, by definition I of the Optics, it will be the place of the image. This image I first consider thus: whether, by always remaining on \(IE\) parallel to the surface \(BC\), it provides a handle for measuring refractions. But that this is false, the sense of the eyes itself gave testimony. For the more obliquely you look into the water, the more the images rise to the surface. If you look directly down from \(A\) to \(C\), the depth of the bottom \(K\) will be seen. Which is to say that this measure at length coincides with the one described above. For it has been proved that if \(DL, HM, J, E\), which are two times the lengths of the images, be in the same line parallel to \(BC\) and \(IK\), and \(DL, HM, J\), \(E\) are perpendiculars, as is supposed here as well, then \(KC, NF, MG, I, E\), meet at the same point \(K\); thus this is one of the rejected measures.

Again, I asked whether the images were at an equal distance from the points

25 If \(AE\) is taken as the radius, with magnitude 100; 000 for Kepler, or unity for us. the length of \(EI\) is that of the angle of incidence in \(G\), which is equal to \(IAE\).

26 Chapter 3, Definition 1 of the present work.
of their refractions, and the ratio of the dense media were a measure of the minimum distance. As, if $E$ be the image, $C$ the surface of the water, $K$ the bottom, and $CE$ to $CK$ as medium to medium with respect to density; and afterwards, $F$, $G$, $B$ be three other points of refraction, and the images at $S$, $T$, $V$, and $CE$, $FS$, $GT$, $BV$ equal. But by this procedure, some altitude was set up for the image $E$ on the perpendicular $AK$, which is refuted by experience, not to mention other procedures of enquiry.

Third, looked whether, as medium is to medium, so (if $H$ be the place of the image) is $FH$ to $FX$. By no means. For thus $CE$ would be to $CK$ in the same ratio, and consequently the depth of the image would be always the same, which we have now refuted.

Fourth, I asked whether, as $CK$ is to $FX$, so is the depth of the image at $K$ to the depth at $H$? By no means. For either the images would never begin to go upwards, or where they had once begun, they would go upwards to infinity, because $FX$ at last becomes infinite.

Fifth, I asked whether the images go upwards in proportion to the sines of the inclinations? By no means. For the ratio of ascent would be the same in all media.

I therefore asked whether, sixth, they might at first be raised up in a perpendicular radiation in proportion to the media, and afterwards might more and more go upwards in proportion to the sines of the inclinations? For thus the ratio of upward motions would be compounded, and would be made different for different media. Nothing of the sort. For the computation disagreed with experience. And in general our consideration of the image, or of the place of the image, was in vain, for the very reason that it is an image. For what it is that happens to the sense of vision, from whose error the image results, has nothing to do with the density of the medium, nothing to do with that real affect$^{25}$ of light, the bending back$^{26}$ of light.

3. Preparation for the true measurement of refractions

Hitherto, we have followed an almost blind plan of enquiry, and have called upon luck. From now on let us open the other eye, proceeding with a sure method.

So, I had carefully ascertained the fact that the image of an object viewed beneath water is so near to the real measure of refractions that it almost measures refractions; it is low when the object is viewed from the perpendicular; it becomes gradually higher when the eye bends down towards the horizon of the water. On the other hand, the reasons just stated deny that the measure is to be sought in the image, because the image does not exist purely from the nature of the object, but at the same time exists because of a deception of the sense of vision, which is accidental to the object itself. When I considered these things, and compared the conflicting arguments, it finally occurred to me to look into the causes themselves of the image having been established in water, and in these causes to seek the measure of refractions. This opinion was all the

$^{25}$ affect
$^{26}$ bending back
more confirmed in me because I saw that the cause of the image as it appeared in both mirrors and in water, was not so pointed out by the optical writers. And this is the origin of the labor that we took up above in ch. 3. Nor was it a trivial matter, in that among the principles in such a complex subject I follow other false opinions in place of the false traditions of the optical writers, in that thrice and four times I set out upon a new road, and repeat the whole procedure anew; and, as it happens, being rashly persuaded, I so many times embrace in my mind the very thing that had been sought after with such zeal, as already discovered.

And indeed, this very difficult Guidonian knot of catoptrics I finally cut by analogy alone, in that manner which I have described above: when I consider what would happen in mirrors, and what tingly should happen in water following this similarity. For in mirrors, it is not at all matter, but only reflection from a polished surface, that places images outside of the place of the object seen. Therefore, it followed that in water, too, images ascend, and approach the surface, not according to the laws of density in water, greater or less according as the perception is more direct or more oblique, but only because of the refraction on the spark of light that has flowed from the object into the eye. When this was stated thus, whatever I tentatively proposed above for the measure of refraction through the image and its rising up, entirely collapsed. This was all the more so after I found the real cause for the image's being on the same perpendicular with the object itself, both in mirrors and in denser media.

So since the use of an analogy of this kind turned out happily in that most difficult demonstration of the place of the image, I began to pursue this analogy further, enticed by the desire to measure refractions. For I wished to lay hold of a measure of refractions, however blinding it was, provided that it was something, in the confident hope that it would happen that once the measure were truly known, the cause would also become evident. Therefore proceeded as follows:

In the same manner that the image of an object is made smaller in convex mirrors, it is also thus in denser media; and as the image is made greater in concave mirrors, it is also thus in denser media. In convex mirrors, the middle parts of the image get closer than the surrounding parts, while in concave mirrors they get farther away. The same thing occurs in different media, so that in water the bottom seems lowered, the surrounding parts raised. Hence it was apparent that a denser medium corresponds to a concave mirror surface, and a rarer medium to a convex surface. It was therefore evident that the plane surface of water assumes a certain type of curvature. This required that causes be thought up to reconcile this effect of curvature with the plane surface, so for instance if a cause were given why the parts of water surrounding a perpendicular incident ray should represent a greater density than that of the water directly beneath the perpendicular. Thus the question kept returning to the consequences described above; which, since they are related by reason and experience, were to be replaced by an enquiry into the cause itself. So it passed on to the measure. And since there are many kinds of media, differing in density, what was called for was to establish some kind of analogy with concave mirrors for this multitude as well. Thus, in order that the image be made greater in a concave mirror, the eye has to be inside the center, and the closer it is to the center, and farther from the surface, the greater will be
the image. And so, when denser media make e greater image in the same way, the different media corresponded to different places of the eye in the mirror’s diameter.

I therefore accommodated the extremes in a concave mirror, the place of the eye, the surface, and the center; in media, the densest medium, and the medium that is plainly equal to that medium in which the eye is. If the eye were located at the center, the same thing were required to happen that happen upon the object’s being viewed in a medium of infinite density.

Here new forks in the road arose. For if you should consider what ought to happen given a medium that is quite the densest (or of infinite density), you will understand, by analogy with the other media, that if there were such a thing, all rays whatever coming falling upon such a surface from a single point must be completely refracted; that is, after refraction they coincide with the perpendiculars themselves, and thus are made parallel. For in the other media, the denser any one of them is, the closer the refracted rays approach their perpendiculars.

And yet in a spherical concave surface, the rays from the center flowing in all directions to the surface are not parallel after reflection, but are gathered together again towards their origin. What had to be looked for, then, was a concave surface, and a point on its diameter, from which all rays going forth to the surface would be reflected into plain parallels. Had I not at this point had some kind of forethought of conics, I never would have arrived whither I was striving. But I was mindful of those things that Wield had written of the parabolic burning mirror, in Book 9 prop. 39, 40, 41, 42, 43, and 44.60 For those things that Apollonius had demonstrated concerning the hyperbola and the ellipse in Book 3 prop. 48 and a few other neighboring ones, were omitted for the parabola; and Wield to some extent supplies these in the places mentioned. In them he demonstrates a certain point from which any number of lines sent forth to the section or curved line make with the tangent angles equal to those angles which lines drawn from the same points parallel to the axis make with the same tangent. And it is this that we were seeking. However, because the consideration of the sections is difficult, because it is too little pursued, it would be good to say a few words about it in a mechanical, analogical, and popular vein. Gratefully, be indulgent!

4. On the sections of a cone

Cones are of various kinds: right angled, acute, obtuse, again, right or regular cones and scalene or irregular or compressed cones: for which see Apollonius and the commentaries of Eutocius.61 The sections62 of all of these, regardless

60 Theon, p. 398–402.
61 Eutocius was born in Ascalon (now Ashkelon) about 480 C. E. and wrote commentaries on works of Archimedes, Apollonius, and other mathematicians. His commentary was included by Commandino in the Latin translation of the Conics that Kepler used.
62 A "conic section" (from the Latin secta "a cutting") is created by the intersection of a
of kind, fall into five species. For the line on the surface of a cone established by a section is either straight, or a circle, or a parabola or hyperbola or ellipse. Speaking metaphorically rather than geometrically, there exists among these lines the following order, by reason of their properties: it passes from the straight line through an infinity of hyperbolas to the parabola, and thence through an infinity of ellipses to the circle. For the most obtuse of all hyperbolas is a straight line; the most acute, a parabola. Likewise, the most acute of all ellipses is a parabola; the most obtuse, a circle. Thus the parabola has on one side two things infinite in nature—the hyperbola and the straight line—and on the other side two things that are finite and return to themselves—the ellipse and the circle. It itself holds itself in the middle place, with a middle nature. For it is also infinite, but assumes a limitation from the other side, for the more it is extended, the more it becomes parallel to itself, and does not expand the arms (so to speak) like a hyperbola, but draws back from the embrace of the infinite, always seeking less although it always embraces more. With the hyperbola, the more it actually embraced between the arms, the more it also seeks. Therefore, the opposite limits are the circle and the straight line: the former is pure curvelessness, the latter pure straightness. The hyperbola, parabola, and ellipse are placed in between, and participate in the straight and the curved; the parabola equally, the hyperbola in more of the straightness, and the ellipse in more of the curvelessness. For that reason, as the hyperbola is extended further, it becomes more similar to a straight line, i.e., to its asymptote. The farther the ellipse is continued beyond the center, the more it imitates circularity, and finally it again comes together with itself. The parabola, in the middle position, is always more curved than the hyperbola, if they be extended by equal intervals, and is always straighter than the ellipse. And since, just as the circle and the straight line bring the extremes to a close, and thus the parabola holds the middle, so also, as all straight lines are similar, as are all circles, so are all parabolas also similar differing only in quantity.

Further, there are in these lines certain points to which special attention is given: these have a precise definition, but no name, unless you take the definition or some property in place of a term. For lines drawn from these points to lines touching the section, to their points of tangency, form angles equal to those that are made when the opposite points are joined with these same points of tangency. For the sake of light, and with an eye turned towards mechanics, we shall call these points “foci." We might have called them "centers," since they are on

plane with a conic surface. The manner in which the construction is to be imagined is presented in detail by Apollonius, Conics, at the beginning of Book I.

53 In the diagram below, the "straight line" is the horizontal line through E, the parabola is KG, and the infinity of hyperbolas lying between them is represented by LK. The idea of an infinitely distant focus for the parabola, and the conception of conics as forming a continuum generated by a moving focus, are Kepler's original and substantial contributions to the theory of conics. This is the source of our present use of the word "focus." The term had appeared in Apollonius, Conics III 35-52, but were named "the points arising out of the application" by the mathematician Pappus, because of the way Apollonius had defined them. The word "focus" means "hearts," and refers to the properties of
the axes of the sections, but in the hyperbola and the ellipse the writers on conic sections called another point the center. So in a circle, there is one focus, and this is the same point as the center; in an ellipse there are two foci, B and C, equally removed from the center of the figure, and more so in a more acute one. In a parabola one focus D is within the section, the other is to be supposed on the axis, either outside or inside the section, removed at an infinite distance from the former one, so that a line HG or 1G drawn from that hidden focus to any point of the section G is parallel to the axis DK. In the hyperbola, the external focus F is nearer to the internal focus E to the extent that the hyperbola is more obtuse. And the one that is external to one of the opposite sections is internal to the other, and vice versa.

It therefore follows by analogy that in a straight line the pair of foci (we speak thus of the straight line, contrary to custom, only to fill out the analogy) coincides with the straight line itself, and is single, as in the circle. Thus in the circle, the focus is right at the center, receding as far as possible from the neighboring circumference; in the ellipse it recedes less, and in the parabola much less. Finally, in the straight line, it recedes from it by the least amount; that is, it falls upon it. And so in the limiting cases, the circle and the straight line, the foci come together, standing at the greatest distance in the former, and falling conoidal mirrors of focusing light rays at one of these points when the rays originate at the other point (in the parabola, the originating "focus" is infinitely distant, and the rays coming from it are parallel).
right on the line in the latter. In the middle, the parabola, they are an infinite distance apart, while in the ellipse and hyperbola, which see at the sides, the foci, paired in their function, are a measured distance apart, the other [focus] being inside in the ellipse, and outside in the hyperbola. In all cases the accounts are opposite.

The line \( MN \), which marks out the focus on the axis, standing perpendicularly to the axis, we call the chord.\(^{35}\) and the line that shows the altitude of the focus from the vertex from the nearest part of the section, namely, the part of the axis \( BR \) or \( DK \) or \( ES \),\(^{36}\) we call the sagitta \(^{37}\) or axis. Thus in the circle, the sagitta is equal to half the chord; in the ellipse the half-chord \( BP \) is greater than the sagitta \( BR \), and the sagitta \( BR \) is also greater than half of the half-chord \( BP \) or the fourth part of the chord. In the parabola, as Wiele demonstrated,\(^{38}\) the sagitta \( DK \) is exactly equal to one fourth of the chord \( MN \); that is, \( DN \) is twice \( DK \). In the hyperbola \( EQ \) is more than twice \( ES \); that is, the sagitta \( ES \) is less than one fourth of the chord \( EQ \), and is ever less and less, in all ratios, until it vanishes in the straight line, where, the focus falling directly upon the line, the altitude of the focus, or the sagitta, vanishes, and the chord at the same time is made infinite since it coincides with its own arc (to speak thus improperly, since the arc is a straight line). For geometrical terms ought to be at our service for analogy. I love analogies most of all they are my most faithful teacher, aware of all the hidden secrets of nature. In geometry in particular they are to be taken up, since they restrict the infinity of cases between their respective extremes and the mean with however many absurd phrases, and place the whole essence of any subject vividly before the eyes.

Furthermore, in the description of the sections also, analogy has been of the greatest help to me. For from Apollonius III 51 and 52, the description of the hyperbola and the ellipse is accomplished very easily: it can be done even with a thread.\(^{39}\) For the foci being given, and between them the vertex \( C \), let pins be placed at foci \( A \) and \( B \) and to \( A \) let a thread with length \( 4C \) be tied, and to \( B \) a thread with length \( BC \). Let each thread be lengthened with equal additions, so that if you grasp the doubled thread with the fingers, and as they move away from \( C \) you little by little pay out the two threads, and with the other hand you draw the path of the angle that the two threads make at the fingers, that drawing will be a hyperbola. The ellipse is described more easily. Let the foci be \( A \) and \( B \); the vertex \( C \). Put sturdy pins at \( A \) and \( B \), to embrace the two with a thread in a single

\(^{35}\) Note the Kepler's usage does not coincide with ours: his chord must pass through the focus and be perpendicular to the axis. Thus in a circle, Kepler's chord is a diameter.

\(^{36}\) For the ellipse, parabola, and hyperbola, respectively.

\(^{37}\) Literally, "arm." Kepler's use of this term here is an extension of its previous application to the circle, where it denoted the curved line.

\(^{38}\) Wiele IX 45, Thesaurus II pp. 400-401. The proposition had been proved by Alhazen in his work on the parabolic burning mirror, which Wiele used.

\(^{39}\) The following constructions for the hyperbola and the parabola were invented by Kepler. Although the ellipse construction had already been described, Kepler probably also developed his construction independently.
embrace, so that there be no thread between \(A\) and \(B\). Let the length of the thread be twice \(AC\), and let the ends of the thread be joined with a knot. Now insert the stylus \(D\) in the same loop of thread with \(A\) and \(B\), and, with the thread stretched as tight as it allows, draw a line around \(A\) and \(B\); this will be an ellipse. Since this description was so easy, not requiring those laborious compasses that some, by the using of which, hunt for people's admiration, I was long disappointed that a parabola could not also be described thus. Finally, the analogy showed (and geometry countenanced) how to draw this too, without much more labor. Let there be set out the focus \(A\), the vertex \(C\); so that \(AC\) should be the axis; let this be extended in the direction of \(E\) all the way to infinity, or as far as it is wished to draw the parabola; let it be wished to draw the line \(E\) to \(E\). Accordingly, let a pin be fixed at \(A\), and from \(A\) here be tied a thread with length \(AC\). \(CE\). With one hand, you should hold one end of the thread \(E\), and with the other stretch the stylus with the thread all the way to \(C\). Also, let \(EF\) be erected perpendicularly to \(CE\), and then with the stylus \(C\) and the other hand \(E\) move equal distances away from the line \(AE\); in such a way that the other hand and the end of the thread always remain on \(EF\), and the thread \(DG\) always parallel to \(AE\); the paths \(CD\) which you will have drawn with the stylus will be a parabola.

I have said these things about conic sections all the more willingly, because not only does the measuring of refractives require them here, but their use will also be apparent below in the anatomy of the eye. Thus too, mention of them will be made in this or other places among the observational problems. Finally, a consideration of them is completely necessary for the most excellent optical devices, for setting up an image hanging in air, for increasing images proportionally, for igniting fires, for burning things at infinity.\(^{10}\)

5. What kind of quantity measures refractions?

\(^{10}\) This last remark is a reference to Pitta's account of the parabolic burning mirror, in Megon nomina, X-VII, p. 275, where he writes, "This mirror burns, not at a thousand places...but at infinity." The other "optical devices" mentioned by Kepler are presented by Pitta in Megon nomina, X-VIII, pp. 267-278.
was yet removed. For we had proposed, for measuring refractions, infinitely many forms of media, differing in density, from that which is imagined to have the ratio of infinite density to that which has no density at all. However, in the preceding it was demonstrated that the increase of the angles of refraction, through various angles of incidence of light upon denser media, is to be sought (at analogy's urging) from the angles of reflections through various angles of incidence of light upon concave mirrors. Now one extreme in the analogy was in good accord, namely, the analogy of the densest medium to the placing of light at the focus of the parabola. For, as we rays in the densest medium after refraction, so those which come from the focus of the parabola after reflection, are made parallel.

But concerning the media next in order, which are less dense, it was not yet established which way they were to be accommodated: whether by different positions of the light in the same parabola below the focus, so that the medium in which no refraction occurs corresponds to the position of the light right on the concave surface or the vertex of the parabola, or whether instead, since the parabola is the extreme of both hyperbolas and ellipses, the media different in degrees from the most dense are to be accommodated either to various hyperbolas or to various ellipses, so that the position of light might always remain at the focus of the section. In this way that medium which lacked refraction, would, in the former case, call for the straight line or plane mirror, and in the latter, for the spherical concave. For this reason, I did not leave it untied to ask whether for any medium there might be its own hyperbola. For if you designate with points the places of the images in water through all angles of inclination, a hyperbola will be approximately foreshadowed, which increases my confidence.

Thus, for example, for the refraction of water, at an inclination of 80° let a ray of light at 30° be reflected by 90°; then let B be the focus and A the opposite one; on the line BC at the point B let the angle 80° be set up, and let this be CBA. Likewise, on the line BA at the point A let the reflected angle 30° be set up, let this be CBA, and let AC and BC meet at C. Now since B is 80° and A is 30°; the remaining angle C will also be 50°; and AB and BC will be equal. Thus if AB be 100,000, AC will be 128,558, but BC is also 100,000. Therefore, the excess of AC over BC is 28,558. Therefore, by Apollonius III 5, 28,558 is the line between the vertices of the opposite sections, or the axis DE. Subtract DE, 28,558, from AR, 100,000, and the remainder is 71,442, whose half, 35,721, is AD or ER. And so E is the vertex of a hyperbola, and D is the vertex of the opposite section. Now let us see whether the refractions

---

1) Wittius's table for water is in Prop. 8 of this chapter, on page 128.

2) As Kepler says below, D is the vertex of the opposite part of the hyperbola, which is not drawn in the diagram. Therefore, AD = BE.
of the other inclinations set forth by Witelo will follow. Let $EBF$ be $70^\circ$, at which inclination Witelo places a refraction of $24^30'$ and a refracted angle of $45^\circ30'$. Thus the question is whether, in this hyperbola just constructed, $FAB$ is $45^\circ30'$. Since $AB$ is 100,000 and $ABF$ is $70^\circ$ and $AF$ is longer than $FB$ by the space $DE$, let us proceed by false position.\(^{21}\) and let $FAB$ be $45^\circ30'$. $AFB$ will be $64^30'$. Consequently, as the sine of $AFB$ is to $AB$, so is the sine of $FAB$ to $FB$, and the sine of $FBA$ to $FA$. Therefore, $FBA$ comes out to be $79.023$, and $FA$ $194.111$, and the difference of these is $25.088$. But it ought to have been $28.558$. Therefore $FB$ is too long with respect to $FA$. It is diminished, if angle $FAB$ be diminished. This is one iteration. Now, for the second, let $FAB$ be $44^29'$. $AFB$ will be $65^31'$, and consequently $FB$ will be $76.993$ and $FA$ $103.254$. The difference, $26.261$ ought to have been $28.558$. You see that by the diminution of angle $FAB$ by $1^\circ$ we gained $1.173$ towards what is required. We were still off by $2.297$. Therefore, that angle, which was to have represented the refraction for us, still has to be diminished by about two degrees. And in return, the refraction itself has to be increased by the same amount, about three degrees, so as to make it $27^30'$, closer to what it formerly was at $80^\circ$. Therefore, the hyperbolic mirror does not measure the angles of refraction at different inclinations. And in general, because $BA$ represents the perpendicular in water, and the ray that is set in motion from $B$ at right angles to $BA$, represents the ray parallel to the surface of the water; or to the horizon, it must be noted that in the hyperbola, about the vertical angles of incidence, the angles of reflection suddenly increase, but to so hardly at all about the horizon, and reach a maximum quickly. In refractions it is otherwise, for near the horizon both the angles of refraction and the increments of those angles increase. Therefore, we seek the measure of refractions between the foci of hyperbolics in vain.

You could now guess immediately that because the hyperbola gives the opposite of the refractions,\(^{21}\) the ellipse, being the hyperbola’s opposite, is going to do the same as the refractions, and will accommodate itself to the measure. This guess is made the more probable in that on this basis the analogy gives the correct spherical mirror to the medium lacking refraction, and in this manner the rays going out from the center and those bent back concord, and do not endure any angles. Let $B$ be the focus of the ellipse. $A$ the opposite focus, $DAB$ the axis. Again, let $IBF$ be $80^\circ$ and $BAF$ $50^\circ$. Here, $AFB$ will be $30^\circ$ representing the actual refraction; and let $AB$ be $100,000$. Accordingly, $AF$ will be $196.562$ and $BF$ $153.208$, and by Apollonius III 52 the sum of the two, $350.170$, will be the quantity of the axis $D1$. Therefore, $AD$ or $BF$ will be $125.085$. The ellipse being thus established, let $IBC$ now be $70^\circ$. I know that $BC$, $CA$ are equal to the axis.

\(^{21}\) That is, an iterate procedure. See e.g. *Mathematica’s Dictionary* v. ‘False’ and ‘Regula Falsi’. In the triangle $ABF$, the side $AB$ and the angle $AFB$ are given, and the difference between sides $AF$ and $BF$ is known. Kepler did not know how to solve the problem directly. Henry Briggs, reading this passage, came up with a direct procedure, which he communicated to Kepler in a letter of 20 February 1625 (No. 1602) in *JRGW* XVIII, ed. pp. 225–8).

\(^{22}\) All editions have a proof here, but the sense seems to require a comma.
350,170. Therefore, in the refraction, the angle $B'CA$, be 24° 30', as Witeilo puts it. $AC$ comes out to be 226,999, 3C 171,995. The sum is 398,594. This is too great. For it was to have been only 350,170. Accordingly, $BAC$ is too great, and $B'CA$ is less than it should be. So let it be graver, i.e. 27°, and let $BAC$ be 45°. $AC$ comes out to be 206,965, and $EC$ 150,223. The sum is 357,208, still a little greater than it should be. Accordingly, the angle $B'CA$ still has to be increased a little. And so it will become greater than the Witibolian refraction. Therefore, the measure of refraction is also not to be sought in the focus of an ellipse. For in general this too increases the said angle with large increments from the vertex, but where a perpendicular to the axis goes forth from the focus, the increments of the angles are small, also nearly as in the hyperbola itself.

Therefore let another way be consulted, that for all media, in the matter of refractions, the measures are in the parabola alone, and let the measure of the densest medium be established by straight lines drawn from the focus, but let the measure of the most tenuous medium, or one lacking refraction, be taken (analogically speaking) by drawing straight lines from the point that is at the very bottom (or, generically, the vertex) of the parabola, situated at the very lowest level; let the remaining media lying between have assigned to them the points on the axis likewise lying in between. For example, in the refractions of water, let $A$ be the focus, in the following diagram. $AB$ the axis, below $A$ let a point $C$ be taken, and let $DC$ be inclined to $AB$ at the angle $DCE$, 80°. And at $D$ let $DF$ be tangent to the sector, cutting $AB$ at $E$, and let $DEF$ be made equal to angle $E'DC$, and let $GD$ be extended until it meets $AB$ at $I$. The first question is, how high should the point $C$ be from the bottom or the vertex $B$, in order that $GIA$ become 50°, as great as Witeilo puts the refracted angle at an inclination of 80°. Therefore, in $C'GI$, $D'IC$ is 50°, $ICD$ is 80°, and thus $D'IC$ is also 50°.

Consequently, $CD$ and $CI$ are equal. Therefore, where $C'1$ or $CD$ is 100,000, $D'1$ is 128,588. And since $DC$ and $FDG$ are set equal, and likewise $D'E$ and $F'DG$ are vertical angles therefore $D'E$ and $FDC$ are equal, and consequently, by Euclid VI. 3, as $CD$ is to $O'D$, so is $CE$ to $E'1$. And so $CE$ will be $3,753$, $E1$ 56,247. Now let a perpendicular be sent down from $D$ to $EC$, and let this
be $DE$, $KDC$ will be 10°, and thus $DK$ will be 98.481 and $KC = 17.365$, and the remainder $EK$ will be 26.388. And by Apollonius I 33, $EB$, and also $BK$, will be 13.394 and $BC$ will be 30.559. But it was proved by Wituilo the $BA$ is half $AM$, and by Apollonius I 20, as $BK$ is to $BA$, so is the square on $KD$ to the square on $AM$. Therefore, where $AB$ is $183.770$, $CB$ is $30.559$.\(^{45}\) Thus the parabola is found and set up, with its point, from the Wituiloan refraction of the eighteenth degree, and the height of the point is very nearly the sixth part of the height of the focus, or of the sagitta. And because as $AB$ is to $CB$, so is the square on $AM$ to the square on $CO$. Therefore, when $AB$ is multiplied by $CB$, and the root is found, which is $74.940$, $AB$ will be to the root as $AM$ is to $CO$. And since $AM$ is $367.540$, $CO$ becomes $149.990$.\(^{46}\)

Let us now see how great the reflection will be at $O$. Let a straight line touch the section at $O$, and let it be $O\ell$ (drawn in the imagination) in triangle $O\ell C$, two sides and the included right angle are given. For $CO$ is at right angles to $IC$. And because $O\ell$ is a tangent, $CB$ and $\ell I$ will be equal, and thus $C\ell$ is $61.118$. But $CO$ is $149.990$. Therefore, the angle $O\ell C$ is $67\ddeg50\arcmin$, and $I\ell C$ $22\ddeg10\arcmin$. If an equal angle be added to this,\(^{47}\) or it be subtracted from $O\ell C$, $67\ddeg50\arcmin$ there remains $45\ddeg40\arcmin$, the angle which the reflected ray\(^{48}\) makes with the axis, representing the horizontal refraction of water.

Now if it is desired to test the rest of the refractions in Wituilo, we shall proceed thus. First, for the sake of an easier computation, let $AM$ receive the round dimension of $100.000$, and $CB$ will become $8.314.1$. Now let $BC$ be $70\ddeg$, at which inclination Wituilo puts a refraction of $24\ddeg30\arcmin$ and a refracted angle of $45\ddeg30\arcmin$. Let this be represented by $C\ell D$. $KD$ must be taken large enough so that together with $KC$ it makes the angle $K\ell C\ell U$ $70\ddeg$, as it is supposed to be, and that at the

\(^{45}\) In the previous sentence, Kepler says that

$$BA = \frac{1}{2} AM \quad \text{and} \quad BK : BA = K D^2 : AM^2.$$

Combining these,

$$BK : BA = K D^2 : AB \ell ^2,$$

and since $BK$ and $KD$ are known, $BA$ can be found. One could proceed arithmetically, treating all these magnitudes as numbers and the proportion as an equation; or geometrically, by finding a mean proportional between $BK$ and $BA$ so as to remove the squares. Numerically, the result is the same.

\(^{46}\) Let $I$ be found on $AB$ such that $AB : BC = BI : CR$. Then since $AB : CR = AM^2 : CO^2$,

$$\frac{AB}{BI} = \frac{AB}{CR} \quad \text{or} \quad BI = \sqrt{AB \cdot CR}.\quad \text{This is the "root" that Kepler mentions. Thus}

$$AB : CR = AM : CO.

\(^{47}\) If $I\ell C$ be doubled and its complement be found, this complement will be the angle sought. Kepler seems to have omitted a step here.
same time it is to $AM$ in the subduplicate of the ratio that $BK$ (the remainder of $BC$) has to $BA$. In order to pursue this artfully, always follow this rule: square the tangent of the angle $KD'C$. To this square add the magnitude made of twice $BC$ multiplied by $AM$, which in these units is always 3,662,900,000. Agains diminish the root of the sum by the same tangent of the angle $KD'C$. The remainder will be $KD$, 18,262, at this inclination. But since $KD$ is a mean proportional between $EK$ and $AM$, and since $KD = 18,262$ where $EK$ is 100,000, therefore where $KD$ is 100,000, $K'C'$ will be 18,262, which is the tangent of angle $KD'E$, 10 $^\circ$ 21'. $K'D'E = 20'$, added to this makes $C'D'E = 30'$ 21'. Twice this, $C'D'I$, is 60 $^\circ$. And, $ICD= 70'$. Therefore, the remainder $C'ID$ to make two right angles is 49 $^\circ$ 16'. Corresponding to this, Vitelo puts 45' 30'. Thus a refraction of 20 $^\circ$ 42' would be shown here: for Vitelo, it is 24 $^\circ$ 36'. Therefore, this measure too is flawed, for it falls off too much near the horizon, the opposite of what happened above in the ellipse and the hyperbola. For it also shows a horizontal refraction of 44 $^\circ$ 26', of which, although Vitelo omitted it, experience ostensibly testifies that it barely goes up to 37'. Nonetheless, it shows the required measure qualitatively, in that it makes the refractions grow with increasing additions near the horizon. For from 79 to 80' the refraction increases by 9 $^\circ$ 18', and from 80 to 90' by 54 $^\circ$ 20'.

---

42 As in footnote 26, let $I'$ be chosen on $BA$ so that $BI'$ is a mean proportional between $BK$ and $BA$. Because $BK : BA = K'D' : AM = 1 : 2AM$. Then $BI = K'D' = 2AM$. But since $BI$ is a mean proportional, $BI^2 = BK \cdot BA$ and so $BK : BA = BI : IA = 1 : 2K'D'$. Moreover, since $AM = 2BA$, $K'D' = 2BK : AM$. Further,

$$BK = EC - CK = BC - K'D \tan K'D'C \cdot 100,000$$

End. (The tangent must be divided by 100,000 because the radius in this case $C'D'$ is taken at 100,000 rather than as unity, as we would have it combining the two equations.

$$KD^2 = 2 \left( K'D - K'D \tan K'D'C \cdot 100,000 \right) - AM = 2BC \cdot A'M - \frac{2K'D \tan K'D'C}{100,000}.$$  

Collecting the $KD$ terms and completing the square,

$$KD^2 = 2K'D \cdot AM \tan K'D'C \cdot 100,000$$

$$= 2BC \cdot AM + \frac{AM^2 \tan^2 K'D'C}{100,000^2}$$

or

$$\left( K'D + \frac{AM \tan K'D'C}{100,000} \right)^2 = 2BC \cdot AM + \frac{AM^2 \tan^2 K'D'C}{100,000^2}.$$  

Solved for $K'D$, this becomes

$$K'D = \sqrt{2BC \cdot AM + \frac{AM^2 \tan^2 K'D'C}{100,000^2}} - \frac{AM \tan K'D'C}{100,000}.$$  

But since Kepler has made $AM$ equal to 100,000, all the $AM$'s but the first one drop out, along with the 100,000's, and this equation expresses Kepler's "rule" exactly.
Further, since the parabola is the most acute of all hyperbolas, it is easy for us to return to hyperbolas with this form of enquiry. It will thus remain at the focus as long as the medium in question is the densest, and with this only the parabola will square. Hyperbolas and ellipses will be at the service of other media, not at the focus as before, but at some point above or below the focus. For according to the various positions of light on the sagitta of the figure, there also occur various reflections, in the other sections no less than just now in the parabola. Moreover, the parabola will lead us by the hand. For in it, if we had remained at the focus, the reflection would have increased in proportion to the inclinations. Now, where a descent was made below the focus, the reflections were made to fall off near the horizon. From which it is given to understand that when the falling off is too great, the descent below the focus is too great. Again, if we had remained at the focus, the reflection of the horizon would have been greatest at 90°; when we went down, we found 45° 30'. Therefore, if we go up again above the original position, we shall have more at the horizon (or at an inclination of 90°) — more, I say, than 45° 30'.

Thus there is more at 80° than at 30°. It is therefore too much. But hyperbolas diminish the horizontal reflections, even from the focus. They also decrease the falling off of the reflections of the horizon, so that there they increase with a lesser ratio than do the inclinations, although at the beginning, about the vertex, they increase in a greater ratio. The ellipses, too, do the same thing. So we shall have to descend below the focus of the hyperbola.

There is no geometrical method of enquiry: luck has to be tried; a hyperbola a little more obtuse than a parabola must be taken, and two reflections investigated. If this does not do the job, another must be chosen, as if by a regula falsi, until we meet with the true one.

So, because in a parabola the half chord is twice the sagitta, let there be a hyperbola whose half chord BC is three times the sagitta; and on the sagitta EB let a point be chosen below the focus B, and let this be M, from which let the lines to be reflected be drawn. Let MF be drawn out such that $FMG$ is 80°; and because $FGM$ is 50° and $GM$ is likewise 50°, the tangent $FH$ to the section at $F$ will make $FHM$ 75°; since it bisects $GFM$. Therefore, by Apollonius II, 50, because the hyperbola is given, by the ratio $EB$ to $BC$ a tangent $HF$ will be able to be drawn making a required angle with $EB$. For let $A$ be the opposite focus; $AC$ will be longer than $CB$ by the space $DE$, the axis. And at the same time, the power of $AC$ is equal to both the squares on $AB, BC$. Therefore

41 The second power, i.e., the square.
where $AD = 1$ and $EF = 1$, $DE = 2$, or $AB = 4$ and $BC = 3$ and $AC = 5$. Next, bisect $AB$ at $1$; I will be the center, and the meeting place of the asymptotes, by Apollonius II. But, in order to know the angle of the $\omega$-points, note that, by Apollonius III. 52, as rectangle $DB$, $BE$ is to the square on $BC$, so is the transverse side $DE$ to the latus rectum. And conversely, Further, as $DE$ is to the latus rectum, so the figure $53$ is to the square of the latus rectum. Therefore, as the figure is to the square of the latus rectum, so is the rectangle $DB$, $BE$ to the square on $BC$, and, permuted, as the figure is to rectangle $DB$, $BE$, so is the square of the latus rectum to the square on $BC$. But the figure is four times the rectangle $DB$, $BE$, therefore, the square of the latus rectum is also four times the square on $BC$: that is, the latus rectum is twice $BC$. This is indeed always true, even in all three sections. Therefore, the latus rectum is 6 and the transverse side $DE = 2$. Therefore the figure is 12, one fourth of which is 3, whose root is $\sqrt[3]{3}$. 54 Therefore, by Apollonius II, 1, the angle between the asymptote $TF$ and $P1$ is 60°.

Now, then, in triangle $HFX$ (for $X$ being sent down perpendicular to $FM$), because $FHX = 75°$, $HFX$ will be 15°. Therefore, where $FX = 100,000$, $XY = 26,795$. But at the same time, by Apollonius I. 57, as the transverse side of the figure is to the upright [i.e., the latus rectum], so is the rectangle $1X$, $XY$ to the square on $XF$. So when one third of the square on $FX$ ($100,000$) is divided by $XY = 26,795$, the result is $1X = 12,401$, where $FX = 100,000$. But by Apollonius I. 21, as the latus rectum of the figure is to the transverse side, so is the square on $FX$ to the rectangle $DA$, $EX$. Therefore, the rectangle $DA$, $EX$ is equal to the rectangle $1X$, $XY$, or one third of the square on $FX$. Accordingly, subtract one third of the square on $FX$, from the square on $XY$, and the root of the remainder is the semicircle $1E$. Therefore, the value of $EX$, which comes out of this is 14, 209. And because $X$. $F$. $E$ is 80°, $XFM$ will be 10°. Consequently, where $XF = 100,000$, $XM$ will be 17,653. Therefore, $EM$, the height of the pair, sought, is 31,842. But $IE = 110,392$, half the axis, as with which the height of the focus $EB$ was demonstrated equal at this place. Therefore, where $EB = 100,000$, $EM$ is 28,807.

Now that a point has been found in the chosen hypotenuse, from which a straight line going forth to the surface, making an angle of 80° with the sagitta, would be reflected into another line which makes an angle of 50° with the same axis, let us now again enquire how great the angle $FGM$ becomes if $FMG$ be 70°. The perpendicular $FX$ should be made of such a size that, together with

52 In the equation $AC^2 = BC^2 + AB^2$, replace $AC$ with $DE$, $AD$ with $DE$, and $AC$ with $3EB$, $BC$ with $3B$, and $AB$ with $DE + 3EB$ or $DE + 3EB$. The resulting equation in two unknowns reduces to $2EB = DE$.

53 The proposition states that if $\theta$ is 12°, then $\theta = 10°$.

54 Apollonius uses the term "the figure" to denote the rectangle formed by the transverse side and the latus rectum, i.e., the "upright side." Thus, calling the latus rectum $L$, we have here $DE = L = DE = L$, an application of Euclid VI. 1.

55 In Apollonius's III. 45, the "point of application" (i.e., point $B$) is defined by this equality, and its other properties are then derived from this and the foregoing propositions.
$XM$ cut off; it includes the angle $XMF$. $70°$; and that the remainder $XE$ make with $XD$ a rectangle equal to one third of the square on $FX$.

The procedure of the computation has difficulty. The line $DE$, $220,384$, has to be added to the line $EM$, $31,842$. The sum, $252,226$, must be multiplied, both by $EM$, $31,842$, and by the tangent of the angle $20°$, namely, $36,397$. Again, this $36,397$ must be multiplied by $EM$, $31,842$, and by itself. Next, the product of $36,397$ and $252,226$, and the product of the same $36,397$ and $31,842$, are to be joined, so as to make three factors in place of four; and let three times all of these be taken. Now, from three times the square of $36,397$ let the last live digit on the right be cut off, and let this abridged number be subtracted from $100,000$. Divide three times the composite by the remainder, and divide three times the product of $252,226$ and $31,842$ (increased by 3 zeroes at the right) by the same. Once this is done, square half of the prior quotient, and to this squared number add the posterior quotient. You shall diminish the side or root of the sums by the quantity of half the prior squared quotient. The remainder shows $FX$ in this second establishing of it, as large as we are ordered to make it by the command of the hyperbola. And by the procedure, $FX$ will become $68,550$, and $XM$ $24,950$ of the same units. For when $FX$ was $100,000$, $XM$ was $36,397$. Before, however, $EM$ was $31,842$. So when $XM$ is subtracted from this, it leaves $EX$.

54 The first product is $(DE + EM)\tan 20°$, and the second is $EM\tan 20°$. When the two are added, the sum is $(DE + 2EM)\tan 20°$. This whole expression is to be multiplied by $i$.

55 The number $36,397$ is the tangent of $20°$ (in Kepler’s units), and the expression developed here is $100,000 - \frac{31,842^2 \tan 20°}{100,000}$.

For future reference, we shall call this $i$.

56 That is, three times the expression $(DE + 2EM)\tan 20°$.

57 That is, by $i$.

58 This expression is $\frac{3DE + EMEM}{i} = 100,000$.

59 Kepler calls it the “squared quotient” only to identify it; half of the quotient itself is subtracted, not the square of itself.

60 A careful reading of this nearly imperceptible description will show that Kepler has constructed the following equation:

$$FX = \sqrt{\frac{3DE + 2EM\tan 20°}{2} + \frac{3DE + EMEM}{i}} - \frac{100,000}{2}$$

He undoubtedly derives this expression using what he describes below up 120° as “Tanner’s auxilium,” that is, algebra.
6892; and when $OE$, 220,384, is added to this, it establishes $DX$ as 227,276; and this, multiplied by $EX$ (and this will serve us as a test), produces a quantity equal to that obtained if you should square the new value of $FX$, 68,556, and take the third part of its square. And you shall divide this third part of the square of $FX$ by $IX$, 117,964 (that is, the sum of $IE$, previously known to be 110,192, and $EX$, now found to be 6,892). In this way the quantity $HIT$, 13,379, will be established, whose ratio to $FX$, 68,548, is as 19.518 to 100,000. It therefore shows the angle $HFX$ to be 41° 3', and therefore the sum $HFM$ is 31° 3'. From what was said before, $GFM$ is twice this, and thus is 62° 6'. But $GMF$ was by hypothesis established as 70°. Therefore, the remaining angle $FGM$ comes out to be 47° 54'. But from Wicel's observations it should have been 45° 30', in the parabola it came out 49° 18'. See how we have come down from the parabola to this obtuseness of the hyperbola; we have come one degree and 24' nearer to the target; and since we are still off, we have to carry or to more obtuse hyperbolas.

That also shows that we are on the way. In the parabola, the height of the point was one sixteenth of the sagitta; here it is a little less than one third. For $IE$, or in this hyperbola, $EB$, which is equal to it, is 110,192 parts, while $EM$ is 109,424. We have therefore gone up. And this is what we said before ought to happen. Therefore, where we select another, more obtuse, hyperbola, we shall go farther up.

So, let there be a more obtuse hyperbola, the ratio of whose sagitta to half the chord is one to four. $EB$ will be the sagitta and $ED$ the transverse side of the figure, or the equal axes,61 and the focus rectum of the figure is eight times the transverse (in the previous one it was three times). For that reason, where before you had tripled, you now multiply by eight. Briefly, so as to repeat the whole previous process in a word, the angle $FGM$ comes out to be 44° 58', which previously was 47° 54'. From Wicel, it ought to have been 45° 30'. And since $EB$ is 60,688, the height $EM$ of the required point becomes 33,850, more than half, which previously was one third, and in the parabola was one sixth. You now see that we have passed the limit in the obtuseness of the hyperbola, but just barely. Come now, if proportional compuation deluges you, or try new and intermediate hyperbolas, and the rest of Wicel's angles. I say that what will happen is that you will find such a hyperbola, and in it such a point, from which the images of things positioned behind it will appear in exactly that form in which they appear under water, that is, that by this hyperbola and point is contained the measure of all the refractions of water; and besides, that by other hyperbolas are contained the measures of media that are different with respect to refractiveness, and these are by means of the angles of reflection that occur in concave hyperbolic mirrors.

61 Substituting the appropriate magnitudes into the equation for the sides of the right triangle $ABC$, as before, one finds that $DF$ and $EB$ are equal in this hyperbola. These three are the "axes" in the sense that they are collinear parts of the axes $AB$. Kepler could not be referring to the conjugate axes here, since it would be $\frac{1}{\sqrt{2}}$ times the transverse side $DE$. 
That was able to satisfy even the most exquisite mind, except that mechanics, and the consideration of the eye that follows in chapter 5, had drawn me back into a new labor. For we have hitherto used hyperbolas, and the mirror reflections from them, especially to represent the distorted images that water show us, when a refraction of rays of objects immersed in it has occurred. And in fact, those things, reflection and refraction, are almost entirely different in kind. The question therefore was raised: what sort of single, continuous surface of water it would be, that would prevent all the radiation received from some nearby point, and diverging in different directions, from diverging once refraction has occurred, but would send them onwards parallel. Whether it would be a parabola, or a hyperbola, or an ellipse, was for a long time in doubt. In favor of the parabola was the equidistance which the parabola shows in reflection. For the hyperbola, there spoke anatomical experience, which is treated below in the consideration of the eye.

God immortal, how much time have the assurances of Geber\footnote{That is, Al-Asama (from al-Geber). However, Frisch (1809 II 406) conjectures that this remark had something to do with an actual work by Geber that was published by Peter Apianus in 1534 as an appendix to his In quaternionem primi mobile, and notes that} lost me! 107
shall nonetheless add a diagram with a problem, if anyone be perchance averse with desire for this cross, i shall prefix a geometrical demonstration that a surface similar to the hyperboloid is required. from point $A$ let radiations $fG$, $Gg$, $Gh$, $Gk$, $Gm$, $Gn$, $Gq$, $Gr$, $Gt$, $Gv$, $Gx$, $Gy$, $Gz$, $Gw$, $Gv$ be 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, 90°. what is required is to say what sort of surface it is upon which these radiations in this position coming forth from $A$ strike; so that they strike just as do here in the lines $Gk$, $Gy$, and so on, so that these lines are either tangents to that surface, or lines equidistant to the tangents. for, such a surface of water being given, it is certain that the radiations diverging from $A$ will turn out parallel after reflection. For a straight line striking upon parallels makes equal angles on the same side, and consequently. Therefore, since these are refracted at such an angle as those coming from $A$, they pass over into parallels after refraction. But they are refracted in an angle of such a magnitude, if they strike thus, as was said, which the experience of Witek showed us in place of a hypothesis. Now, if $SB$ be tangent to the required surface at $S$, while the surface is to encounter the radiation $At$ in obliquity $t$, it therefore must be bent from $S$ below $t$ to $t$, and on extended will encounter the surface beyond $t$ (at $u$, say). But at the point of incidence $t$ there will touch some line parallel to $t$. And whatever line that is going to be, it must not cut $AS$ below $S$. For it would cut the surface extending all the way to $S$, to which it is nonetheless supposed to be tangent. Therefore, a tangent to this surface at $t$ cuts the tangent $S$ in an intermediate place between $S$ and $t$. Let it be extended until it also cuts $Ob$ below $O$. Therefore, again, because the required surface must meet $AO$ at the obliquity of the line $pQ$, and has as a tangent at $A$ a line parallel to $t$, it therefore must pass below the intersection to, and the line that is tangent to it at the place of meeting ($q$ that is), will pass equidistant to $pQ$ between $t$ and $u$, and will cut $q$ below $q$, and there again, moving emulating the obliquity $q$, the surface will meet with $rQ$ below this intersection. that is, at $q$. And a tangent drawn through a equidistant to $t$ will meet with $rQ$ extended at $q$. But the surface will pass below $u$, as so as to meet $q$ at the point $q$, where a line parallel to $tQ$ is tangent. the same is to be understood of $Q$, $F$, $D$, and $B$. In this way, a pair of tangents to the surface always cut each other at places is between the points of tangency. And such an intersection is always given, since $Aq$, $qy$, $vQ$, and the remaining pairs always converge towards the direction $SB$, as is obvious from the angles. For the exterior angle $SQB$ is equal to the interior and opposite angles, which are the angle of

Henry Briggs, in the letter cited above (p. 112), understood Kepler's remark in this way.

Note that the points $q$ and $t$ coincide here, and are denoted by the point $S$. Kepler nevertheless meets them in though they contained an angle.
refraction and the angle of altitude $\Delta \theta$. Angle $\lambda$ is $40^\circ$, $\mu$ is $44^\circ 30'$, $\nu$ is $49^\circ 30'$, $\xi$ is $55^\circ$, $\alpha$ is $61^\circ$, $\beta$ is $67^\circ 30'$, $\gamma$ is $74^\circ 30'$, $\delta$ is $82^\circ 15'$. So, since the lower is always greater, and goes to the same line $AS$, the remaining parts will therefore meet on the right side. Thus since $\Delta \theta$ always becomes less in the direction of $X$ until it finally vanishes at $X$, where $SAX$ represents the horizontal refraction, and the altitude of $AX$ is zero and further, since between $\beta$ and $X$ there can be infinite radiations, each striking at a more inclined tangent; and since the tangents, too, the more they meet obliquely and at a steeper angle with the radiations and the surface, the more distinctly they touch (as point, $BD$ are more distant than $DE'$). Therefore, the surface will be infinite, beginning, that is, (from the magnitude etc., accumulating $\Delta \theta$ and so on, greater than $\Delta \theta$, an infinity of times; and will always situate itself more nearly to the straight line $AX$, because it always has longer parts with a smaller inclination—that is, straight! These, moreover, are found only in the hyperbola, not in the parabola, which tends toward a straight line parallel to the axis, not one meeting the axis, as $AX$ here. Consequently, if the surface is indeed a hyperbola, $AX$ either is an asymptote or is equidistant from an asymptote $PV$ that is further in (which) computation shows. Moreover, that this surface is not only similar to a hyperbola, but is also really a conic section, would immediately be proved if the converse of Apollonius II 29 and 20 were taken. For when a tangent $TV$ is drawn, it can either be drawn through $S$ or through $V$, and therefore through all the intermediate points; and therefore, both passing through that point through which the straight line has been drawn, and lying upon an intermediate point between $S$ and $V$, it is directed into the center of the figure. This is to be said of all other tangents. But because all these are also contained in other kindred surfaces turning slightly aside from the hyperbola, the theorem cannot be completely deduced.

Therefore, if it is wished to enquire whether this figure contain the rest of the properties of the hyperbola, I have got on to this theory, with the following things given. From $B$, $D$, $F$, let perpendiculars $BH$, $DI$, $JK$, fall to $AS$, applied ordinatewise, and let $BC$ be drawn equidistant from $BJ$. Likewise, let $DE$ be drawn equidistant from $GI$, and $FG$ equidistant from $BI$, tangents to the surface at $E$, $D$, and $F$. Thus three angles of refraction from Witelo, with the same number of their inclinations or altitudes, give three classes of numbers, independent of each other and unconnected. If these plainly allowed of being so connected together that at the same time they should submit themselves to the rules of the hyperbola, the hyperbola will be given, and the hyperbolic surface will complete the refractions. For first, from the condition of Witelo's angles, where $AH$ is $100^\circ 00'$, $IB$ will be $57^\circ 35'$; $CH$, $68^\circ 06'$. Likewise, where $AD$ is $100^\circ 00'$, $JD$ will be $45^\circ 53'$; $EF$, $46^\circ 37'$. And where $AK$ is $100^\circ 00'$, $KF$ will be $35^\circ 41'$; $KG$, $30^\circ 24'$. 842, 845

842 Algobit.
845 Apollonius I definition 4: "Of any curved line which is in one plane, I call that straight line the diameter which, drawn from the curved line, bisects all straight lines drawn to this curved line parallel to some straight line... and I say that each of these parallels is drawn ordinatewise to the diameter."
Now we need to find the ratio of the sides of the figure, and the distance $AP$ of the center $P$ from $A$. And the square is $PS$, from these laws, because we are in the hyperbola, as $HHB$ (which for lack of brevity shall be the notation for the square of $HR$) is to $1DI$ and $KEX$, as is the rectangle $CHP$ to $EIP$ and $GHP$, that is, as the latiss rectum of the figure is to the transverse side. And at the same time, $CPH$, $EPI$, $GPK$, and $PSP$ are equal, by Apollonius I 37. Also among the beginnings, for instance, are Apollonius II 29 and 30, because by its power you will be able to establish for yourself the bounds within which the unknown $HJ$ lies. If you prevail in showing that through these angles of this kind what is ordered is more than sufficient, and that the case of a certain hyperbola can be conclusively proved by only two, the operation will be all the more clear, and it will immediately become evident in another combination of angles whether a surface of this kind is a hyperbola. Mechanics shows that it has no better resemblance to anything else; nevertheless, it is a little more scarce than the hyperbola itself near the vertex. When you shall have acquired perfect knowledge of this surface by some procedure, know that you have achieved something great in mechanics. 

6. Causes of the quantity of refractions

For now, reader, I have kept you and myself hanging long enough now, while I tried to gather the measures of different refractions in a single packet, meanwhile acknowledging that the cause is not in this measure. For what do refractions, which we have established, to be fundamentally in the plane surfaces of transparent media, have in common with conic sections, which are mixed lines? For that reason—may God look kindly upon us—we shall now also busy ourselves with the causes of this passage. For even if we shall perhaps still stray somewhat from the goal, it is nonetheless preferable to show our industry in looking around, rather than our inactivity in reaction. If among the optical propositions above we have explained the cause of refractions correctly in general, the specifics must also be correctly derived from the same source. But in prop. 20 above, we proposed as cause the resistance of the medium, by which the spreading of light is hindered, by material necessity. Now it must be seen whether we are able to arrive by following these tracks.

Proposition 1

Where light strikes more obliquely, it is refracted in a greater angle. This is Wielo X 34, but faulty and obscurely demonstrated. If one were to deny this, he would indeed be taking on a great deal of toil in order to demonstrate it legitimately. Therefore, I must make an attempt at another demonstration, which goes like this. Unless the refractory angle were to increase continually with the obliquity of incidence, there would be no universal cause of refraction. For since the oblique incidence is the basis for the spreading of rays, when the former increases, the spreading will increase. But if the angle of refraction were to stop increasing at some incidence (80 degrees, say), at incidences of both 80° and 82° it would be 30 degrees; therefore, the medium would resist the spreading up to an inclination of 80°, from which, over the 2 degrees to 82°, it would no
longer resist. Bnt Ch. 1 Prop. 20 of this book demonstrates that it always resists; therefore, the angle of refraction always increases as the obliquity of incidence increases.

Hence, there is a corollary:

If the medium itself were to be considered in isolation, in respect to its density, the angles of refraction would become proportional to the angles of incidence.

**Proposition 2**

When light strikes more obliquely, the resistance from the same medium becomes greater than at a more direct incidence, even with respect to the same medium. For since refraction is a product (Affectus) of motion when light strikes upon the surface of a denser medium, and the surface, because of its infinite points, terminates, or rather affects here, the infinite motions of the infinite points of light, while it takes on a consideration of density in this regard, not less than the corporeal itself, it will accordingly produce a greater effect if it in a certain respect a denser medium encounters the light. But a denser medium encounters light obliquely. Let A be the light, B C the denser medium, A B, K M parallels, or nearly so, from the sun, whose perpendicular distance is M L. So, since B L M is right, and L B M is by supposition an acute oblique angle, L B M will therefore be less than B L M; and the side L M opposite the smaller angle B, will be less than the side B M which is opposite the greater angle L. But L M is the measure of the breadth of the medium encountering the light coming down directly, because B L M is right, while B M measures the breadth of the medium encountering the light obliquely. Therefore, there is more density in B M than in L M. Therefore, the resistance, in this respect, is greater.

**Proposition 3**

The angles of refractions increase with greater incremental proportions than does the obliquity of incidence. For by the corollary to the first, if even the density alone be considered, the angles of refractions will be proportional to the incidences. Now however, by the second, the rate of L M to M R will also mix itself in. Therefore, the angle of refraction is compounded of one ratio, which is proportional to the incidences, and another, which is proportional to the lines B M. But the lines B M initially increase very little, at a low incidence increase greatly, as the able of secants shows, where ever greater and greater secants correspond to equal degrees. Therefore, part of the angle of refractions is proportional to the incidences, and part increases with greater increments of proportion. Thus the whole angle increases with greater increments.

**Proposition 4**

There is no difference between the refraction of weak light and denser light, other things being equal. For if the light that is weaker were more overcome by
the dense medium, the refraction would be a characteristic of light itself, owing to its essence, and not of motion. But refraction is only the diminution or inhibition of the motion of spreading, and this motion does not accept any modification from the density or dilution of light, i.e., from its strength or weakness, nearness or remoteness, since all motion of light, without exception, is perfectly swift, and simply instantaneous. Therefore, refraction is not in all varied by the weakness of light. The force of the proposition is more evident in another example, in the preceding diagram. Let the surface of the medium enclosed between the perpendicular AC and the oblique radiation AB be represented by BC. Now let A descend on the same radiation AB to S, and a perpendicular SX being dropped, BN will be less than BC, but BN represents the surface combined between the perpendicular SX and the oblique radiation SB. And there is the same amount of light on BN as there was before on BC, which is greater, when the point A was more distant. Therefore, if as a consequence of the light of A or BC being more dilute than the light of S on BN, it were that much more refracted, there would as a result occur different refractions at the same incidence AB by the same medium, according to the approach or departure of A on the line AB. This, however, is in conflict with experience. For the rays of all illuminations, whether approaching or receding, that fall upon the surface of the same medium at the same angle are refracted at the same angle. This is most readily experienced in water, whose refractions are most evident. In air, Tycho Brahe put it to a test, in finding the same refractions of the sun and moon, the former of which is twenty times as far away as the latter. And although in the fixed stars a very few minutes are wanting, no one person would assign the cause to their distance, if he were to consider that Tycho ascribes the same refractions to the fixed stars and to the planets, some of which are nearer than the sun. I shall accordingly give consideration to the cause of this below. It is therefore in vain that Rothmann, in Tycho's Epistolar astronomicae, vol. 1 p. 124,569 notates that unless the cause of refraction be first upon an adventitious vapor, which should not stand above the observer's zenith, it would happen that the refractions of the fixed stars would come out different from the sun and the planets. And although Brahe, in p. 280 of the Promagnusam, suspected that the moon's refractions are greater because of its proximity, he nonetheless found, on p. 124 (which he wrote later), the same refractions as the solar ones, as I have just said.57

Proposition 5

Near the horizon, refractions increase in suddenly decreasing proportion to the inclinations. For since two causes alone combine, there is no third (by the preceding).58 Therefore, one cause is proportional to the inclinations, and the

57. TBOO II pp. 287 and p. 136. However, despite Kepler's claim, the lunar refractions in the table on p. 136 are slightly different from the solar refractions on p. 64 (p. 79 of the 1602 edition).
58. Propostion 3.
other increases with the secants of the inclinations. For because BM, above, is right, and SCA also; and LBM is common to the two triangles, the triangles will be similar. Therefore, as CA is to AB so is LM to MB. But AB are the secants of the inclinations ABC, which at the end of the quadrant suddenly increase. Therefore, BM do so too. But we have said 70 that it is in the ratio of BM to ML, or approximately so, that the refractions increase, apart from their increasing in proportion to the inclinations themselves. Consequently, the refractions also increase suddenly.

Proposition 6

BM, however, 71 is taken into consideration twice, 72 according as, when it makes one or another set of angles with the rays undergo refraction in either a rarer or a denser medium. Therefore, it must be known that it is the secants of those angles of incidence that are set up at the surface in the denser medium that contribute to the measure of the refractions. This is clear from a [reduction to] absurdity. For since near the horizon the secant becomes infinite, if the rarer medium be considered, its effect should also be infinite, and all horizontal radiations should be refracted at an infinite angle. Which is absurd to state. Therefore, it is not the secants of the angles in the rarer medium that are to be taken as a measure; that is, it is not the ratio BM to ML, but the ratio BM to BR. Besides, even if it were only a whole right angle (not more) that is set up by so infinite secant, a single ray from the depth of water or any other medium, existing perpendicular at a single point of the surface, would be scattered over the whole hemisphere in the free air. For the path of the entering and departing ray is the same. And here there would be two paths supposed for the entering ray, one from the perpendicular, the other parallel to the surface of the water, and this in all directions. 73 In demonstrating the true cause of this directly and a priori, I am slack. Perhaps it is this, that since the angles of refractions are in the dense surface, and so is the cause resisting the scattering of light, it is appropriate also to seek the measure within.

Proposition 7

In the densest medium, all refractions take place on the perpendiculars themselves, and are equal to the inclinations. In less dense media, those that have

---

69 In proposition 5, Kepler states that "the angle of refraction is compounded of one ratio, which is proportional to the incidence, and another, which is proportional to the lines BM." He does not specify that it is the ratio BM to MB that is to be taken, and for good reason: in proposition 6 he will show that one must use the secant of the angle that the refracted ray makes with the perpendicular. This is equal to angle RBM, whose secant is the ratio of BM to BR. It is not the same ratio as BM to ML, but the two ratios are approximately the same; hence, Kepler adds "or approximately so" below.

70 The primed text has only the letter "a," here, undoubtedly an abbreviation for "antei"., which is what Frick, JGOU, vol. II p. 298, gives.

71 Cf. Proposition 5 above.

72 The marginal note is a parody of Vergil, Eclogue 3, 104: "Dix quilbus in terris, et eris michi magnum Apollo."
a smaller refraction make it suddenly diminish near the horizon, and begin to be perceived later. For the highest density, by reason of its infinity, causes so great a refraction as to put a stop to all dispersion of light. Therefore, it makes light descend perpendicularly, and appropriates every angle of incidence leaving as a result nothing for the ratio of the secants. For, by the premise, the angles beneath the water will be forever right, and therefore the secant forever the same. All that remains, then, is the ratio of the incidence. But where the refraction is small, by the premise, there is too too little removed from the quadrant for finding the last secants, at a time when they now increase vigorously. On the other hand, where the density is great, the horizontal refraction is great, and too much is removed from the quadrant for extracting the secants of the last refractions. Thus in the former instance, the last refractions increase in a great proportion, while by the latter, in a lesser proportion. Finally, if the greatest refraction of a medium of low density, such as air, is so many times as great as the inclinations of the refraction, and as the smaller secants of the refraction, it is in fact, as supposed, very small indeed; and therefore, any fraction of it that corresponds to the inclinations that are not yet completely horizontal, will be much less perceptible. And so it will begin to be perceived slowly and only near the end.

Proposition 8

Problem I. From a known composite refraction\(^{23}\) of any inclination, to hunt the elements of refraction, and the composite or whole refractions of the remaining inclinations. Let the medium be water, the inclination 80\(^\circ\). The refraction, from Winckel, is 30\(^\circ\). I shall use this since it is larger and less subject to experimental error. So, that which is under water is inclined 50 degrees from the perpendicular, the secant of which angle is 155,572. As this is to the secant of the angle 0\(^\circ\), that is, to the right sine,\(^{74}\) so is the composite refraction, 30\(^\circ\), to the proportional refraction of the inclination 80\(^\circ\). For this has been demonstrated in the preceding. Thus the refraction that is simple or proportional to the inclination, for an inclination of 80\(^\circ\), is 19\(^\circ\) 17', to which is added 10\(^\circ\) 43' because under water, in refraction, it encounters a medium that is rarer than in a straight motion, by the ratio of 155,572 to 100,000. Once the simple refraction of an inclination of 80\(^\circ\) is obtained, let there be a distribution to the other inclinations, since the simple refraction is proportional to the inclinations, angle for angle. Next, let any one be multiplied by the secant of the refracted ray, which is not yet fully known. And

\(^{23}\) From the preceding propositions, the refraction is the continued effect of the angle of incidence, and what might be called the apparent density, which is measured by the lines BM (or, more accurately, by the secants of the angles of the refracted rays), hence, Kepler calls it the "composite refraction".

\(^{74}\) The sine of 80\(^\circ\) is 0.9063.
This tiny discrepancy should not move you: believe me: below such a degree of precision, experience does not go in this not very well-fitt'd business. You see that there is a large inequality in the differences of my figures and Witelo's. But my refractions progress from uniformity and in order. Therefore, the fault

59 Per Croissant. "Civis" (from the Italian "cosa," "thing") originally denoted the unknown in an equation, and later came to refer to the theory of equations as a whole.

60 Witelo 10.8.
lies in Witeelo's refractions. You will believe this all the more, if you look to the increments of the increments in Witeelo. For they increase through 30 minutes. It is therefore certain that Witeelo laid his hand upon his refractions gathered from experience so as to bring them into order through an equality of the second increments. For the straight lines that are taken from a circle, or from any sort of thing of a circular nature, of which kind refractions are, have increments that vary infinitely, and never become equal.

Proposition 9

Problem 2. Given two refractions of the stars arising from the air in a certain altitude above the ground, to seek out the refractions of the other inclinations.\(^7\)

The business is veiled in many perplexities. Let \(A\) be the center of the earth's orb \(BC\), and likewise the center of the sphere of air \(DE\). If both the surfaces \(BC\) and \(DE\) were plane and parallel from a given angle \(DBE\), and a given refraction at that angle upon the surface of the earth let this be \(GEF\), the inclination above the surface of the air of the free ray \(EF\) in the ether, world also be given. Now, because \(BC\) and \(DE\) are spheres, let \(BEG\) be the tangent to the earth at \(B\), and \(EIH\) the tangent to the air at \(E\). So, from the given inclination \(DBE\) of \(BE\) above the surface \(BC\), it does not follow that either the inclination \(GEH\) of the satie \(GEB\), or the inclination \(FEH\) of the refracted ray \(FE\), above the surface of the air, is immediately given. But if only the ratio \(AE\) to \(AC\), or the altitude of the airy surface \(EC\) that introduces the refraction, were known, the angle too would immediately be known. For since \(EB\) is tangent to \(BC\) at \(B\), the line \(AB\) drawn from the center \(A\) to the point of tangency \(B\), will be perpendicular to \(BE\), and likewise \(AE\) will be perpendicular to \(EH\). Thus \(EBA\) is right, and \(EAE\) is also right. And \(BBA\) and \(BUE\) together are equal to a right angle. Therefore, the common angle \(BBA\) being subtracted, \(EEB\) or \(GEH\) and \(BSE\) will be equal. But from the altitude of the air \(CE\) being given in proportion to \(BA\), \(EA\) and \(BA\) and \(EBA\) would also be given; and consequently also \(EAB\) and hence also \(GEH\), equal to the latter. And, when the refraction \(GEF\) is subtracted, there would remain \(FEH\), the inclination of the refracted ray \(FE\) above the tangent \(EIH\). But in fact, \(E\), the altitude of the air making the refraction is unknown. For as for the optical writers' demonstration that the

\(^7\) Here Kepler refers to the note introduced on his p. 79. on the effects of vapors. The note is on p. 90 of this edition.
altitude of the matter of twilight is equal to twelve miles, it does not immediately follow that it is the same matter that makes the twilight and the refractions. We might now suppose that what refracts the rays of the stars is a liquid matter, moist and heavy, distinct from the matter of water only in degrees. It will thus follow that if there exists here on earth something dry, smoky, and fiery, it is going to force its way above this humid body no differently from the way air forces its way up out of water. On this account, the smoky matter will occupy places that are higher than this refractive matter; and, being there, will be illuminated (since it is material) to the brightness of twilight, while not bringing about any refraction. 

For the definition of the transparent, by ch. 1 prop. 11, does not belong to smoky matter; and hence neither does refraction.

Surrounded by these difficulties, let us try approaches, not the geometrical ones that we would like; rather use the ones, comparing pairs of refractions of pairs of inclinations. And at the beginning let us establish reliable inclinations of the same ray in air and on earth, whose discrepancy is imperceptible. For to the extent that the ratio $EC$ to $CA$ is smaller, this discrepancy of angles is smaller, and is perceived more slowly. So, for example, let $EC$ be 95 when $AC$ is 100,000. The incidence will be as follows.

<table>
<thead>
<tr>
<th>On Earth</th>
<th>In Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>87.30'</td>
</tr>
<tr>
<td>89</td>
<td>87.19'</td>
</tr>
<tr>
<td>88</td>
<td>86.48'</td>
</tr>
<tr>
<td>87</td>
<td>86.06'</td>
</tr>
<tr>
<td>86</td>
<td>85.17'</td>
</tr>
<tr>
<td>85</td>
<td>84.25'</td>
</tr>
<tr>
<td>60</td>
<td>59.54'</td>
</tr>
</tbody>
</table>

Let us therefore suppose that at an inclination of 60°, or an altitude of 30°, the rays do not differ perceptibly. At this altitude, however, Tycho, *Pravgammasata* book 1 vol. 79, supposes a refraction of the sun of 1° 25'. And the secant of the angle 60° is twice the radius. Therefore, the simple refraction proportional to the angles is 43°. Therefore, to one degree correspond 43°. Now let us pass on to consider the remaining inclinations. At an altitude of 1° or an inclination of 89°, the sun is refracted by 26°, as is stated on the same page. It is required to find how the incidence of the day to use the occasional Greek word, in much the same way that we misuse French. Where possible, I adopt this analogy in the translation.

It should be kept in mind that these are angles of incidence of the same ray $BEC$ to the respective circles $B A$ and $E C$ that is, refraction is ignored for the moment. Kepler arrives at a first approximation, but the altitude of the surface of the air is 5° where the earth's radius is 100,000. From this, he calculates the angle of incidence of the ray $GE$ at the surface of the air, which is the same as the angle $BEA$. He does this for seven values of the angle $EBE$, which is now no longer needed to be right. The results are in the following table.

118 *TBOO II* p. 64.
how much the altitude of the sun is refracted above the air.\[53\] It is required to find the secant of that number of degrees, which, multiplied by 43\(^{\circ}\), and the product multiplied by the secant of that number of degrees, with the last five digits dropped, amounts to 26.\[52\] Let it be 84 \(\cdot 16\), \(\frac{24}{16}\) multiplied by 43\(^{\circ}\) gives a proportional refraction of \(\frac{1}{0} \cdot 23\).\[26\].\[43\] The secant is ten times the radius.\[84\] Therefore, the refraction is 10 \(\cdot 3\).\[53\] It should have been 26. Therefore let the trial angle be 87 \(\cdot 8\). Multiply \(\frac{27}{8}\) by 43\(^{\circ}\); the simple refraction comes out to be 1 \(\cdot 3\).\[66\] The secant is twenty times the radius. Therefore, the refraction is 20 \(\cdot 3\).\[53\] It should have been 26. Again, let the trial angle be 87 \(\cdot 48\). Multiply \(\frac{27}{48}\) by 43\(^{\circ}\): the simple refraction comes out to be 1 \(\cdot 3\).\[66\] The secant is twenty-six times the radius. Therefore the refraction is 26 \(\cdot 3\).\[53\] It should have been 26. Therefore, in air that ray is inclined in refraction by \(87 \cdot 47\).\[85\]

If the two refractions were correctly determined here, we would now have reached the goal. But because the very small refraction of 1 \(\cdot 25\)\(^{\circ}\) at 30 \(\cdot\), which we have taken over from Tycho, is very easily subject to observational error, not large in itself, but intolerable after subsequent multiplication, it must now be seen whether the rest also correspond. So in the diagram, since it is supposed that \(E\) \(\cdot A\) is 91 \(\cdot\), because \(D\) \(\cdot B\) is 89 \(\cdot\), and the angle in air corresponding to it was found to be 87 \(\cdot 47\), the difference \(B\) \(\cdot A\) is therefore 1 \(\cdot 13\). Consequently, \(B\) \(\cdot E\) is 87 \(\cdot 47\), and as the sine of \(B\) \(\cdot E\) \(\cdot 87\) \(\cdot 47\), \(99925\), is to \(A\) \(\cdot B\), so is the sine \(B\) \(\cdot E\) \(\cdot 87\) \(\cdot 47\), \(99985\), to \(A\) \(\cdot E\), 100,000. So if we now use this to seek the horizontal

\[\text{53 This is a confusing way to state the problem, so a clarification is in order. It is known}\]
\[\text{that when a star appears as 80 from the zenith angle \(D\) \(\cdot \) \(B\) \(\cdot \) in the diagram, it is}\]
\[\text{actually 26 lower, at 89 \(\cdot\). (That is, in the diagram, the angle \(F\) \(\cdot E\) is 26 \(\cdot\). It is also}\]
\[\text{known that for each degree, the part of the refraction is proportional to the}\]
\[\text{angle of refraction that is proportional to angle \(B\) \(\cdot A\) \(\cdot \) in \(3\).} \]
\[\text{From this it is possible to compute through an iteration a value for the angle \(B\) \(\cdot A\), and this in turn}\]
\[\text{will yield a new altitude of the air \(E\) \(\cdot C\), which can be used to compute refractions at}\]
\[\text{other incidences.}\]

\[\text{52 What Kepler needs is the ratio of the secant to the radius. Since the radius was by}\]
\[\text{convention taken to be 100,000, the required ratio is obtained by dropping the last five}\]
\[\text{digits.}\]

\[\text{51 The unit indicated by the symbol} \(\cdot 1\) \(\cdot 60\) \(\cdot\) \(\cdot 16\) \(\cdot 16\) \(\cdot 16\) \(\cdot 45\) \(\cdot 45\) \(\cdot 45\) \(\cdot 60\) \(\cdot 60\).\]
\[\text{which comes out as Kepler says.}\]

\[\text{54 This is the secant of the trial angle of 84 \(\cdot\) 86.}\]

\[\text{55 As in Proposition 8 above, the part of the refraction that is proportional to the}\]
\[\text{inertia is multiplied by the secant of the refracted angle to obtain the total refraction.}\]

\[\text{56 This should be 1 \(\cdot 3\).} \] However, Kepler rounds it off to 1 \(\cdot 3\), knowing that the main}\]
\[\text{determinant of the result is the magnitude of the secant.}\]

\[\text{57 This obviously should have been 37, but the error has no effect upon the final result.}\]

\[\text{58 This is the inclination of the refracted ray \(E\) \(\cdot B\) from the vertical \(E\) \(\cdot A\); that is, the angle}\]
\[\text{\(B\) \(\cdot A\).}\]
refraction on earth, and $EBA$ becomes 90°; then the secant 100.060 results in the angle $BAE$ of 2°.\(^{98}\) So the secant of the angle 88° is 2,665.370. And the simple refraction of the inclination is 63°.\(^{99}\) Therefore the total refraction comes out to be 30° 5'; from Tycho's inclination it should have been 34°. So, although we have not quite hit the target this time, the light is nonetheless open to us, so that we may tell whether it has to be diminished or increased. The increments of the refractions are in fact too large enough. So one should resort to a sharper drop in the secants. But the moments we go to this, the refraction at 89° will be increased too much. Therefore, the simple refraction, which we have derived from the altitude of 30°, has to be decreased.

In sum, it should be noted that the proportional refraction from 89° to 90° is going to be extremely small in the difference of, because, while the inclination on earth varies by one degree, in air it varies by barely 13 minutes.\(^{101}\) And to one degree there belong 43°; therefore, to 13' there belong barely 10°. Therefore, the proportional refraction, being so small, remains perceptibly the same. And it must be multiplied by two secants, so that one makes 26' and the other 34°. And since the ratio of equnmultiples is the same, look for two secants, about 1° apart from each other, that are to each other as 26 to 34. Such, approximately, there is from 88° 54' to 89° 20', by 15 or 16 minutes. Note whether the inclinations in air corresponding to the inclinations on earth of 90° and 89° differ by 16°. If $BAE$ is 0° 50', $A$ will be 10,001.058, where $B$ is right.\(^{102}\) But if $EBA$ is 91°, $BA$ will be to say $BEA$ as $EA$ is to sine $EBA$: $BEA$ comes out to be 88° 45'; before it was 89° 10'; the difference is 25'. They are 25' apart, and the secant of 88° 45' now shows a smaller refraction than 26' at 89°. So we have already passed the mark. The truth is between the secant of 88° and 89° 10' for a refraction of 34', and likewise between 87° 47' and 88° 45' for a refraction of 26'. I shall try 89° for air, corresponding to 90° for earth. Thus $BAE$ is 1°, $AE$ 10,001.524. As this is to 9,996.473,\(^{103}\) so is 10,000,000 to the sine of $BEA$. $BEA$ comes out to be 88° 35'. The secants are 573, etc., for the former.

98 The angle here should be 1° 59', which makes the total refraction 30° 19' instead of 30° 5'.
99 That is, 43° x 88°
101 Without comment, Kepler has made A.B 10,001,000 units in length, to allow greater precision in the evaluation of the secants.
102 This is the sine of 91°. Having established a magnitude for angle $BAE$, Kepler takes the sqy. $BEG$ to an altitude of 1°, and calculates the corresponding angle $BEA$.\(^{102}\)
and 404, for the latter. If 34 gives 573, what does 26 give? It comes out to 438; it should have been 404. We have come closer, but the secants nevertheless still drop off too fast, as you see.

We shall go down another 10 minutes, making $BAE = 19'$, $AE$ will be $10,002.074$. As this is to $3,988,477$, so is the whole sine to the sine of $BEA = 88' 28'$. But the secant of the complement of angle $BAE$ is $49' 6'$. The secant of $BEA$ is $37' 4'$. The ratio of 34 to 26 required that it be $37'$. We have therefore come very close and nonetheless we are still a little off, which will overcome by going down another 5 minutes, an amount that suffices for the refractions of 26' and 34'.

In fact, since the refractions presented by Tycho could not attain such precision from observation as is here required, you accordingly see them rounded off to exactly 34 minutes at altitude 0' and 26 minutes at altitude 1'. Tycho himself, on pp. 79 and 124 does not try to hide this, thus, before the measure is established, others that are a little higher and more easily perceptible have to be compared.

Above, for a refraction of 1' 25' at altitude 30' and a refraction of 26' at altitude 1', a 2' difference of angles on earth and in air suffices. Again, for refractions of 26' and 34' at altitude 1' and 0', a 1' 15' difference of angles sufficed. It must be seen whether a refraction of 8', which Tycho gives for an altitude of 14', likewise results from this hypothesis. So, because the refracted ray inclined in air by 88' 45' makes a refraction of 34', so that the one in the aether is inclined 89' 19', while the secant of 88' 45' is 4,584.023, as this is to 100,000, so is the composite refraction of 34' to the simple refraction of the inclination, which is 44'. When a distribution of this into 58' is made, to one degree there comes 80'. Now let $BAE = 1' 15'$. Therefore $AE = 10,002.581$. Again, let $EBA = 104'$. Its complement, 76', makes $BEA = 75' 57'$. To this there should correspond a refraction of 8', so as to make the inclination of the unrefracted ray in air 76' 5'. The difference of this and 89' 19' is $13' 14'$, to which a simple refraction of 6' 38' corresponds, which is to be subtracted from 44', so that there remains 38' proportional to the inclination of 76' 5'. This, multiplied by the secant of 75' 57', which is 411.015, gives 2' 35''. From Tycho's indication, this ought to have been 8'. Therefore, this latter hypothesis does not suffice for the intermediate refractions, which are perceptible enough. For the secants have dropped off too much; the refraction that is proportional to the inclinations has been made too small.

We shall, however, also try the former hypothesis for the refraction at 14' of altitude. There, to one degree corresponds a simple refraction of 41'. And, that we not demand excessive precision, the secant of the angle in air, and the angle in aether, and likewise also the angle on earth and at the air's surface, hardly differ. Let it accordingly be 76', or 1' 16' ; multiply by 43 this, and the product is the simple refraction, 54' 1'' seconds. But the secant here is four times the radius, plus...
one tenth of it, so the product here is about 4 minutes, and it ought to have been 8, and thus these secants also drop off too much.

Although this disturbance contains much that is disturbing, you still do not yet have cause to suspect a flaw in the general hypothesis, as long as we can suspect something unequal in the Tychoitic refractions. The two certainly change daily, as Tycho Brahe notes on p. 39 of the Præcessi-
nummate, and as will be shown below in its own chapter. Tycho, however, investigated refractions at different times, and indeed a variation in altitude did occur that was perceptible in this matter. For the surface of air that brings about refractions is very low. Next, consider Tycho's differences of refractions; there immediately appears from them something unequal, namely, that it cannot have its focus in any alignment, whether it imitates a circle or any other geometric configuration whatever. See Table 3 below.

 Someone might therefore wish to see how much the refractions at these horizontal inclinations vary in one and the same day. This may be sought in many places from Tycho's observations themselves. I shall give one example. In 1587, 16 January, while the sun was setting, the fixed to 3.58; the sun's declination in observed nine times, from altitude 2° 30' to 9° 35'. It is an easy task to get the refraction for the altitude from these. For the triangle between the sun, the zenith, and the pole, is given, in which four elements are known: therefore, the angle at the sun cannot remain hidden, from which is usually obtained the ratio of the observed refraction to declination to that refraction which belongs to the altitude. Now the side between the zenith and the pole is 34°. 5° 15', the complement of the altitude of the pole at Hven. The observed declination, added to a quadrant, constitutes the side between the pole and the sun; the observed altitude constitutes the side between the zenith and the sun. Finally, the triangle contains the angle at the pole. The place of the sun at the fourth hour of that day is 6° 18' Aquariorum. The true declination of this is 18° 45' 10", and half an hour before is only 20' greater. But something must be added to these declinations for the sun's gnomonix, which is 3 minutes at such a small altitude, so that it may appear how large the declinations of the sun were.

---

53 7 Trolo p. 64. The text has 28 instead of 79, evidently a typographical error.
54 The observations for the following example may be found in Trolo p. 101.
55 Hven, an island near Copenhagen, was the site of most of Tycho's observations.
56 The convention of astronomers of that time was to begin the day at noon, hence, the fourth hour is 4 pm.
57 Kepler followed astronomical tradition in setting the sun's horizontal parallel at 3', which is too large by a factor of twelve. This error had a dramatic effect upon Kepler's determination of the eccentricity of the earth's orbit, which in turn created problems for all other planetary theories. (On this subject see Curtis Wilson, "The Error in Kepler's Acpuency for Mars," Gemin Series 13 [1969], 261-8, reprinted in Wilson's Astronomy from Kepler to Newton: Lunae, Venus, 1999.) Here, its effect was to make all the refractions in the following table 3' too large.
<table>
<thead>
<tr>
<th>Depth of Atmosphere</th>
<th>Refraction</th>
<th>Decrease</th>
<th>Refraction</th>
<th>Decrease</th>
<th>Refraction</th>
<th>Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>in the Sun</td>
<td>M. S.</td>
<td>M. S.</td>
<td>M. S.</td>
<td>M. S.</td>
<td>M. S.</td>
<td>M. S.</td>
</tr>
<tr>
<td>50</td>
<td>34.0</td>
<td>8.0</td>
<td>26.0</td>
<td>5.0</td>
<td>11.0</td>
<td>1.0</td>
</tr>
<tr>
<td>40</td>
<td>26.0</td>
<td>4.0</td>
<td>19.0</td>
<td>4.0</td>
<td>7.0</td>
<td>1.0</td>
</tr>
<tr>
<td>30</td>
<td>17.0</td>
<td>3.0</td>
<td>11.5</td>
<td>3.0</td>
<td>4.5</td>
<td>1.0</td>
</tr>
<tr>
<td>20</td>
<td>10.5</td>
<td>2.0</td>
<td>6.75</td>
<td>2.0</td>
<td>2.75</td>
<td>1.0</td>
</tr>
<tr>
<td>15</td>
<td>6.75</td>
<td>2.0</td>
<td>4.25</td>
<td>1.5</td>
<td>1.75</td>
<td>1.0</td>
</tr>
<tr>
<td>10</td>
<td>4.25</td>
<td>1.5</td>
<td>2.75</td>
<td>1.5</td>
<td>1.25</td>
<td>1.0</td>
</tr>
<tr>
<td>8</td>
<td>2.75</td>
<td>1.5</td>
<td>1.75</td>
<td>1.0</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
<td>1.75</td>
<td>1.0</td>
<td>1.25</td>
<td>1.0</td>
<td>0.75</td>
<td>0.5</td>
</tr>
<tr>
<td>5</td>
<td>1.25</td>
<td>1.0</td>
<td>1.0</td>
<td>0.5</td>
<td>0.625</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>0.5</td>
<td>0.75</td>
<td>0.5</td>
<td>0.45</td>
<td>0.25</td>
</tr>
<tr>
<td>3</td>
<td>0.75</td>
<td>0.5</td>
<td>0.625</td>
<td>0.25</td>
<td>0.375</td>
<td>0.125</td>
</tr>
<tr>
<td>2</td>
<td>0.625</td>
<td>0.25</td>
<td>0.45</td>
<td>0.25</td>
<td>0.25</td>
<td>0.125</td>
</tr>
<tr>
<td>1</td>
<td>0.45</td>
<td>0.25</td>
<td>0.375</td>
<td>0.125</td>
<td>0.25</td>
<td>0.125</td>
</tr>
<tr>
<td>0.5</td>
<td>0.375</td>
<td>0.125</td>
<td>0.25</td>
<td>0.125</td>
<td>0.125</td>
<td>0.0625</td>
</tr>
</tbody>
</table>

Tycho Brahe's refraction table.
going to be if the air had not caused any refraction, See the following table of the entire operation.\textsuperscript{101}

<table>
<thead>
<tr>
<th>Series of Observations of Altitude of Sun</th>
<th>Declination</th>
<th>Declination Without Parallels</th>
<th>Declination of Declination of Altitude</th>
<th>True Declination</th>
<th>Refraction of Declination</th>
<th>Total Declination</th>
<th>To Find Our true Declination of Altitude Sought</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. 50</td>
<td>18 35 30.6</td>
<td>2. 42</td>
<td>18 32 48</td>
<td>18 45 30</td>
<td>12. 57</td>
<td>1 30</td>
<td>14 22</td>
</tr>
<tr>
<td>3. 30</td>
<td>18 34.0</td>
<td>2. 2</td>
<td>18 31.0</td>
<td>18 45 28</td>
<td>14. 10</td>
<td>1. 35</td>
<td>15 45</td>
</tr>
<tr>
<td>3. 20</td>
<td>18 32 30.0</td>
<td>2. 1</td>
<td>18 29 40</td>
<td>18 45 26</td>
<td>13. 37</td>
<td>1. 37</td>
<td>17 44</td>
</tr>
<tr>
<td>2. 50</td>
<td>18 31 45.0</td>
<td>2. 40</td>
<td>18 29 5</td>
<td>18 45 24</td>
<td>16. 39</td>
<td>2. 2</td>
<td>18 21</td>
</tr>
<tr>
<td>2. 40</td>
<td>18 30 30.0</td>
<td>2. 40</td>
<td>18 27 50</td>
<td>18 45 22</td>
<td>17. 32</td>
<td>2. 12</td>
<td>19 44</td>
</tr>
<tr>
<td>2. 5</td>
<td>18 29 0</td>
<td>2. 39</td>
<td>18 26 21</td>
<td>18 45 20</td>
<td>19. 0</td>
<td>2. 23</td>
<td>21 23</td>
</tr>
<tr>
<td>1. 15</td>
<td>18 25 30.6</td>
<td>2. 50</td>
<td>18 20 52</td>
<td>18 45 16</td>
<td>22. 24</td>
<td>3. 29</td>
<td>27 55</td>
</tr>
<tr>
<td>1. 0</td>
<td>18 22 30.0</td>
<td>2. 38</td>
<td>18 19 52</td>
<td>18 45 14</td>
<td>25. 22</td>
<td>3. 33</td>
<td>29 36</td>
</tr>
<tr>
<td>0. 35</td>
<td>18 20 0</td>
<td>2. 38</td>
<td>18 17 52</td>
<td>18 45 12</td>
<td>27. 20</td>
<td>3. 34</td>
<td>31 10</td>
</tr>
</tbody>
</table>

Now to these refractions, which were taken on the same day by the method just now investigated above, we shall compare the measurement, hunting it from the first and last refraction, and comparing it with our in the diagram, let EBA be 35\textdegree above 96\textdegree, at which altitude we have here found the refraction to be 31\textdegree 10\textarcmin. Now let the corresponding angle in the air, BEA is 87\textdegree 30\textarcmin.\textsuperscript{102} so that, when GEF, 31\textdegree 10\textarcmin, is subtracted from GFE, 2\textdegree 30\textarcmin, the remainder FEH is 1\textdegree 58\textarcmin 50\textarcsec, or its complement, 58\textdegree 1\textarcmin 10\textarcsec. The secant of 87\textdegree 30\textarcmin is 2,292,558. Divide this into 31\textdegree 10\textarcmin, or 1870\textarcsec, and the result is 81\textdegree 1\textarcmin 10\textarcsec.\textsuperscript{103} This portion, divided into 88, establishes the simple refraction for one degree as 55\textarcsec 10\textarcsec. And because as sin BEA is to BSA, so is sin EBA to EIA, EIA consequently becomes 10,000,000. Now let EBA be 3\textdegree 50\textarcmin above 90\textdegree, at which altitude we find the refraction here to be 14\textdegree 22\textarcmin. As I.A is to sin EBA, so is BSA to sin 90\textdegree, which becomes 86\textdegree 59\textarcmin.\textsuperscript{104} This differs from the previous value by half a degree, and consequently the inclination of F.E. will be 87\textdegree 14\textarcmin, less than the previous value by three fourths, to which belongs 42\textdegree of simple refraction. There therefore remains a simple refraction of 1' 22'. But the secant of 86\textdegree 59\textarcmin multiplies this little sum by 19. Therefore, it will make

---

\textsuperscript{101} Since the full refraction occurs along a given circle perpendicular to the horizon, the amounts of the refractions measured in the declinations, which are at an angle to the horizon, need to be increased by a factor of the cosecant of the angle at the zenith (i.e., the angle zenith-hour pole).

\textsuperscript{102} Kepler knew that this would be too small. As before, it is the starting point for the iterative process that will determine values for AF and BF, and the rest will correspond to this particular series of observations.

\textsuperscript{103} The quotient, of course, be multiplied by the radius, which is 100,000, to obtain this result. Kepler is applying the refraction computation in reverse, dividing the total refraction measured by the secant of the angle from the zenith in order to find the "simple refraction."

\textsuperscript{104} This should have been 85 28', an error first pointed out by Delaunay, Historie, I. p. 365-6.
the refraction too large, and the ratio of secants is too close. One must carry on towards the horizon. Thus the supposition that $BEA$ is $87.30'$ is false.

Therefore, since $EBA$ is $35'$ above $90'$, let $BEA$ be $89'$, so that the inclination $E.F.$ becomes $89.31'$, and, if the secant of $arc. 89' = 5,729.871$, be made $100,000$, what would $31'10''$, or $1870''$ be? The simple refraction comes out to be $32.13'$, and for one degree, $22''$. Hence, $E.A$ becomes $10,001.005$. Now let $EBA$ be $3'.50'$ above $90'$, then $BEA$ will become $86.5'$, and $F.E.A$ $86.26'$, differing from the previous value by $2'.40''$. That many times $22''$, subtracted from $32.13'$, leaves $31.73'$. But the secant of $86.5'$ is $1,464.011$, [which], multiplied by $31.73'$, produces $7'.40''$. It ought to have been $14'.22''$.

Therefore, the setting of $BEA$ at $99'$ was false, and the secants drop off too sharply. And so we have now gone past the true value. Accordingly, let $BEA$ be $88'.EA$ will become $10,005.578$. The simple refraction, divided into $88.5'$, produces a simple refraction of $44$ for one degree. And where $E.B.A$ is $93'.50'$, $BEA$ will become $85'.43'$, $86$ without refraction; previously it was $88.5'$. Therefore, subtract two times and one half times $44'$, or $1'.50''$, from $65'$; the remainder is $63'.10''$. But the secant of $85'.43'$ is $55'.100'00\times$ times the radius, and therefore the refraction becomes $14'.49''$. Therefore, by setting $BEA$ at $88'$, we have hit the target well enough. The simple refraction should be investigated, when the inclination in air is $90'$. To a simple refraction of $1'.5'$ for $88.5'$ must be added about $1$ second for $1'$ degree, so that it becomes $1'.6'$. From this, the simple refractions for all degrees of air are easily derived.

By the simple refractions, accommodated to the refraction-free inclinations in air, the table is carried through thus. The whole refraction of the preceding degree is subtracted from the following degree; secants are taken of the remainder, casting off the last six digits, since the whole sine in no zeros; and they are multiplied by the seconds expressing the simple refraction of that degree; and the product is divided by 600. If something large comes out, differing from the previous refraction, the operation will have to be repeated twice, sometimes thrice, and so on.

At $60'$ let the whole refraction be $1'.28''$; subtract from $61'$; and there remains $58'.32''$. The secant, $206$ and some, multiplied by the simple refraction, $41'$, gives a product of $9.064$, which, divided by $600$, makes $1'.31''$, the whole

---

126 That is, the secants now do not drop off too quickly; successive secants are too close to each other.
<table>
<thead>
<tr>
<th>Indication of Refraction in Air</th>
<th>Indication of Refraction in Water</th>
<th>Indication of Refraction in Earth</th>
<th>Table's Refraction App.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rel. of Fresnel</td>
<td>Compos.</td>
<td>Refr.</td>
<td>Rel. of Fresnel</td>
</tr>
<tr>
<td>Dec 1</td>
<td>0&quot;</td>
<td>0&quot;</td>
<td>45&quot;</td>
</tr>
<tr>
<td>Dec 2</td>
<td>1&quot;</td>
<td>1&quot;</td>
<td>46&quot;</td>
</tr>
<tr>
<td>Dec 3</td>
<td>2&quot;</td>
<td>2&quot;</td>
<td>47&quot;</td>
</tr>
<tr>
<td>Dec 4</td>
<td>3&quot;</td>
<td>3&quot;</td>
<td>48&quot;</td>
</tr>
<tr>
<td>Dec 5</td>
<td>4&quot;</td>
<td>4&quot;</td>
<td>49&quot;</td>
</tr>
<tr>
<td>Dec 6</td>
<td>5&quot;</td>
<td>5&quot;</td>
<td>50&quot;</td>
</tr>
<tr>
<td>Dec 7</td>
<td>6&quot;</td>
<td>6&quot;</td>
<td>51&quot;</td>
</tr>
<tr>
<td>Dec 8</td>
<td>7&quot;</td>
<td>7&quot;</td>
<td>52&quot;</td>
</tr>
<tr>
<td>Dec 9</td>
<td>8&quot;</td>
<td>8&quot;</td>
<td>53&quot;</td>
</tr>
<tr>
<td>Dec 10</td>
<td>9&quot;</td>
<td>9&quot;</td>
<td>54&quot;</td>
</tr>
<tr>
<td>Dec 11</td>
<td>10&quot;</td>
<td>10&quot;</td>
<td>55&quot;</td>
</tr>
<tr>
<td>Dec 12</td>
<td>11&quot;</td>
<td>11&quot;</td>
<td>56&quot;</td>
</tr>
<tr>
<td>Dec 13</td>
<td>12&quot;</td>
<td>12&quot;</td>
<td>57&quot;</td>
</tr>
<tr>
<td>Dec 14</td>
<td>13&quot;</td>
<td>13&quot;</td>
<td>58&quot;</td>
</tr>
<tr>
<td>Dec 15</td>
<td>14&quot;</td>
<td>14&quot;</td>
<td>59&quot;</td>
</tr>
<tr>
<td>Dec 16</td>
<td>15&quot;</td>
<td>15&quot;</td>
<td>60&quot;</td>
</tr>
<tr>
<td>Dec 17</td>
<td>16&quot;</td>
<td>16&quot;</td>
<td>61&quot;</td>
</tr>
<tr>
<td>Dec 18</td>
<td>17&quot;</td>
<td>17&quot;</td>
<td>62&quot;</td>
</tr>
<tr>
<td>Dec 19</td>
<td>18&quot;</td>
<td>18&quot;</td>
<td>63&quot;</td>
</tr>
<tr>
<td>Dec 20</td>
<td>19&quot;</td>
<td>19&quot;</td>
<td>64&quot;</td>
</tr>
<tr>
<td>Dec 21</td>
<td>20&quot;</td>
<td>20&quot;</td>
<td>65&quot;</td>
</tr>
<tr>
<td>Dec 22</td>
<td>21&quot;</td>
<td>21&quot;</td>
<td>66&quot;</td>
</tr>
<tr>
<td>Dec 23</td>
<td>22&quot;</td>
<td>22&quot;</td>
<td>67&quot;</td>
</tr>
<tr>
<td>Dec 24</td>
<td>23&quot;</td>
<td>23&quot;</td>
<td>68&quot;</td>
</tr>
<tr>
<td>Dec 25</td>
<td>24&quot;</td>
<td>24&quot;</td>
<td>69&quot;</td>
</tr>
<tr>
<td>Dec 26</td>
<td>25&quot;</td>
<td>25&quot;</td>
<td>70&quot;</td>
</tr>
<tr>
<td>Dec 27</td>
<td>26&quot;</td>
<td>26&quot;</td>
<td>71&quot;</td>
</tr>
<tr>
<td>Dec 28</td>
<td>27&quot;</td>
<td>27&quot;</td>
<td>72&quot;</td>
</tr>
<tr>
<td>Dec 29</td>
<td>28&quot;</td>
<td>28&quot;</td>
<td>73&quot;</td>
</tr>
<tr>
<td>Dec 30</td>
<td>29&quot;</td>
<td>29&quot;</td>
<td>74&quot;</td>
</tr>
<tr>
<td>Dec 31</td>
<td>30&quot;</td>
<td>30&quot;</td>
<td>75&quot;</td>
</tr>
</tbody>
</table>

Below the earth.
for degree 61. When this is subtracted from 62, the remainder is 61:58:29"; whose sexant is 21 and some. This multiplied by 45", the refractions of degree 62, leaves a refraction of 1° 35" for degree 62. Again, at 84 the refraction be 9° 43". Subtract from 85. The sexant of the remainder, 111 and some, multiplied by the simple refraction 63", and divided by 600, gives 11° 39", which differs in position from the previous one, 9° 43", by nearly 2 minutes, where the sexants are now increasing vigorously. Therefore, repeat the operation, and in place of 9° 43" of the previous degree, now subtract 11° 39" from its degree of 85. The sexant, 1101, multiplied by 63", gives 11° 36", the correct refraction.

Moreover, in order that the inclinations of the refracted rays be accommodated to earth, always multiply E.A. 10.005,578, by the sex of E.A. or the inclination of the refracted lines in air.

I conclude this tiresome investigation with the setting out of this table, in which we come very close to the Tychoitic solar refractions, differing by only four minutes near a solar altitude of 15°, and at the same time we satisfy those that were observed by Tycho within a single day. They would perhaps not differ even by that much if what was said in proposition 6 of this chapter had been correctly carried out. Again, I beg the reader not to pursue this very stupendous investigation of mine into the refractions provided by Tyche, for undermining the whole treatment of refractions, for the confirming of which I undertook it. And if I judge well of these matters, I have fully confirmed it, while thus which Tycho considered to result from the unequal thickness of the air is its varying density, I myself demonstrate this, or something not much different, from the ratio of the circle and of sexants, from principles that are clearly demonstrable. Let it be enough for me to have given notice, I know how today many blind persons would argue about colors, and how they desire to bring forth someone who would in some way help their rash insulatio upon Tycho and even upon this very matter of refractions. 107 Had they held their childish errors and their sheer private ignorance, they would have been without blame, since that happens to many great men, but because they publish, and make marvellous attacks with thick books and fine sounding titles for the profit of the ignorant (so that today there is more danger from the abundance of bad books than there was once from the scarcity of good ones), let them know accordingly that room is available to them to emend publicly their own public errors, so that, where they have prostrated doing so for a very long time, it will be permissible for me or anyone else to do the same for them (having unfortunately entered upon geometrical matters) as they have taken upon themselves to do against the most prominent men. Although this task is disdained; and is sure to be involved in a worthless topic of absurdities, it will, however, be as much more necessary than what they themselves had taken up

106 The moon is supplied conjunctually. One would expect "rays here, but "refraction" is feminine, so the best plausible candidate is "linearum."

107 It is evident from the correspondance of Kepler and David Faustius that the present query is very directed against Jacob Christman, a Heidelberg professor who wrote a book entitled Observationum solarium libri III, Israel, (161). He argued that the vamps that refract the solar rays and make the solar hole bigger have no effect upon the measurement of the mo solar altitude: the solar paradox alone suffices for this.
against others, in proportion to the greater public harm that is done by one who endeavors to undermine good and necessary discoveries of others, in defense of the truth, that by one who prejudices himself that he has found something that cannot be found. Let them meanwhile cease in the sciences of others, to boast in their own obscurity.\footnote{108}

\section*{Proposition 10}

\textbf{Problem III. From the quantity of refractions to investigate the ratio of media to each other: such as that of air to water, as concerns their density. In Proposition 6 of this chapter it was pointed out that the simple refraction of that inclination that belongs to the ray in the denser medium above the common surface, is multiplied by the secant of that inclination that belongs to the refracted ray in the denser medium above the common surface. And so let the simple refractions of the same inclinations be compared, so that one might not place different positions in the two media upon the same incident ray. As, in Prop. 8 of this chapter, at an inclination of $90^\circ$, the simple refraction of a ray from air to water is $19^\circ 17'$. At the same inclination of $90^\circ$ in Prop. 9 of this chapter, the simple refraction of a ray from water to air is $59^\circ 7'$. Therefore, from air to water there would be a simple refraction of $19^\circ 18'$. Thus the ratio is the same as that of unity to $11772$.\footnote{109} And as a cause for refractions of this kind, there was alleged only that density which is considered on the straight line, because the surface of refraction, as named by the optical writers, intersecting the surface of the denser medium, establishes an intersection line. And although the same surface intersecting the denser body establishes an intersection surface, it was nonetheless demonstrated in the first chapter, that no refraction occurs in the denser body, but it all happens on the surface. Thus if you were to make a cube from the two terms of the ratio, you will find that ratio of the bodies as concerns their densities, which is between $1$ and $1633,304,360$.\footnote{110} Nor is there any doubt that if one were to start in pure water and were from here to pierce out one dipper of water, and then sixteen myriads of myriads of dippers of air, that these would weigh the same.\footnote{111} And in a chamber or cube 12 feet long, wide, and high, there is contained no more matter, when it is filled with that pure air that is contiguous with the water, than there is in a cubicle that extends for the eighth part of an inch in all directions. He should, however, doubt whether it might not be enough to square the terms of the ratio, or rather multiply them by their ratio.\footnote{128}}
Proposition 11

Problem 4. From the refractions to investigate the altitude of the air from earth. Now because the matter of the air is fluid, as is argued by the evidence of winds, which are, in my opinion, nothing but an abundant boiling up from the highest mountains, from where, by its own nature, matter, in again seeking the low places (which is not out of keeping with Aristotle, Problems, Section 25 Prob.I.3), is poured around the mountain in a circle; and from the initial impulse, one circle arises from another, as also occurs in pools of water. What follows from this is what was used in Proposition 9 of this chapter, that the matter of air embraces the orb of the earth circularly. Therefore, for investigating the measure of the refractions of air, there was obviously need of supposing an altitude for the sphere of the air. The altitude of the air will accordingly be most nearly that by supposition of which we most nearly approached the measure of refractions. Now E.A. above, was 10,005,578, where the distance from the center of the earth A to its surface was 10,000,000. Therefore, by the rule of proportions, if

spaces is $125^{1/3} : 1$, or a bit more than Hellen myths of myths, nearly the ratio of densities given above.

That is, to take the $1/3$ power.

Physics: These are not "physicists" as we use the word today; rather, they are those who profess one or another variant of the natural philosophy of Aristotle.

Aristotle considered air to embody heat and wetness, water being cold and wet and fire being hot and dry. Cf. On Generation and Corruption II.3, 330b 4, trs. Forster pp. 274–

10,000,000 becomes 860 German miles. according to the received tradition of the geographers, the altitude of the air, 5578, will give 1.777 miles. That is, in the arc of Uraniborg,115 where the observations of the heavens were made, the altitude of the air was half a German mile, no more.

Here the reader might also be advised of the controversy which Tycho had with Rothmann concerning the substance of the air and the aether, in volume I of the *Epistolar astronomicae*, which Tycho concluded on p. 92 of the *Phaenomena*.118 Rothmann and Pena had said that from earth to heaven there is nothing but air, with the exception of a little vapor.119 Tycho joined them in eliminating the Aristotelian fire, and admitted vapors around earth different from air, but extended the air to the boundaries of the moon, and said that there it gradually comes to an end in aether. I, moved by the present experiments, hold the middle. First, what they called vapors, I call air, and I end it with the peaks of the mountains above that, the smoky exhalations, lamps of the twilight, are set, and the aether follows immediately.

7. Consideration of those things that Witelo advised were necessary for astronomy.

This would now be time for me to insert Witelo XI 49 and the following propositions, had they not already been well enough brought home by Witelo himself: in Prop. 49 he shows by experiments that refraction of light occurs among observations of the stars.121 Tycho Brahe proved the same thing with instruments of the most exquisite accuracy. See the various ways on p. 15 and p. 53 of Tycho's *Phaenomena*.122 Others have proved the same, for which see below. And indeed, by aspherical spheres, when the pole is high, the stars appear closer near the horizon than on the meridian; and this experimental procedure is a good one, used by Brahe along with the others. However, he was unable to believe that it was properly carried out by

117 The German mile was five Roman miles, or about 7.4 kilometers. Kepler describes how to measure the radius of the earth in the *Epitome astronomiae Copernicanae* I (Linz: 1601), p. 39, where he comes up with a slightly greater number.

118 Tycho Brahe's observatory on the island of Hven (q.v.)

119 For this debate, see Section 1 of this chapter (p. 93 above). The relevant part of the *Epistolar* begins on p. 134 of *TBO* VI. The passage from the *Phaenomena* is on p. 77 of the same.

120 In his introduction to his 1557 edition and translation of Euclid's *Optics* (Euclidis *Opticae et conopticae, cum planit antichae graece ordinis. Eadem latine redditio. Paris 1557*), Jeanes Pena argued on the basis of lack of refractions that the heavens cannot consist of "spheres" of different substances. Frisch includes extensive quotations from Pena's work in *TBO* II (p. 573.) In the traditional cosmology, there was a sphere of fire between the air and the heavens.

121 *Phaenomena* II p. 444.

122 *TBO* II pp. 16 and 78-82.
Witelo in that age, or by Alhazen sp. 91 of the *Progymnomaematica*. But that Witelo added another way, that the place of the moon be computed and compared with observation, is a splendid fiction, really from Witelo's time. There is, of course, no doubt that the same thing happens to the moon that happens to the other stars. But there is so much uncertainty in the parallelaxes and in the mean motions of the moon, that the refractions would often have ended up in the wrong direction, even right at the horizon, where the refraction was greatest.

Thus Witelo's X 50, that aether is rarer than air is also pertinent here, as Rothmann protests in vain, denying the difference and placing the blame for refraction upon fortuitous vapors which would not cover our heads. But he was in error, not considering that the actual cause of refractions makes them imperceptible at high altitude. This proposition, moreover, Witelo demonstrates legitimately, from the fact that the altitude of the stars above the horizon appears not less, but greater than it should.

Now, Prop. 51, 125 that the distances of stars at the zenith overhead appear less than they should, is in fact true, but whatever there is of this effect is imperceptible. For, by 9 of this chapter, the refractions at the zenith are also imperceptible.

Thus also, in Proposition 52, 126 that the distances of a pair of stars, or their diameters, appear smaller than they are in actual fact when they are parallel to the horizon and near it, is again true, but completely imperceptible, unless the stars are separated by a semicircle or a little less. Let AB be a portion of the horizon, C the zenith, CA, CB quadrants, and let A appear by refraction at D, and B at E, in such a way that the distance AB should appear to be DE; DE will indeed be less than AB. But because the arcs AD, BE, are not greater than 30 minutes. AB and DE will be perceptibly equal. For let ACB be a half quadrant, a distance that is seldom actually taken with instruments, never, of course, by the Tychoean sextants, never by astronomical radii or staffs, to keep the eyes from having to turn around too much and make the observation untrustworthy. Therefore, perpendicular CF being dropped, CFE will be right, FCE 45° because DCE is isosceles. 127 and CE is 89° 26'. And as the sine of CFE is to the sine of CEF, so is the sine of FCE to the sine of FE, whose double is DE. Thus in the case in which AB is 90°, the distance DE involved in refractions be

---

123 *TBOO* II p. 76.
125 *Thesaurus* II pp. 415-6.
126 *Thesaurus* II pp. 446-7.
127 Note that arc DE is a great circle arc, and is not parallel to AB. Therefore, the angles at D and E are not right, but they must be equal. Under these circumstances, a perpendicular dropped from C to DE must bisect DE; hence, angle FCE is 45°.
Paralipomena to Vitelo

89 \textsuperscript{59-63}: the difference is \textsuperscript{17}. It will be even smaller where a smaller distance \textit{AB} is taken.

Those things that are said is Proposition \textsuperscript{53}.\textsuperscript{129} however, that the distance of the stars or the diameters of the heavenly bodies set up from the zenith towards the horizon, are less than they should be, especially if one end is near the horizon—these things, I say, are both true and evident to the senses, and are in the highest degree necessary for the astronomer to know. So much so that I do not believe that this proposition has been read by these who accuse the Brahman refractions, concerning whom see the end of Prop. \textsuperscript{9} of this chapter. I therefore refer them to this proposition.

Therefore, what Vitelo presents in the following Proposition \textsuperscript{54}.\textsuperscript{129} that all stars near the horizon appear smaller than they should, is true, but his contention that they also appear round, is false. For it is not true that all of their diameters appear uniformly smaller—one that is set up towards the zenith comes out perceptibly smaller, while one that is equidistant from the horizon is diminished by an imperceptible amount. Thus the refracted figures acquire an oval figure, like that of a coin thrown into water, if viewed from a very oblique angle. And this is the sole proposition on which those who deny refractions founded their apprentice-work in optics: that there is no force to the refractions, except that of expanding or contracting bodies seen under refraction. What they subsequently build upon it, concerning the directing of vision to the center of the heavenly body, cannot be any more solid than this proposition of Vitelo’s. Let it be that the lower edge of the sun graze the horizon: its refraction will be \textsuperscript{34}. But the refraction of something higher, such as has an attitude of half a degree, will be \textsuperscript{29}. Thus, in place of the sun’s diameter, \textsuperscript{30}, there will be seen a quantity of \textsuperscript{25} minutes, while the transverse diameter, or that parallel to the horizon, will be \textsuperscript{30}. The \textit{semidiameter} of the sun \textsuperscript{12}, when the sight is directed to it, besides error from the whole body, will also err from the center of the body seen under refraction by \textsuperscript{2} minutes.

In the same Proposition \textsuperscript{54}, Vitelo proposes two other causes that alter the diameters of the luminaries. The first, he says, is a thick vapor bound on this side and that by two surfaces, one of which is opposite the star, and the other opposite our vision, like a convex lens.\textsuperscript{130} Of this, certain books of spherics have imitations, even using the same words about a coin thrown into water. Therefore, reader of these matters, be studiously careful not to get confused. The action of ordinary refraction is in itself such that a decrease of diameters follows upon it, not an increase—so far, Witelo has it right. Or rather, if on the contrary the diameters should increase anywhere because of refraction, it is necessary that the distances also increase there, as in certain examining glasses, as well as in the coin thrown into water, if it be viewed from the perpendicular. But here there occurs exactly the opposite to that case that exists in which the coin is thrown out into water. For you are viewing from a rarer medium the coin that is lying in a

\textsuperscript{129} Thesaurus II pp. 447-8.
\textsuperscript{129} Thesaurus II pp. 419-9.
\textsuperscript{130} Peripatetikon.
deeper medium but we see from the deeper medium of the air, the stars existing in the rarer medium of the ether. Thus, when Wieland following Cleomedes, 131 and others, following him, explain the reason why the stars sometimes appear larger, be adduces, not the air, in which we otherwise exist, as is done in the usual explanation of refraction, but a thick vapor, which stays in the middle of the air like a cloud, so that the ray of a star viewed thus has a transition, first from the ether into the air, from the air into the convex vapor, again from the convex vapor into the air, and finally from the air into the eye, something this could delude you.

The cause of the delusion I believe many find in a typographical error, which all copies have, even the corrected ones published by Frideric Riiser. For instead of what we read, that all stars appear rounder, greater, and so on, “smaller” should be read. 132

However that may be, if I may unveil my own opinion here, this cause ad
duced by Wieland, following Cleomedes, is hardly sound one for this event (even though I am going to introduce it below in demonstrating another phenomenon). The causes are these: It is a very common event, that when winds, or warm air after a hot freeze, arise, the lights of the stars appear enormous to us. Aristotle plainly asserts this in Sect. 26 Q. 31, 133 saying that When Eurus blows, everything looks bigger. And so since this happens so often, it could not exist because of a chance intervention of vapor between us and the stars. Besides, all the stars of the whole hemisphere are seen in this way at once, but some phlegm vapor saturated from the rest in one place were the cause, this would be seen to occur sometimes in one part of the heavens. Thus, I ask, by what force does a phlegm vapor, much heavier than air, stay suspended in air, in order that it, like lenses made of glass or crystal, could magnify the stars? The instance of clouds has no force. For clouds hanged in air, and moist and things of this sort, introduce darkness, and are not phlegm; and since they appear to be of a somewhat dry nature after they rain (for when they are heavy with humidity they do not hang, but fall down as drops), it is no marvel that they float upon air, as wood upon water. But how will vapor that is humid, phlegm, fluid thick, and heavy, be held suspended in air, which is much lighter than itself? Why when this happens, is it not rather evidence that bo
ing matter ascends from the lowest viscera of the earth, and this that the parts that are lower and contiguous to both the earth and to our eyes are filled up first: the result is the same for us as for those who sleep, for in both cases the eye is in physical contact with the humid. Furthermore, the same things usually happen in refra
cion, to both the diameters and the distances of the luminaries. But this accident

132 Riiser was the editor of the Thesaurus. It should perhaps be remarked that the chip proposed by Kepler would require the substitution of “smaller” for “fainter” for some stars in at least four places, and a substantial emendation of Wieland’s geometrical argument.
does not affect the distances. For it has never been observed that the distances appear greater than they ought to; they are always either as they should be, or less than they should be. Finally, this thing that happens to die stars in a some contrary to that of the refractions, happens at the moon to less than at the horizon. Therefore, it is necessary that this does not belong to that class of refractions of which we have spoken so far. It will thus not be an accident of quantity, or of the visual angles, but of light alone, roughly the same sort of thing as haloes, rainbows, scintillation, and the like. Of these meteorological phenomena, the causes do not hitherto appear well enough explained; but since they are hardly necessary to astronomical considerations, I have not hitherto given them special attention.

A third cause by which Witelo says the apparent diameters and distances of the stars vary, is truly optical, and does not have to the light, as does the second, nor to the visual angle and the actual quantity of the image, as does the first. Nor does it make the ones near the horizon smaller, but it does have to the estimation of quantity, and deceives this faculty of vision into imagining for itself a larger object than ought to appear by virtue of the visual angle. For when the eyes are turned upwards, nothing, in between encounters them by which they may estimate the distances of the heavenly bodies. We therefore reckon those stars overhead to be extremely close, and consequently also smaller, the angle remaining the same. The opposite happens at the horizon. For then, if the interposed tract of the earth on one horizon are comprehended in a single view, they instinct the vision to some extent about the enormous distance, from which the quantity of the object viewed (whether it be the distance of a pair of heavily bodies or the diameter of one heavenly body) appears rather large, the angle remaining the same. For of those things that are perceived under the same angle, those that are farther away are greater, and thus that are less [far], smaller. This cause does not affect astronomers much, because the observation that is carried out with instruments does introduce any error from this source. The only important thing is this: when we read that the ancients carried out their observations, not with instruments, but by estimation of the distances, we know that they might have been in error in this matter, since the faculty of estimation itself is egregiously in error through this cause. This was taught by Tycho Brahe, and by Ptolemy himself in Book 9 Ch. 2.

8. Whether the refractions are the same in all times and places.

The refractions in maritime locations are more constant; for those inland they are sometimes nearly nothing, and sometimes prodigious. For in maritime locations the air keeps about the same altitude, and it is deep enough that even if some altitude were added to it, that would not be so perceptible. In inland locations, however, the air is lower, so much so that on certain mountains one could not even stay alive unless one put a sponge to the nostrils, as Arateo attests of Olympus in the Hyperaspis. The Peruvians relate something similar of
mountains that one cannot approach because the bottom spirit fails. And Bodin relates from an Indian history that a great many of the Spaniards died of cold when they moved across the highest ridges of the mountains beneath the equator, while the skies were burning with heat. For the highest mountains project out of the air, like rocks out of the sea, and the inland region, near to the sources of the rivers, because of the altitude above the shores of the sea, which provokes the downwards flow of the rivers, correspond to some extent to the shallows in the sea. Hence the common opinion that the air in the Alps is more beautiful, because it is thinner and more clear of impurities, the thicker air settling down into the valleys, and so in some places there will be no refractions and in other places they will be very slight, and all nearly at the horizon itself. For it is in accord with this that there are many places that are elevated half a mile above the surface of the sea, and this, by 11 of this chapter, is the elevation of the surface of the air in the Danish strait. Vitruvius, in Book 8 Ch. 7, specifies the two hundredth part of the course for the gradient in aqueducts. There is no doubt that such a height will be renowned for navigation. And so if you allow a hundred miles for the meanderings of the Elbe, there will be much less than half a mile left for the elevation of the Austrian plain, arising from which, it flows into the Ocean, because it flows gently and forms pools. But the Danube, carried down through almost another hundred miles again delimits the plain of Swabia, higher than that of Austria. So it will be brought about, that the Vosges and the continuous Alps of Raetic nearly surpass the altitude of the air, which brings about the refractions. The perpetual snows confirm this, for they indicate that their strata are not clothed in the vaporous air. Therefore, to the greatest extent there will be no refractions there. But if storms threaten, and sudden vortices arise out of the neighboring mountains, before they flow down and disperse themselves into the uniformity of the atmosphere, it is fitting that they perform their function in the matter of refractions and that this would vary very greatly because of the surface.

23 Jean Bodin, Methodes ou les lois historiariam cognitionem, many ed. It is not certain which passage Kepler is referring to, possibly he had confused this book with another. Cf. Epistola 1.3 (J. KB VII p. 48s). where he presents the same example and, in the Errata, says he doesn't know the author.

24 Marcus Vitruvius Pollio, fl. 1st C. B.C. I.e., known entirely for his De architectura, which covers upon architecture but contains excursions into clocks and timekeeping, navigation, and astronomy.

25 A former Roman province including much of the Tirol, Switzerland, and Bavaria.

26 Here Kepler refers to the following example:

To p. 135 I. 31: A perfectly clear example: On 1685 May 10 (20), the sun set from a far number of degrees of latitude without any rays, with a dull face, as if through water, the sky being in the entire clear, I saw in the part of Styria which they call by the German word that means "The Hills" to the west are the highest mountains. Therefore, the sun's distinct appearance, and the uniform color of the air, were arguing for a gaseous matter, but devote, as might be proportionate to sunlight that is very much weakened, its light being drunk up, so that the sun could not draw a shadow around anything. In three days there followed a tremendous rain, and a nimbus of overflowing of those regions.
that, in the boiling up of the vapors, is uneven. In that case, then, a place sees greater refractions in proportion as it is nearer to the boundary of the air, 136 because air is circular, and (as a consequence) it refracts at the greatest angle the rays of the sun falling upon it. This observation is described by 81 [1], as was just now clear in Prop. 9. But those rays, thus refracted, traverse only the highest places of the air; those, on the other hand, that penetrate to lower places, are refracted with a smaller angle, because they also strike the air more directly. See the diagram in Ch. 7 No. 5 [p. 230 below].

This same thing is confirmed by the testimony of learned men and by experience. Certainly, Rothmann in Hesse, which is near the sources of the Weser, constantly affirms that the refractions of the stars commonly appear less 140 than those that Tycho Brahe observed in Denmark (pp. 29 and 85-6 of Tycho’s Epistolae astronomicae), 141 and he urged Tycho to reckon the refractions to be diminished at places to the south, which is indeed true of Hesse, but not in proportion to its removal southward from Denmark, but in proportion to its greater elevation above the center of the earth and its greater nearness to the surface of the air. See vol. 1 of Tycho Brahe’s Epistolae, pp. 65, 64, 65, and 112, where Tycho, refuting Rothmann’s argument, confutes my argument. 142 Indeed, his having experienced

136 Here Kepler refers to the following endnote:

On line 25 following: When I attribute greater refractions to high places, you should understand “horizontal” refractions. For those that occur at some altitude of the heavens are proportionately smaller, and drop off proportionally more rapidly, as was demonstrated above.

140 Here Kepler refers to the following endnote:

Below this on line 34: In Hesse, which is high, the refractions are said to be smaller, that is, they are rather high, and are those at which the stars can be observed. For everywhere they observe, or completely horizontally, or elsewhere to be enormous, one right at the horizon they rarely captured the stars with instruments. For these two things must be carefully distinguished: First, that in some region or time, the refractions can be smaller because of the thinness of the air, and here, upon the diminution of those that occur at some altitude of the heavenly bodies, there does indeed also follow the diminution of the whole refraction, that is, of the refraction that occurs in the ray that is tangent to the sphere of the air. But, second, in this same region and time, the horizontal refraction can nonetheless appear greatly. For the reason that the observer’s nearer the surface of the air because of the altitude of the region, and is blessed with the ray tangent to the sphere of the air, most greatly refraction, which ray would have passed far above his head, if the observer had been standing in a lower place. For it does not happen in all places that the greatest amount that the refraction attains is also that which the setting sun has, as I have taken pains to impress upon you.

And it is clear per se that there is some part of a smaller quantity, nearer to the whole, which can be greater than some part of a greater quantity, more distant from it whole. Therefore, even though the whole refraction of a tangent ray is sometimes smaller in one place than in another, nevertheless, that part of it which the setting sun has, can be greater than the sun at another place.

141 TIBO VI pp. 51 and 114

142 TIBO VI pp. 92-3 and 140-1. The text here has 85 instead of 65, which cannot be correct as 65 is in a letter of Rothmann 65 is a conjectural emendation.
colder winters in Bavaria (which he afterward also asserted of Bohemia) clearly argues that the depth of the air is less in the mountains than on the coasts. For the coasts of the Atlantic Ocean, they bring in another cause, that warm winds are stirred up by the daily sides of the sea, which spread over the coasts, so that the sun rarely lasts long in the Baltic, however, there are no waves, or very slight ones; and the cause I proposed stands intact: nor, however, that which Rothmann brings, that the air in Denmark is thicker because of the slightly increased elevation of the pole; but this, that the air is deeper in this part of the sea in which the Næstved poured north than it is in the plain whence that river arises (for the level of its Ocean is the same as that of the Baltic, which flows together in a nearly straight). I mean we do not deny that it can also be thicker, but this is the highest North. On the same p. 112, Tycho opines that even in different regions of the same horizon, the refractions can change because of different vapors; much more so, therefore, in different places. The Landgrave of Hesse, in turn (p. 22 of Tycho’s Essays), asserts that on a certain night the star Venus was carefully observed by him to remain stationary on the horizon for about a quarter of an hour, as if it were not being carried at all by the motion of the primary mobile, while it would have seen a further two degrees. Afterwards, it suddenly vanished. This unusual sight arose from no other source than a gradually rising thick vapor, that was immediately again diffused in the level of the air.

A not dissimilar experience was related by my teacher Maestlin in the Theses de eclipsebus which he published in 1656. These are his words in Thesis 55.134

In the year 1590, July 7, as the center of the sun was emerging above the horizon, we here at Tubingen saw the moon, already two digits135 eclipsed from the south, at an elevation of almost two degrees, and on the other hand, as the center of the moon was descending beneath the rising point, we noted that the sun’s altitude was two degrees above the rising point. Moreover, the moon was before the eclipse reached its maximum darkness.

From these it is deduced that the horizontal refraction on that day was much greater than two degrees, half of which was due to the sun, and the rest to the moon. For, with the center of the sun rising, it was fitting that the center of the shadow should be setting, and since the moon had now at that time begun to enter the shadow, while it was usual for it to enter from the west side, the center of the moon was therefore more westward than the center of the shadow; and since this was setting, the center of the moon had already set. And yet to the sight it was still raised by two degrees. Therefore, there was more than two degrees in the aggregate refraction of the sun and the moon together, not to mention that the horizon of Tubingen does not lack mountains, whose level is a little higher than the gradient of water, and so when the sun was halved, if the mountains had not

134 IBOO VI p. 50.
135 Maestlin: Michael: Dissertation de eclipsebus nunc etiamae, 1596, in M. Ursus et Boreitfeld, Tubingen 1596, p. 19. (Quoted from JGWH II p. 447.)
136 See the note to Ch. 2 Prop. 2, p. 71 above.
been there, it would have been entirely clear of the horizon. This the refraction of one star was greater than a whole degree. Moreover, that part of Swabia is not far removed from the sources of the Neckar and the Danube, and Mount Hoehberg near Horb is notorious for sorceries and gatherings of witches, since the credulity of the people treats the wit of Nature and the closely spaced evolutions of storms, as portents.

But that at different times, refractions are different even in the same place, Rothmann argues with only, on pp. 85-6 of Tycho’s Epistolae astronomicae, and Tycho too grants it without difficulty in the Promotissimae pp. 79 and 280.138 And in vol. 1 of the Epistolae p. 64, he requires that the air be rather pure and calm when he makes a test of refractions.139 Thus in the Observations, I have found an annotation for January 1587, “refractions near the wintice appear to be greater.”140 Nevertheless, in my opinion, the cause of the increase refraction on the day just mentioned is not to be assigned to the winter wintice, but to the warmer air created by Jupiter and Mars being in quadrature, as they were then. For that Tycho found a smaller refraction in the fixed stars than in the sun, there seems to be no other cause than this, that the refractions of the sun are observed in summer and those of the fixed stars in winter, most correctly, but in summer the air is more humid and higher than in winter. For it appears on p. 64 of Book I of the Epistolae, and on p. 94 of the Promotissimae it is openly taught, that refractions must above all be observed when the sun both passes through the meridian fairly high and does not change the declination perceptibly within the same day, which happens at the summer solstice.141 For in the wintertime the sun does not surpass the altitude of refractions. On the other hand, the refractions of the fixed stars are difficult to observe in summer, partly because of the brightness of the air all night in Denmark, and partly because the thick air near the horizon intercepts the view of the stars. In winter, then, and on a very clear night, when the sun is lower and why would the refractions not increase in east winds, since, as on p. 122 of the Tycho’s Epistolae, Rothmann states that when he stood in the middle of a heated observatory he himself had very often seen the stars refracted through the vapor of the hypocaust.142

And so, to finish up this section, let this be certain, that in different places and weather the refractions are different, and that on extraordinary occasions they are extraordinary. For example, if the altitude of the place be high, the refraction will be none: if there is an exceptional releasing of vapors, the refraction will be prodigious. If, on the other hand, the place and weather keep themselves moderate, the refractions will be approximately the same.

138 Oberhohenberg, in the western part of the Swabian Alps, near Konstanz.
139 As Kepler’s time, this was the seat of the county of Hohenberg.
140 TROO VI p. 112 II pp. 84 and 287.
141 TROO VI p. 92.
142 Observation of Jupiter, 15 January 1587, TROO XI p. 175.
143 TROO VI p. 92 (where the passage referred to appears to be on p. 63 of the first edition; and II p. 79.
144 TROO VI p. 152.
9. On the observation of the Dutch in the far North

The story is familiar to everyone, of the journey of the Netherlanders, that is, the description of the sea voyage through the northern Ocean to the desert regions called Nova Zembla, in search of the strait by which a passage might be made to the Scythian and Eastern Ocean.153 In this book, among other memorable things, they relate that when night overtook them, stuck in the ice, and they saw the sun for the last time on 1596 3 November, New Style, although from the altitude of the pole, which they reckoned to be 76 degrees, they considered it certain from astronomical principles that the sun would not return before 1597 11 February, it actually happened that they saw the sun again, its highest edge right at the meridian point, on 24 January, seventeen days before the proper time. A few hours after this time, they noted the conjunction of Jupiter and the Moon at 2 degrees of Taurus; lest anyone think that because of the continuous darkness they had failed to observe the correct intervals of days and nights. Moreover, to remove all doubt, they saw the whole sun standing clear on 27 January. Therefore, on 25 January, the center had risen. Taken by wonder at this fact, many people everywhere consulted many mathematicians, of whom the others answered something else,154 while I answered thus. Since the account of the Netherlanders appears to be trustworthy, the entire cause must not be assigned through suspicion to an error of the seamen about investigating the altitude of the pole, which the others tried to do. For if you draw the seamen this, that they cannot have any certitude about the altitude of the pole within five degrees (which is in fact about the size of the arc by which the sun was still below the horizon on the stated day, in the truth of the matter), you would be overturning nearly all the nautical knowledge of this age, nor will these Parnassians bear without indignation this being said of them.155 Besides, as their account states, the setting of the sun's center between November 2 and 3 does not at all correspond to its rising. Therefore, that whole error cannot be in the altitude of the pole. On 2 November the sun was at 11° 37' Scorpio, when they did not see the whole, and on 3 November it was at 12° 38' Scorpio, where they could barely see the top edge. Therefore, the center set at 12° 7' Scorpio, whose declination is 15° 27', which is the same as what the sun has when it enters 17° 53' Aquarius on February 6, not when the sun was at 5° 28' Aquarius on January 25. But it is also not true that the place or show in which they were held captive by chains of ice was floating in the Ocean (to reply here to Bodin's fellows), so that they

153 This was Willem Barent's expedition of 1596-97. The description of the voyage mentioned by Kepler is Gerrit de Veer's Dierictae navium seu descripsit trium navigantium abominandum. Amsterdam, 1598 (and other eds.), a work which gained a wide readership.

154 Kepler had corresponded with a number of colleagues concerning the problem raised by this observation, most notably David Fabricius and Michael Maestlin. See Letters, 132 and 214, JKG XIV, pp. 55 and 729.

155 Palmarini was a pilot of Aenaeus's ship, chiefly known for having been lost overboard after being overcome by sleep at the helm. CT Vergil. Aenid. V 827-871 and VI 337-383.
might have been carried through sixty miles from north to south during those three months. For when, shortly thereafter, the altitude of the pole as well as that of the sun were diminished, the circle of the horizon, they traversed the same course that they had made going out. What is left, therefore, is that refraction alone bears the responsibility for this phenomenon.\footnote{To the marginal note in his Observationum mathematicarum libri III. Basel, 1601, mentioned above (see page 139), Christiaan had argued that certain trigonometric theorems of Philipp Lansberg in Triangulorum geometricarum libri IV. Amsterdam, 1591, and 1631, were false. Against Christians, Daniel Mixtus (English by birth, though he practiced medicine in various places in the Netherlands) wrote a book entitled Apologia pro Lansberigen et L. Christiannum. Middelburg, 1602, which Kepler cites here.} But to make such a prodigious refraction probable, of so many degrees, I prescribed first a look at the examples just presented of Tillinghen and Mose; where trustworthy authors had noted a horizon- zental refraction greater in the former case than one degree, in the latter than two.

For although the Netherlanders’ place was maritime, where according to the nature of humidity the altitude of air would be about the same as at land in Denmark, nevertheless, another of the causes, namely the density of that air, was able to boost the refractions. If it is true that air is condensed in darkness and expanded in light, the darkness in those places was indeed α sufficient duration for so great a refraction, being of about three months. I also added another example, of the drops in a cloud of Greenland, of an eternally incredible size, as someone or other left attested. Before I add anything as these at present, let their altitude of the pole first be examined. Let $EF$ be the horizon, $CD$ the equator, whose poles are $A, B$; the sun at $F$, on day $2^\text{d}$ November, with declination $15^\circ 27’$. But on April $30^\text{th}$ let the sun be at $E$, on which day (I meant at $12^\text{th}$ hours before noon) they relate that they first saw the whole sun above the horizon. It was at $9^\circ 20’$ Taurus with declination $1^\circ 39’$, which is $CE$. Subtract the semidiameter, $15^\circ$. Therefore, at declination $4^\circ 24’,$ the center of the sun could have been on the horizon. So $AE$ is $75^\circ 36’$, and $DF$ $15^\circ 27’$. The sum sought to have been $90^\circ$, but it surpasses $90^\circ$ by $1^\circ 3’$, which is the standard of measurement of the two refractions at $E$ and $F$ together. Indeed, rather small. But if the observation just cited had been on $30^\text{th}$ April at $12^\text{th}$ hours after noon, this aggregate would become less, barely $44$ minutes. Unless perhaps the sun at $E$ had attained some altitude above the horizon, which they do not add. But if this aggregate refraction be bisected, and the half be subtracted from $AE$, and the remainder from $DF$, that remains, in the former case, the altitude of the pole, $75^\circ$, and in the latter, that of the equator, $15^\circ$, more or less by a few minutes. Or, more probably, let the refraction in the former be $20’$, because of the length of the day, and in the latter $41^\circ$, because of its shortness, the altitude of the pole will become $75^\circ 16’$, of the equator, $14^\circ 44’$. And the declination of center of the sun when first seen is $18^\circ 58’$. Therefore, the refraction is $4^\circ 14’$, by bisection of the sum of the refractions it would be $4^\circ 3’$. Let me now supply the previous computation of the
altitude and density of the air, what is the least it could be to bring about such a refraction. And we would be assuming a minimum density, if we should stipulate that this refraction of 4° 14′ is entirely horizontal; that is, that the ray of the sun from the aether that is tangent to the surface of the air, being refracted in air, is again tangent to the surface of the earth. Then within the air there will be set up an angle of 85° 46′ with the surface of the air, whose secant is 1,354,677. Hence, the simple refraction of 90° is 19°. But above, air was normally refracting from 90° through its density with a simple refraction of 1° 56′. Therefore, this density that it has is nearly seventeen times as great, but the sphere of the air is nearly four times higher than before, almost two miles. It is in fact not credible even to me that this altitude should be so great. But you cannot assume a smaller one. About center A let the greatest circle of the surface of the earth BC, and of the air E, be described; and at some point E let a ray of the sun FE be tangent to the surface of the air E, and let it be refracted there into EB, so that EB is tangent to the surface of the earth at B; and let the angle of refraction be 4° 14′. By what was demonstrated above (in Prop. 8), BAE will be equal to this, and the altitude CE will become fixed. If you now should wish to depict the surface of the air as lower, then let a circle inside E be described about center A; it will cut the refracted ray EB; let it cut it at D; and from the point D parallel to EF let DC be drawn out, so that the angle of refraction again be 4° 14′, that is, so that BEF is equal to BDC. I say that DC is no longer in the aether, but is going to be in the air, that is, that it is going to follow the circle through D in the direction EF. For let points D, A be joined. Then, because EF, DC are parallel, DCA, FEA will be equal and right, because EF is the tangent to the circle at E. Therefore, in the right triangle DCA, the right angle DCA is equal to the remaining angles CDA, CAE, together. Thus DCA is greater than CDA alone. Thus DA is greater than CA, by Euclid I 18. And since DA is the semidiameter of the most recently described circle of the air through D, the circumference D is therefore farther away from A than is point C. Therefore, DC cuts the circumference in the direction C no less than at the point D. So it will happen that CD is beneath the air, not in the free aether. Consequently, other this phenomenon will not be of the class of genuine refractions, or we have to believe that the altitude of the air was 2 miles. I shall therefore show two other ways by which such a phenomenon might come to be possible, both drawn out from Cleomedes, if his words at the end.

---

357 This is 90 times the simple refraction of 4° 14′ per degree. See the refraction table on p. 138 above.
358 This is proved immediately below. Since the angle of refraction is about four times as great as that in the table, the angle dAE in the following diagram must also be four times as great, and the height of the air CE must increase accordingly.
of Book 2 be well compared with these.\footnote{Clavsius, De motu corruptur B II pp. 222-4. French ed. Goulet p. 173.} The first consists in this, that if the observation occurs at point $B$, $BC$ is drawn tangent to the earth's surface, and containing with $DB$ the required angle, so that, because the refraction is to be $4° 14'$. $BDC$ is $175° 46'$. Let $B$ be tangent to the earth at $C$, and cut $BE$ at $D$. And suppose now that there is no air at $C$. Therefore, in places lying between $B$ and $D$, there is some boiling up of dark vapors with an irregular surface, which neverthelesss is in such a state that through the mediation of the density of pellicid matter it might reflect the ray $CD$ of the sun approaching from the tree aether through $4° 14'$. I believe this is what Cleomedes meant when he compares vaporous air near the horizon to water into which a coin is thrown. For just as, in the former, the eye is placed outside the denser medium, so also, in the latter, Cleomedes locates the eye of the observer in the air outside that thicker vapor. Therefore, since $DBA$, $DCA$ are right, and $DAB$, $DCA$ are equal, each being $25\frac{1}{2}$ degrees, we have now some closer note to the previous measurement, for in the refractions that usually are seen, the angle $BDA$ was $2°$. If it should appear to anyone that the altitude of the point $D$ or of the vapors is quite too high, the vapor can still be lowered by doubling the refraction. For let a lower point $G$ be chosen on $BR$, and from $G$ let $GH$ be drawn tangent to the earth, so that it may cut the air at $H$ and be refracted there. Finally, from $H$ let $HI$ be drawn parallel to $EF$. There will be the usual refraction at $H$, and afterwards the extraordinary refraction at $G$. However, if you consider that in turning their faces to the south, the Dutch settlers looked towards mountainous, wooded, and very high Tarrytown, whence flows the river $OH$, so that $EF$ is raised up by about sixty miles $n$ that inland region, it will probably not be incredible to you that the point $E$ on the surface of the air was so greatly raised above $BC$, the level of the Netherlands' quarters at that moment. There will be much on this inequality below.

The other way is based upon reflection caused either by a dense and uniform cloud (as Cleomedes maintains), or, as I maintain, by the upper surface of the air surrounding us, such as at $IIB$ were refracted into $HIO$, and at point $G$ of the concave surface passing through $G$ it were reflected to $H$. This can happen through the laws of reflection, if $CD, DB$ are tangent to the same spherical surface. Nor is there fear that the surface may transmit rays, for the reason that after that concave surface it encounters a rarer medium. For we see in glass that both surfaces, both the exterior one at the top and the interior one at the bottom, reflect rays, so much so that when a glass mirror is set facing the sun, it comes to a third or a fourth reflection. Let there be a glass, whose upper surface is $ABC$, lower surface $EDA$, by which the mirror is bordered by an application of white lead. And let the sun be at $F$, the incident be $FC$, the consequent reflection being $CG$, at equal angles.
But because glass is transparent, the most powerful part of the solar ray passes through the surface at \(C\) and is refracted into \(CD\), and thus the remaining part, which is scattered through \(CG\), is rather faint. Next, \(CD\) is reflected from the point \(D\) strongly (because the mirror is bounded at \(D\)), and at equal angles, into \(DB\); and, going out into the air there, that is, at \(B\), it is refracted into \(BH\), and here the ray \(BH\) is strongest. Again, because \(DB\) encounters the polished surface \(R\), it is partly (but weakly) reflected at equal angles into \(BE\), notwithstanding the presence of air above \(B\); and at \(E\) it is again strongly reflected into \(EA\). And, going out into the air at \(A\), it is refracted into \(AI\), but here it is very weak, because part of it went away at \(C\), but much more of it at \(B\); the former of these depatures was scattered into \(CG\), the latter into \(BH\), so that at \(AI\) it is very much weakened. And nonetheless, even at \(A\) some part of \(EA\) is reflected into \(AM\), and from \(M\) into \(MK\), and is refracted into \(KL\). But now this fourth ray can seldom be perceived, only when cast forth into dark places, because of its excessive refraction. Therefore, in that way in which the rays are reflected in glass by the upper surface, even though it is not bounded, they can also be reflected in air by its highest surface, so that, in place of the sun, its image could have been seen by the Netherlanders in Nova Zembla. The learned should see whether the phenomena of twilight, too, may be justified in this way, so as to make it unnecessary for the matter that is so illuminated to ascend to a height of 12 miles; it would suffice that once the sun's ray has entered the surface of the air, from which it again departs, it be reflected at equal internal angles, and that this occur twice, thrice, or four times, until the illumination of the air made by the reflected rays is completely obliterated and no longer enters into the eyes.

10. Conjectures from antiquity concerning refractions

Much to astronomy's loss, it happened that in the investigation of the sun's motion and the equinoxes, refractions were ignored by the ancients. If it could be proved by reliable arguments that the rays of the heavenly bodies are refracted in all places and times, but loss might partly be made good by us. I shall make a try at accomplishing this, as much as I can. First, that refractions are different in different places, we have just now proved clearly enough. Further, from the same argument it was evident that one or another magnitude of refraction resulted from taking one or another density of air and altitude of its sphere above the earth. Next, Egypt and Rhodes, homelands of Ptolemy and Hipparchus, respectively, are maritime places, and are therefore immersed in the depth of the air. On the other hand, while cold condenses, heat expands, and there is a greater heat in those climates, so the air will also be thinner, which, when denser and heavier air comes in underneath the North, is driven upward, and places itself above, by which position it is made to slope, and in turn seeks the north, pouring itself

144 Idolum. This is the Latinized form of the Greek \(\text{i}d\)o\(\lambda\)\(n\)a\(\nu\)s, a somewhat unusual word in classical Lati. Although it can mean "ghost, apparition", it was also used by the Epicureans to render the small images given off by illuminated bodies. These images enter the eye of an observer and provide the information about their source that consti\(\tau\)uates vision. Could this be the origin of the Greek word \(\text{i}d\)?

Could this be the origin of the Greek word \(\text{i}d\)?
down onto a lower surface of the air, unless it is driven back by a stronger force (such as if the entire level of the air were set in motion by the initial impulse), and, being driven back, overflows and ponds, as it were. And in sum, as regards density, there is a perpetual intermingling of our air with the southern air, and in turn of that air with ours, and this is because of the continual blowing of winds. And so with respect to density, there will not be a very great difference between us Europeans and Egypt or Rhodes, but in what follows, in Chapter 7, we shall make use of the liberty to seek further whether in all places the altitude of the air be the same, and the density the same. And so, once the cause is set in place, it is necessary that the effect follow, and since it is not just today that the air is poured around the earth in a circle, but this law of nature, as is fitting, endures from the beginning of things all the way to us, it should be thought reasonable and fitting that there was never a time when there were no refractions. These supports deduced by reasoning are to be bolstered by testimony and conjecture fetched from antiquity, from which it may appear that, over the continuing succession of apses, refractions of the stars in air were either noticed or at least perceived by the sense.

Pliny provides clear testimony in the Natural History II 13.161 His words are as follows: "For what reason is it that when, as the sun rises, that shadow that dulls [sic., that causes lunar eclipses] ought to be beneath the earth, it happened once already that the moon was eclipsed as it set, both bodies being visible above the earth?" Here you have at the same time the sun and the moon, although they are at opposite positions, distant by a semicircle, nevertheless appearing by refraction above the horizon, and at a distance that is less than a semicircle. This is in agreement with the theory of refractions as hitherto presented.

Clomedes relates in Book 2 that certain more ancient mathematicians thought they could unite this knot by saying that the earth, because of its roundness, is like a sort of mountain, from which the same observer easily looks upon what is being done in two valleys.162 However, Cleomedes refutes them by showing the dissimilarity. For if the place in the mountains from which the valleys are viewed is a rather high place, it is also necessary that the mountain be everywhere precipitous, conical like a top. But we are settled upon the surface of the earth, so much so that it appears flat to us. To Cleomedes' refutation, I add that if we look down from some mountain, that happens along lines that descend below the level of water. But if this phenomenon just described should happen to us, the lines of vision on both sides are necessarily raised above the level of water (to which the visible horizon appears parallel). For in the similar eclipse mentioned above, and in explaining this very passage of Pliny, Maeslin testifies that each was alternately seen to shine at an elevation of two degrees.163 Therefore, at the start, Cleomedes calls this account of Pliny's into doubt. He says, "What if we should say that this sort of account was fabricated by certain people whose aim was to throw into doubt those astronomers and philosophers

163 See p. 149 above.
who devoted themselves to this investigation. But more than once, Tycho and Messudis strongly restrained the incredulous Cleomedes by observations. Nor unjustly, Messudis is amazed that Cleomedes was able to assert that 'no one who professes to be a mathematician has ever left written testimony that this had been seen by him, even though all eclipses, from the Chaldean, Egyptians, and the rest all the way to the time of Cleomedes, had been noted.' Even so, Cleomedes also labored over this; how they could have abandoned his trust and transferred the cause of the phenomenon to the sense of vision, and how this sense, from a reflection from a cloud or refraction by thicker air, might have erred, and might have looked upon the sun's image in place of the sun. We have aired out both ways above, and have not rejected them completely. In refraction of unusual magnitude, but the legitimate and commonly encountered cause of this phenomenon is sufficient. Surely, the refraction of the light of the two hemispheres in the surface of the air that is poured around the earth. Therefore, there were once refractions in Italy, and this testimony does not allow any exception. There follow weaker conjectures. In Book III of the Great Work, Plato asserts that he had often observed the equinoxes twice on the same day on the Alexandrine armillaries. He places the blame upon the instrument's position having settled to that extent from the beginning of its installation. Hipparchus, however, who was closer to the first installation, and who affirmed that he had observed the equinoxes on the same armillaries—Plutarch, I say, draws him into the same accusation. "For in year 32 of the second Calippic period, on the 27th day of the month of Mecom." (Plutarch speaking), "he observed the equinox in the morning, but the Alexandrine armillaries were equally illuminated on both sides on the fifth hour of the day." Plutarch therefore concludes, "the two observations do not agree within five hours." And indeed, this is driven home by Hipparchus: "these observations are the best which is made using the Alexandrine armillaries, for that is the hour of the true equinoxes in which the two surfaces of the armillary are equally exposed."

164 Oddly, Cleomedes's Greek text, quoted in JOHN II, p. 415, says, "But first one must confound those who say that this account is fabricated by certain astronomers and philosophers wishing to create confusion for those concerned with these matters." Although Kepler's version is better than Talei's (which Fuchs also included), it is still not what Cleomedes wrote.

165 Idolat. See the footnote on p. 155.

166 This is a loose translation of ἑκατάτευχον, one of the titles by which the Almagest was known.


168 FARMER, p. 134.

169 From Plutarch's writing, Toomer concludes (note 9, p. 134) that the former observation was made by Hipparchus, presumably at Rhodes, while the latter was by another observer. Evidently, Kepler believes both observations to have been made by Hipparchus, and takes it as evidence of multiple apparent equinoxes on the same day.
illuminated."\(^{150}\) So even if it coidd indeed happen that this inconsistent observation of the ephemerides originated from a fault in the instrument, and Ptolemy's critique might become, it is nonetheless also true that this same phenomenon (i.e., the equinox appearing twice on the same day) was very often observed in our age by Tycho Brahe using highly accurate and very precisely installed instruments. For in the zooning before the ephemerid's moment has arrived, the rising sun appears higher and closer to the pole of the world because of refraction, so that it may already be seen on the equator while it is still in the south. After noon, when the sun is free from refraction, suppose that it has now completed the western semicircle: thus it will gain at last be both truly and apparently on the equator. The opposite must be said of the inferior equinox. It is therefore uncertain whether the cause for the authors' having persuaded themselves that they saw the equinox twice is to be derived from a fault of the instrument or from refraction.\(^{151}\) I should like to say something that is not really proper to this place, and should instead be deferred to the debate on the determination of the year. For what if this inaccurate manner of refraction alone were the cause for Hipparchus's once finding a greater accuracy of the sun than Tycho finds today, and Al-\(^{147}\)

\(^{150}\) This is not an accurate translation of Ptolemy's quotation of Hipparchus as the Al-\(^{147}\)

\(^{147}\) This explanation was proposed by Karl Mamiotti in his German translation of the Al-\(^{147}\)

\(^{147}\) Alhazen (Latinized Alhacenius), Arab astronomer of the late 10th and early 11th centuries, known chiefly for his work on the motion of the sun and the length of the year. His chief work was published in 1027 and it was used by other astronomers in their subsequent work.\(^{147}\)

\(^{147}\) Ptolemy (140–168), Greek mathematician and astronomer, is greatly admired by Kepler, who included a preface to his commentary on Ptolemy's Almagest on the title page of Book I of the Harmonica mundi, Of Men Copernicus, Kepler, pp. 92 and 261–273.

\(^{147}\) On the use of the clepsydra, or water clock, for such measurements, see Ptolemy, Almagest V 2.4.512, Tannenfeld, 525, and Tannenfeld's note referring to Pappus, Hellen, and Ptolemy. On the clepsydra in general see W. K. C. Guthrie's note to his translation of Aratus, On the Revolution, p. 275–8.
from the open hole in the clepsydra into a separate vessel, and once the whole sun stood clear, the remaining water was received by another jar, throughout a whole day and night, until the edge of the sun was again seen to rise in the same place. Since they used this observational procedure to measure the sun’s diameter, what is more likely to be believed than this, that Hipparchus prescribed this day for the equinox upon which the sun should touch two diametrically opposite indicators on the visible horizon, one in rising, the other in setting; and that once those indicators had been established by observation, they would forever after be kept as markers of the equinoxes, if it should stay in the same place. If I have guessed right here, I shall obtain the rest without difficulty. For up to the time when the sun is in the southern semicircle of the zodiac, it is impossible to accomplish what we have just said was seen by Hipparchus, that is, rising, it should touch the point of the horizon diametrically opposite that which it touches in setting. For in both cases it will verge towards the south, and therefore, on the horizon, where we have presupposed that the observation occurs, the sun, appearing higher than is correct because of refractions, in the spring will appear to get to the equator more quickly than is correct, and in the autumn will appear to have sunk down to it more slowly than the sun. And so the time of the summer semicircle will appear to be more lengthy, since both ends are extended. Thus the eccentricity will be greater than is correct. And so one who wishes to believe with a clear mind that the sun’s eccentricity is the same both in the past and today, has the perceptible argument of refractions that Hipparchus had observed.

You would not be entirely ridiculous to add at this point what Ptolemy of Lycia presents in the Sphaerae,117 where the antarctic118 is described by the forward foot of Ursa Major; that is, where the star in that foot appears to touch the horizon daily. In the same place, the summer tropic is cut by the horizon into 5 and 3. These two teachings are not consistent with each other. Suspicion can be plausibly cast upon refractions, this either the foot of Ursa appears to touch the horizon, which in fact it goes beneath; or that the sun, in rising more quickly than is correct and setting more slowly than is correct, indicates on the clepsydra diurnal space that is more drawn out than it really is, whence a portion of the tropic will be thought to stand clear that is greater than is correct, and the altitude of the pole will be considered greater than is correct. I realize that these teachings are not written particularly accurately, and can be taken more roughly. But since this way of taking the altitude of the pole is rather easy, I wanted to advise the learned to Tycho does, on p. 95 of the Phenomenum to consider which places on earth it was, at which Ptolemy established the altitude of the pole by this observation of the longest day.117 This may very well be why. (10)

117 The Greek text, with a Latin translation by Thomas Linacre, was published in Basel in 1547. The passages cited by Kepler are on sp. 3 and 11, according to Frisch (JDOG II p. 415).

118 Or, “antarctic circle.” In either case, the use of the word is puzzling, and I have been unable to check Ptolemy’s text, other than the brief passage quoted by Frisch.

117 Tycho does not appear to have treated any such consideration. He did advise great care in determining the polar altitude, and discussed the effects of refraction, on p. 15 of
year ago, Antonius Maria,170 teacher of Copernicus, thought that the altitudes of the pole had decreased in all places in Italy, namely, if the altitudes belonging to the places of which Ptolemy had once correctly noted the latitudes, were excessively increased in later times, by estimation of the longest day, because of the refractions exceeding their measure. Or, contrariwise, that Ptolemy had investigated them from observation on the shortest day, which, since it is greater than the truth because of refractions, creates the illusion of a greater portion standing above the horizon, and of a lowering of the sphere. William Gibert warms of this same thing in that abstruse study of his on the magnet, in Book 6 ch. 2, in concise words.171 A man of such excellence, in whose divine discoveries it is timing for all students of Nature greatly todelight. I would hope to be able without much difficulty (since it is not a haughty philosophy) to earn my friendship through my eagerness to learn, unless that obsequious Tythus begetread me this.172

Those things that I have brought in from Proclus, things similar to them can be added from Clemens, who in turn gets them from Posidonius, but are also referred to by the same Proclus at the end of the Sphaera, and likewise by Pliny, Book II, that in Egypt, Canopus at noon shows an altitude of $7^\circ$; at Rhodes, it grazes the horizon, on which island the exo of night and day in summer is that of 19 to 29. For since from this the latitude173 of the city of the Rhodians is 36°, while that of Alexandria is 31°, how is it that Canopus, which a Rhodes is barely visible, the highest mountains cannot, on Proclus’s authority, stand $7^\circ$ out, or the fourth part of a sign, in Egypt? For this altitude is given consistently and in the same words by Proclus and Clemens and Pliny, with Ptolemy also supporting, who gives that star a latitude of 75°. Therefore, either all the authors with a simple

the Programmaticus (1800) II pp. 16–17); perhaps a 4 was substituted for a 9 in the typesetting.

170 This was Dominicus Maria of Ferrara (not Antonius Maria), of whom J. G. Rhetius said (in the Novato pronest that Copernicus has been his “assistant and witness of observations.” See Edward Rosen, Three Copernican Treatises, Third Edition (New York: Octagon Books, 1971), p. 111. Maria’s views on the decreasing altitude of the pole were published by Antonio Magini, Tabulario de mundi mundum, Venice, 1585, Canon I, p. 79.
171 Gilbert, De Magnete, trs. Motteux, pp. 316–7. Gilbert considers Maria’s argument at some length and rejects it on the grounds that the observations upon which it is based were not sufficiently accurate. At the very end of the chapter, he remarks that observations of latitudes cannot be made with exactitude save by experts, with the help of large instruments, and by taking account of refraction of light. “Evidently, these are the ‘concise words’ Kepler means.
172 Here, “pompisa ika Tethys” is the North Sea, this remark shows that Kepler was already contemplating a journey to England. Later, in 1606, Kepler sent a copy of the newly published De Stella nova to King James I, and in 1610 he dedicated the Horaeos mondi to that monarch. In 1620, Kepler was visited by Sir Henry Wotton, the English Ambassador, who invited him to come to England; see Wotton’s letter to Francis Bacon, [JOW XVIII, no. 392]. However, he never succeeded in visiting England, and by the time he expressed this hope, Gilbert had died on 1603.
173 Reading “altitudi” for “altitude”.
voicemust be hallucinating, or in Egypt, towards the sources of the Nile, towards the Mountains of the Moon, and those regions that are molested by continous rains, it is necessary that there be interposed for observers in Alexandria as air that causes a refraction at some prodigious number of degrees, and that this air is two thus precisely to the south of the island of Rhodes, as Rhodes is indeed a few degrees west of Alexandria.

What the same Cleomedes confidently alleges, probably possesses less soundness; that there are two stars, distant by the diameter, the Eye of Taurus and the Horn of Scoepius, when either of which sees, the other sets, and they are seen on the horizon at the same time. If anyone ever really saw this, the two must have been elevated above the horizon, since in reality it could not happen otherwise, unless one or both were always beneath the horizon, in this northern part of the world. For even though they are exactly a diameter apart, with respect to longitude, neither of both are in the southern hemisphere, one having a latitude of 5° 31′, the other 4° 27′.

To this point there can be related something, though it is obscure, from Proclus Dodechus's book Hypotheses astronomicæ152 and Question 3 of his Menthae; even though this too appears: that he speaks of the excess of the year, as then accepted, above its true measure. However, his words, in Vulgi's translation, sound as if he were alluding at something that appears in the sun's motion upon the occurrence of refractions: "That the sun is fully seen, and that it accomplishes its rising as if taking a seat." Now William, Landgrafe of Hesse used nearly the same words in his first lecture on astronomical matters to Tycho, explaining the occasions of his taking note of refractions.153 And although those things at which Proclus wandered there can have occurred because of the computation in general use at that time, nevertheless, it gives reasons for them. Proclus waived so, that you might suspect, not unjustly, that in tagging himself in refractions he made the undertaking more bewildering, and that without men it would have been easy for him to unravel it.

Martinus Capella154 is somewhat clearer in the Encredipedia, Book 8, whose words concerning the latitude of the zodiac are these: "For the sun, nowhere leaving the zodiac, is carried in a central equinoctial outside of the proper boundary of Libya. For it defies itself there either to the south or the

152 Aldebaran and Antares, respectively.
153 Giovanni Valla's translation of Proclus's Hypotheses, to which Kepler refers here, is included in Chauji Poliomei Politianis, Alexandriae Oratio quae existat opera, . . . ut E. O. Schreckerbach, Bresl 1551.
154 The Landgrafe's letter is in Encredipedia astronomice pp. 21–23, Thio II pp. 48–51. He discusses refractions and the measurement of the pole's elevation on p. 22 (Thio II p. 50).
155 Martinus Capella lived in the first half of the 6th century C. E. in northern Africa. His encyclopaedic, entitled Seleneum, or On the Marriage of Philology and Mercury and on the Seven Liberal Arts, covered Astronomy in Book VIII. Capella is noted for having placed the orbits of Mercury and Venus around the sun, rather than between it, as Ptolemy had done.
north at approximately the half moment. He expressed the horizontal refraction carefully enough, and therefore the absurdity and strangeness of the statement that the sun, at a single place on the zodiac, inclines in zodiacal latitude, instills me with a suspicion, as follows. Since the equinoxes, chiefly the autumnal one, have been carefully observed, and particularly on the horizon, at sunrise or sunset, because the visible horizon, as was said before, is like an instrument, it happened, from frequent repetition, that the mathematicians finally noticed the refraction that occurred at the horizon, and since they were ignorant of the cause itself, they attributed this little space the amount by which the sun is moved closer to the pole by refraction, to the motion of the sun itself. If any one be less pleased with this interpretation of Capella, let him present a more appropriate one to defend the author.

For the rest, let us leave out the guestwork and turn to the surer testimony of later authors. The Arab Alhazen, to whom Risseter, in a conjecture not lacking in judgement, attributed an age of fifty years, and, active about two centuries after Alhazen, our Witelo, assert fully the same thing in their Optics, bringing in astronomical experience, which nowadays Tycho Brahe, using the most precise instruments, has brought fully out into the light, that stars are seen near the horizon under refraction, on which there is enough above.127

127 Bernhard Walther, in the book of observations that is appended to Regionontumus, Torquetum, gives fully the same testimony, that he had learned from experience that stars often appear above the horizon when in fact they are beneath it. That was about 120 years ago. But the passage deserves to have light shed on it, in order to have this man’s discoveries compared with those of our Tycho. The author’s words are these:

In the year 1489, on March 6, about sunset, while, to be exact, 25 Gemini was at mid heaven, the sun was at 25° 15′ Pegasus, with the amillaries. Venus was found to be at 27° 15′ Aries through another circle, by the ecliptic dividing the sun. But by the circle of latitude mediating the sun, as is usual near the horizon, it was found at another place, namely, at 25° 30′ Aries, the cause for which I shall append later. These things are explained a little later. There follows:

126 I have not found this passage in the Morning of Phoebus and Mercury, though there is something rather like it in par. 848 “although the supposed sun moves along the middle line of the zodiac belt, the obliquity of its course causes it to be depressed or elevated.” c.m. Stahl and Johnson p. 300.
128 Joannis Regionontumus, De torquetu astrolabiio armillarii etc., Nuremberg, 1544. Pages 36-65 contains “Ioannis de Monereji, Georgii Petruschi, Bernardi Waltheri ac aliorum, aequatorium, horidum, planetarium ac thuram observationes.” The observation cited here is on 60°.52′-53′.
On March 7, the sun was 26° 15' Aries by observation of the armillaries. Venus, from the ecliptic, was 28° 15' Aries. From the circle of latitude: 27° 38' Aries.

To these observations, he immediately appended this:

Further, so that it might not be hidden too long from those who are going to read this, how I could proceed, once the position of Venus was found so variously in practically the same instant, it must be noted that, because of broken rays, stars appear above the horizon when according to the truth they are beneath it.

And this was clearly required by the explanation and theory of refractions, historians handheld down, but not its contrary. For when the rays of the sun that strike obliquely upon the surface of the water immerse themselves within the water, they proceed at an angle more inclined to that surface of the water, exactly as if the sun were higher. In clearly the same way, if the same rays of the sun striking upon the surface of the air, which completely covers us humans like a flood, were to follow that path with which they strike upon the surface of the air, they would never strike upon our eyes, but would be carried off far above our heads. But, being refracted at the surface of the air, they now descend more steeply to us, and appear to us to be more elevated. And in fact, if we were dealing with water, the sun could not even be beneath the surface of the water, always above, because the portion of the water surface in which we create the opportunity for this refraction is narrow, so much so that it seems to us perfectly flat. However, since air is round, the part of it that places itself between our vision and the sun is not narrow, and the part is indeed great enough that it now begins to curve towards the sun. And so, although some particular ray of the sun is tangent to the surface of the air at some point that is overhead at our zenith (when the sun shall be fixed right at the horizon) or at a point that is now nearer the sun than that which is above us (when the sun shall have already set), nevertheless, the parts of the airy sphere that trend towards the sun after that point of tangency, because of the air's being massed together into a globe, will send themselves down beneath that tangent, so that what is now setting for us is still elevated for those points. In this way, the same thing happens again that also happens in water, that the sun is never below that point of air in which its ray is refracted, and nonetheless, for us it has in truth already set. But Walther proceeds: "Which very often appeared to me perceptibly with the instrument of the armillaries, before I saw the Perspectives of Alhazen and of Willeus of Thuringia, in which I later found this stated within a hair's breadth." The same thing happened to Tycho Brahe, that he noticed the refractions of the luminaries before he had read these optical writers or the discoveries of Walther, as he states in the Programmatum, Book 1, fol. 91, for the occasions upon which he discovered them. See fol. 15 of the same book and the same fol. 91. For this reason, both deserve all the more trust, in that ignorance, and not caught by any prejudices, they realized that this was so. For it is characteristic of truth, to come together by diverse ways.
There follows, in Walthé, the occasions upon which he himself came to notice this fact. "But in order to escape the differences in the moon's aspect, I even examined Ptolemy's way, in the second chapter of the seventh book." That is, the occasion that showed me the refractions, was this. I was about to investigate wat instruments whither on the eclipses the fixed stars had proceeded in my time. The easiest way to inquire into this is if a perfectly correct position of the moon from the sun, or from the equinox, could be obtained for any desired moment by computation; for then one may see by inspection which stars it is that the moon has approached at that moment. Or, conversely, if at some particular hour of the night the moon should cover some fixed star, there may be computed, by a completely reliable calculation, the moon's true position from the equinox. For that will also be the star's true position from the equinox. Or if anyone should mistrust the calculation, he should carefully observe the middle of some lunar eclipse, and not the distances of the moon from the neighboring fixed stars. For he will thus have the distance of the sun from those fixed stars, and from the sun's meridian altitude and the latitude of the place he will also have the sun's distance from the equinox, through its declination. Thus the distance of those fixed stars from the equinox will also be given.

The procedure could easily be carried out in these ways if the moon's true position could be had, either in eclipse or outside of it. But, among other things, it happens that although we may have some knowledge of the moon's true position, which one might see it holding if he were looking out from the center of the earth, nonetheless, parallax, whose variety is incredible, places the moon in one or another place. And so the moon's distance from the fixed stars is far different than if the moon were perceived in its true place. Therefore, so that the moon's parallax not bring a case against me, I used that method that Ptolemy pioneered in book 7 ch. 2. He, of course, used it for the moon, which I imitated it for Venus, whose parallax is less, and whose diurnal motion is less, and I used an auxiliary instrument," (for a description of which see Tycho Brahe's Mechanic),\(^{(100)}\) "to take the distance of the sun and Venus about midday, or afterwards," (a procedure which Tycho Brahe, unaware of Walthé, imitated many times with the greatest care, as you see in the Præparata Nominis Vol. 1 fol. 152),\(^{(101)}\) "the sun's position being previously acquired by rules or auxiliary instruments, or by both instruments." Both methods are described by Regiomontanus in that same book from which this is transcribed.\(^{(102)}\) Moreover, the order is this. First, from observation of the fixed stars, the altitude of the pole from the equator are obtained. Next, the meridian altitude of the sun is observed, which, compared with the altitude of the equator, gives the declination. And to each declination there corresponds a certain distance of the sun from the cardinal points, or the position on the ecliptic, the maximum declination being presupposed. "But since I had ostensively been examining the positions of the two (the sun and Venus) at sunset, by turning the

\(^{(100)}\) TRIO II p. 159.

\(^{(101)}\) TRIO II p. 159.

\(^{(102)}\) Regiomontanus. Scriptum in astronomiam Libri III. (Nürnberg 1544).
ecliptic armillary, and to be examining the sun’s place on it,” (by turning that part of the eclipse: in which the sun was perceived to remain by the meridian observation), “towards itself;” the sun not, however, by accurately locating the center of the armillary, the found position of the sun, and the center of the true visible sun, on the same straight line, but by turning that position of the sun to one side and the other of the sun’s body, “until both parts of the surface,” both edges of that circle of the inside of the ecliptic armillary, because the material circles extend over the latitudes, “that is, the lower and upper,” or south and north, “were equally illuminated by the sun,” already setting, “also by moving the circle of latitude, until the two parts of its interior surface, that is, the right and left,” or east and west, “received equal illumination from the sun.” By this procedure, if the solar rays had come to the sense of vision,” or the instrument, “without refraction, the circle of latitude,” by its intersection with the ecliptic, “would accordingly have shown the the position of the sun,” which was previously found on the meridian, and was corrected by the addition of the required hour; motion.

“But I found a noticeable difference.” The sun was found in one position by the meridian observation, and in another position by the evening one, “and further,” the difference, “was different,” not always the same: “it was greater when the sun’s declination was increasing or decreasing moderately, but smaller when the declination was changing noticeably, so as to be greatest at solstices, and least at the equinoxes.” This is not because there was that great a variety in the refractions themselves, but because even if the sun’s declination were equally altered in both cases, nevertheless, since the longitude is obtained from the declination, and a false longitude from a false declination, this discrepancy in the declination therefore causes a greater error, where to one degree of longitude there corresponds a single minute of declination, which happens near the solstice, than where to one degree of longitude there corresponds 24° of declination, or at the equinoxes.

“So, when I saw both circles,” the ecliptic and the circle of latitude, “illuminated by the sun at sunset, as was said before, I by no means had either the true place of the sun or, from it, of Venus.” For it was just said, that if this should happen, the true place and this refracted one would be seen to differ. “If, however, with the circle of latitude placed over the sun’s position as found by the meridian altitude and restored by the addition of its intermediate motion, I showed the same circle towards the sun at sunset,” that is, if I found the place that the sun has on the meridian by a different and reliable method, and if I have corrected it by the addition of the part that corresponds to the time from moon to sunset, if, to that place (I say) I were to apply the circle of latitude, and were to turn them, thus affixed to each other, to the setting sun, “until its upper surface,” that of the circle of latitude, “was illuminated in the manner previously described,” it being ignored that the same would happen on the ecliptic. “I have come closer to the earth,” in investigating the place of the sun’s longitude. For the circle of latitude nearly coincided with the vertical, especially at the spring equinox with the sun setting, and so the refraction which as a rule happens on the vertical, nearly all went over to the latitude, the longitude being hardly affected. However, “this way, too, was insufficient,” because the circle of latitude and the vertical or refractory circle would not have entirely coincided. “On account of this fact, from
the opening at the eye," which represented the true place opposite the sun, found by the meridian observation, and reduced to the time of this evening observation.

155 "I added a perpendicular to the primitive," thought a special constivance," which perpendicular represented the vertical circle. "In considering that the point whose form is refracted, whose function in this place is performed by the intersection of the circle of latitude with the ecliptic; the center of vision," represented by the opening mentioned. "Oh Punsh!" on the surface of the air by which the refraction is generated, and the perpendicular from the point of refraction, are in one surface," which we here imagine to be the place of the vertical surface. "And I saw, when the intersection of the ecliptic and the circle of latitude representing the sun in its true position "had come into contact with the horizon," which position corresponded to the sun's true setting: "that the sun, coming through the opening of the other plane," the one towards the sun, which I elsewhere used only for the stars, "still illuminated the thread of the perpendicular." I took care, he wishes to say, that the sun illuminate uniformly, not the circle of latitude, not the ecliptic, but the thread of the perpendicular suspended from the opening of one panacidium through the opening of the diametrically opposite panacidium: "so that the previously described things are in one surface, namely, the center of the true sun that is, the point whose form is refracted, the center of vision." In this place, the opening from which the thread is suspended, or rather, the lower place of the thread, in which it is illuminated by the sun, "the point of refraction in the air and of the perpendicular; likewise also the sun's true place on the ecliptic" of the instrument, and finally, the position of the image, or the visible sun.

Now because the openings are diametrically opposite, and the thread is suspended from one opening, the plane passing through the thread and the openings bisects the instrument, and since the visible sun illuminates the thread through one opening, the visible sun is therefore also in the same surface, which, because of the thread, is perpendicular to the horizon, therefore in a vertical plane, therefore the sun's true position is also in the same plane, since the true place of an object and the place of the image are always in the same surface. When this occurs in an instrument of fact the one opening in the intersection of the ecliptic and the circle of latitude represents the perfectly true place of the sun, as is supposed, it serves as proof that the instrument was so directed that the center, the

155 The primitive, or primitive, usually had a hole in it through which the object was sighted. From what follows, it appears that Regiomontanus added a perpendicularly oriented thread across the hole.

154 Reading "panacium" for "punshus."
openings or the intersection mentioned, and the perfectly true position of the sun in the heaven fall upon one straight line: and that they cannot be arranged differently in such a way that this may occur. It is indeed an ingenious notion (in to eliminate the refractions instrumentally by observation, without calculation from longitude and latitude, and to establish a sound longitude and latitude, indeed also to measure the refractions themselves by all three sides of a right angled triangle. It could also be carried over to the parallels. I, at any rate, have somehow less doubt than did Tycho Brahe, on fol. 94 of the Programmatum, concerning Walther’s carefulness, after I have helped myself out with an explanation of the passage. Besides, if anyone is so careful that he does not want to trust blindly in instruments in such an exciting matter, he has the way of distributing the parallels and refractions, which affect the altitude, into the longitude and latitude, described in Tycho Brahe’s Programmatum, chiefly vol. I fol. 93, 94, and 96, where he also shows how to investigate the whole refraction or the altitudes. In chapter 9, below, you will find a most useful and easy method for this enterprise.\(^\text{159}\)

In going through the examples, I think it is sufficiently established that astronomers of all ages took note of refractions. I therefore anticipate how those who deny the whole matter of refractions, except those that consist only in the enlarging of diameters, would want to excuse their rashness: they would say that the phenomenon that I showed above is very different from the usual matter of refractions. Now it is true that all people consider themselves permitted to rise up against those who appear to profess something new to them. Where this happens without reason, it degenerates into disputation. It is, however, much more shameful that at the same time, people of this sort openly betray their own ignorance, and the smug contempt of others. They should but read only the very recently published disputation of my teacher Mädelin, who uses words from Thesis 58, as the most recent writer, even now by the grace of God alive, are fittingly appended to this chapter in place of a calumny. I append them so that this man’s discoveries may be compared with the Tychochron ones, and the authority of this theory may be all the greater, being propped up by the votes of many, by which the mouths of the naysayers may thus be finally shut. After the testimony of Wielo and Walther is presented, he speaks thus:

If our observations made with the astronomical radius are to be trusted, we

\(\text{159}^{\text{THIO II p. 76. However, Tycho does not mention Walther on this page, though he did consider the observations of Regiomontius and Walther on pp. 59–61.}}\)

\(\text{160}^{\text{THIO II pp. 78–82.}}\)

\(\text{161}^{\text{Chapter 9 Section 4, pp. 32b. The word Kepler uses here is 'nepyprwv', which is the term he uses in the Mars Notensk (KGGW XX.2) for the working out of orbital parameters. I have used a French word because of the analogy between Renaissance humanists' use of Greek and modern English writers' use of French.}}\)

\(\text{162}^{\text{Mädelin, Michael. Disputation de reliquis solis et lunae videl. Marcus ab Hohenfeld.}}\)

Tübingen 1596.\)
have not seldom found, [sometimes in my presence], that the distance of Venus, higher above the horizon, from the sun, placed next to the horizon, is noticeably less than it, on the same day, the distance of the same phainon from the sun were taken higher up and more free from vapors [concerning the opinion of the cause, there is enough above]. Therefore, through vapors [as well as through the surface of the air], it appeared higher than is correct. Therefore, that both it and other stars can likewise appear above the horizon while we are still beneath it, we do not hold as impossible, but conclude as certain.

Further, this is one of the ways by which Tycho very frequently investigated refractions, as will some day be evident, God willing, from his books of observations. Furthermore, Maestlin too, because of his unique keenness of vision, is very experienced at observing Venus by day.

Note to Section 7

Of haloes, of whose circles parhelia are certain parts, as well as parselene. I wished to inform those who are interested in these things with this note. First, the colors of haloes around the sun and of parhelia are the same as those in rainbows, but darker, because of the brightness of the sun shining nearby, and rather weak in haloes around the moon and parselene. Therefore, they are the offspring of refractions, as are rainbows as well. See the experiments through water globes and glass triangles, in ch. 5 below.

Next, the diameter of the rainbow, when the sun is setting, is always 90°; that of a halo, 45°. Therefore, you, whoever you are, who wants to know something beyond Aristotle here, know that the cause of this fact is to be demonstrated by you.

Third, haloes are perceived in clouds that are closest, which is evident from their swift flight. And nevertheless, by absolutely all people, in whatever place they are located, they are perceived as he 221 from the luminaries, as from its center. Therefore, in clouds that are so nearby, everyone sees his own halo, no less than the rainbow, because for a different spectator the heavens’ body is placed among different clouds.

A halo and a parselene have often appeared to me now, while remaining in the same place, when a cloud was suitably located, and have now vanished, when the cloud departed, and have again appeared, when another cloud followed. It is therefore necessary that the rays of the heavens be refracted, and thus be sent onward refracted, in a much higher place, and nonetheless in a matter that is fluid and bounded by a distinct surface, and it always of the same density and depth. Nevertheless, these rays are not seen, unless they be thus received from

201 This is Kepler’s parenthetical insertion, referring to his own presence. Other bracketed words below are also Kepler’s.

202 Ch. 5 Sect. 3, beginning on p. 191.

203 Aristotle considers these phenomena in the Meteorologica III 3.7.
above by a phylactid cloud, in order that this picture might appear from below, that is, through the body of the phylactid cloud.

Fourth, parhelion and parhelic aureole are very near neighbors to haloes themselves, because nearly always there are parhelia in a halo. And whenever these appear, they are perceived at an equal altitude with the sun or moon.

Fifth, in order that a rainbow appear, it is necessary that the place where it can appear be both clothed in matter and shadow. For this reason, whenever there is rain with the sun shining in between, with a cloud appearing immediately above the sun, that is, where the shadow of the cloud has barely gone away, it is then that rainbows are perceived most clearly. Therefore, the waters matter between the spectator and the sun, whether it be rain, or phylactid cloud, or mist (for I have also seen rainbows in mist)—this waters matter, I say, forms and shapes the refractions of the sun's rays, while the matter which is beyond the spectator receives these refracted rays of the sun. But a cloud standing overhead puts the place in shadow, and puts aside the direct rays of the sun, in order that the colored rays be capable of being perceived.

Thus it is not true that the rays, whether of the sun or of the vision, are reflected or refracted in the exact place of the cloud in which the rainbow appears.

Sixth, it must not be thought unimportant that the centers of vision, of the halo, and of the sun or moon are in the same straight line, no less than the centers of vision, of the rainbow, and of the sun.
Chapter 5
On the Means of Vision

While the diameters of the luminaries and the quantities of eclipses of the sun are noted as fundamental by astronomers, there arises a certain deception of the sense of vision, arising partly from the techniques of observation, which we discussed in ch. 2, above, and partly from the simple sense of vision itself, which, insofar as it is not removed, creates a great deal of work for the practitioners, and detracts from the reputation of the art. And so the occasion of looking into error in vision must be sought in the formation and functions of the eye itself. If the optical writers - Alhazen or Witelo, or after them the Anatomists, had treated these subjects clearly, lucidly, and without risk of uncertainty, they would have freed me from this task of continuing the Paradigmata of Witelo in this additional chapter. But, to proceed with this matter methodically, I shall first gather together, as if in the role of principles, the descriptions of the parts of the eye that will come under consideration, from the testimony of the most reputable anatomist, for Witelo's description is untrustworthy and confused. Second, I shall foreshadow in a summary manner the way that vision occurs. Third, I shall demonstrate each and every thing. Fourth, I will uncover that which escaped the reasonings of the optical writers and the physicians concerning this function. Finally, I shall explain the deceptions of vision arising from the construction of the instrument, and shall accommodate them to astronomical use.

1. Anatomy of the Eye

It is conducive to the winning of trust in the demonstration which I am about to bring in, to introduce, not my own experiences, but the public experiments of the most eminent physicians concerning the eye. For what if one were to charge me either with bad faith, as if I were intent upon establishing my own opinion, or with inexperience in the dissections, of which I never before had been either spectator or performer? Therefore, let men of accepted authority speak for me on the subject that is best known to them, up to the point where the undertaking shall have reverted to the limits of my profession. For then they too will ungrudgingly hand the torch over to me at the point where I am going to carry it forward legitimately into mathematics, concerning which the judgement will belong to the expert.

I have consulted chiefly Felix Plater's plates concerning the structure and use of the human body, which, published in 1583, were deservedly reprinted in this year 1603. With these I compared the Anatomia Praemiis of my friend Mr. Johannes Jesenius of Jesen,2 for the reason that he not only professed

---

1 Felix Plater, De partium corporis humani structura et usu libri IIII, Basel, 1583 and 1603. Plater (1555-1614) was Professor of Medicine at Basel from 1586 to his death.
2 Johannes Jesenius a Jesen, Anatomiae Praemii anno 1600 ob se sollemnis admin- istratum historia, Wittenberg, 1601. Jesenius (1556-1621), Professor of Medicine at Wittenberg from 1596-1600, and thereafter at Prague, where Kepler came to know him.
chiefly to follow Aquatendente, but on his own prowess devoted himself chiefly to anatomical labors. If I, being myself, chiefly occupied in the mathematical profession, have passed over any of greater merit in this succession, they will grant me pardon.

The eye [culos] is, from the Greek ὄλυς, whence some φυλή, φυλός, φυλής, φυλοδαίης; because these are the cracks or uncovered orifices reaching from the shadowy head into the bright air, just as, by the opposite reasoning, hymata (wolf) appears to have been said from λύομαι; it is not from σκότος [cover up], for when they are closed they are no use. They are paired in animals, and are purified, and in a suitable distance apart, as it is said above in ch. 3 and below in ch. 9, because they have to note the dispositions of things, and not to have another in case one is lost. For nature does not designate anything for loss. Their location is in the highest place, so that the vision may reach to places that are that much more distant. For since the earth and the sea are most of all spherical, an eye that is not placed at some height would not be able to perceive anything at all of the surface, hindered by the curvature of the globe in all directions. This is shown to us by birds, and especially by griffins, capturing the sight and sound at distant objects all the more distinctly by an erect neck; and above all by eagles, who, with the intention of surveying a whole region for the sake of predation, seek out the highest places. For this reason Lessius is clearer than Pfluter, in saying that the eyes are high up in order to see more things, while the latter says it is in order that they might better see individual things from on high. For as regards individuals, they are better inspected from nearby. And when we are going to scrutinize a coin thrown upon the ground, or a low plant, we bring our eyes downwards. Oddly ethically and wisely says of humans,3

While none of the other animals behold the earth.

A high mouth he gave to hummus to view the heavens.

He ordered them, and so came to the stars,

their sovereign ornament.

But if you should consider physically, the eyes are not at the top, nor is the eye socket upwards, but they lie open more to the sides and downwards, following the nose. And so for the sake of changing the story, they seem to be so located as humans because the human being is a social and political animal. Accordingly, the human makes himself visible to others, in one to another, more than those of animals. So this opinion of Pfluter's that they obtained their anterior place in order to be directly opposite their objects, unlike as it pertains to humans, you may take to apply beautifully to one person opposite another. Another cause is derived from human dignity. Since pasturage is required for other animals, they have eyes.
that are directed downwards to things closest to them. The human, master of creatures, has his face so directed that he should be invited continually to contemplate how far things are the faults of his possession: they are the heavens, itself, contiguous to the mountains, as it appears.

But the cause for the two eyes being directed to the same side is most properly derived from motion; and since this was required to take place in a straight line parallel to the horizon, in every respect the fence of the members and position of the necessary senses were required to be directed to the same region of the circle of the horizon. Further, Galen said that it is for the sake of the eyes that the head is elevated, since the eyes could not be far from the cerebrum, and it is fitting that the neck also be made flexible so that the senses could be turned to see and hear those things that are behind; and hence this line of reasoning also appears to be constructed not badly, and, as it were, from the elements. Thus the eyes are close to the cerebrum because the objects of sight and hearing are the final causes of local motion, and in fact all local motion is placed in the authority of the cerebrum. Therefore, in order that the limits of the body, as a result of some sort of sudden emergency, might get ready for a more swift pursuit of those things whose ghosts might be carried through the senses to the cerebrum, it was fitting that the eyes be set in the very embrace, as it were, of the cerebrum.

Each eye is enclosed in an individual chamber, for safety's sake. However, they protrude, least hindered by the sides of the chamber, they are unable to look out upon the whole hemisphere in a single gaze. But since this total view could not be distant, except in the middle, but would be confused at the sides, it was appropriate that the eyes have a very fast motion, by which the view might better be carried across to the particular parts of the hemispheres.

Further, since there are two eyes, in order to distinguish the distances of things, there was a need for them to have the power of direction, or turning closely together, which is likewise motion. It was not, however, useful that the eyes be moved from their center in this motion. Therefore, in order that they move about their center, they had to be spherical on the outside, except to the extent that they are clothed with oblong muscles within the head, and reach into the cerebrum by means of a certain long process of canals. Moreover, the need for eyelids seemed to require that they be spherical even on the anterior part, on which they are not rubbed by motion with respect to the parts of the face. For because the eye had to be transparent, that could hardly be maintained in a member that would always be hot along with the entire body, unless tears, and air reduced into water and sticking to the outside, and small dust particles flying in, were repeatedly wiped away, and unless the eye were also moistened, wherever it is inclined to dry out and go white. The eyelid appears to accomplish all this, as is manifest in those who weep. The same eyelid also at the same time acquired the function of covering, when danger should threaten or when sleep creeps up, naively also turning aside from the sleeping person the chances of danger that were a threat when the eyes were open. Moreover, to this end of wiping off, the interior of the eyelid is a membranous and soft covering, of the same substance as that

1 Literally, lightning (althenu).
covering which it wipe off, while stone is in an arch at the edge, called the tarsus, smooth and round and of a rather hard substance inside, as are all cartilaginous parts, yielding to some sensible extent at its highest surface, while resisting by the very mass of its body and the condensed hardness. Thus it happens that the upper membranous deserts to be wrinkled up, and is stretched by the globe of the eye; and when the tarsus is lifted up, it folds itself into itself, while when the tarsus is lowered, it never falls away from the globe of the eye, and is not stretched in one place more than in another. Since the whole tarsus moves as one. So it was hardly possible for this tarsus to open and close without getting tired, and to unite with its companion the lower tarsus, otherwise than if the globe of the eye were spherical, while the arches themselves of the tarsus were semicircular, attached on either side as if at opposite poles of the globe. From this mechanism there also arise a pair of angles in each eye, which they call the "canthi" or "interocular". Why it is that the line between these coincides with the line between the two eyes, is said in chapter 3 above, namely, so that the eyes, singly as well as jointly, might imitate the trace of the visible horizon. And so that this might be possible, and the breadth of vision might follow this line, the chamber of the eye, between the brow and the jaw, had to be somewhat depressed, so that nothing stand in the way there. At the nose this was indeed not necessary; rather, a palisade had to be made of its nose, so that the vision of the two eyes might not be confused. This palisade seems in part to accomplish two things: one, that it remove the view of one eye from the other, lest they be blinded by the borrowed brilliancy arising from reflections; the other that by its height it prescribes a boundary of the approach of visible things. But that which has been removed from one eye by the nose is compensated for by the other eye. For nature established the perfection of vision in two eyes. Two other palisades of this sort go off downwards from the chamber of the eye between the jaws and the nose, so that those things that are nearest the mouth and the feet may be open to the eyes. This the necessity of motion and eating requires. The arch of the brow, however, has the greatest protrusion, in order to close off the light of the heavens, by which visible things would be overpowered. For this reason, those touched by the brightness of light wrinkle the brow, and draw that palisade down closer to the eyes. The eyelashes and eyebrows are both for nearly the same purpose. For light that is sent down through them does not strike suddenly upon the eyes with its full ray, but is partly deflected by the eyelashes, and the eyes create the risk for the light approaching through the small gaps in the eyelashes, that it might not be able to be carried along. And when the light is coming from a greater distance, we defend ourselves with the eyebrows and the wrinkling of the brow, while when it is directly opposite, we also make use of the eyelashes' lashes and their squinting. This Aristotle, in sect. 31 prob. 16 thought he had to dispel, but he used an absurd emission of rays, which elsewhere he himself rejected. This is, so to say, the cause for the eyebrows' being curved

7 Literally, "tars" or "fly's pin", respectively.
8 Proposition 9, on p. 30
into an arch, so that they might enclose the circular eyes circularly. Further, as something extra, is that, because they hold up small dust particles, the eyelashes seem to play the role of fans, to sweep out the small dust particles that fly against them; and finally, just here too, as in the whole body, the moist and soft parts appear to be defended by hairs against abrasion. Particularly, when the eyelashes are wiped off, and dust must be pushed out through the double doors by being rubbed off, the hairs function as channels, to divert the dirt through the doors, so that they not be driven into the eye. In fact, all hair-like things, being soft and dry, repel moisture by their roughness and by a certain sort of stiffness, to the extent that that is not overcome. And from these the cause is apparent why the hairs are erect, not lying forward or bent back, not more sparse, nor more dense, nor longer, nor shorter, and moderately curved. It is these which, standing about the eye itself, assist in the function of seeing.

In surveying the globe of the eye itself, just this one thing seems to me to need my care, that I understand the anatomists correctly, and that I have a good command of the nomenclature. So, therefore, the anatomists appear to me to depict and describe the eye as if the overall form of the eye, as it arises from the cerebrum, were to represent the image of an onion, just as the consists of a bulb and a stalk, so the eye consists of the visible globe and of the hidden process reaching in to the cerebrum. And, what is more, in ions there are found nuclei, lying entirely hidden in the bulb, which the coverings common to the bulb and the stalk enclose and conceal. To these nuclei you might not unprofitably compare the humors that are in the sphere of the eye, and that are enclosed in nearly the same way by coverings, which are common to the eye and to the process going to the cerebrum, as well as to the cerebrum itself. For the actual substance of the cerebrum is surrounded by two coverings, the inner being thin, the exterior thick, which they call "membranae." Thus there are in the cerebrum three parts, from each of which is expanded its own covering into the process of the eye, and thence into the globe of the eye itself. The thick meninges produces the exterior covering of the eye, the thin membrane produces the middle covering, while the actual substance of the cerebrum produces the optic nerve, which becomes the inner covering, which is itself not simple. Planet, however, asserts that this is called a covering (membrana) improperly, no doubt because he holds that the optic nerves, which the others assert to be entirely hollow, are similar to the other nerves, and denies that they are hollow.

11 Kepler refers here to the following note.

To p. 163 and 177. Forgive the crude analogy, reader. For when I wrote this, I had not yet hit upon the idea of copying Plutarch's drawings, which depicts the parts of the eye true to life, in print, and including it in the work. As friends later persuaded me to do, so that it could be inserted at p. 177, below. And so I wanted to accomplish here with words and comparisons, what I did not wish to include in a picture.

You should therefore disregard this comparison and repeatedly look at the drawing referred to, and carefully compare what has been said in this entire argument with that drawing.

12 These are the arachnoid membrane and the dura mater respectively.
Let us now divide the globe of the eye into the apparent and hidden hemispheres, and let us further subdivide the one that appears, into the white and the corneal tunic, to speak more clearly. Accordingly, the anatomists say that the exterior covering of the ocular globe, which alone enfolds the whole eye, develops from the thick membra, but according to its different parts it acquires different names and modifications. For the part from the end of the optic nerve through the whole hidden hemisphere, and through the white of the eye, has the name tunic or tunica, 13 and it is hard, thick, opaque, white, and nearly cartilaginous, but that part which covers the iris is a pellucid and prastrade, as Jesenius points out, and experience confirms by a careful inspection of the living eye. Thus the cornea is a portion of a smaller sphere, the sclerae that of a greater: this must be carefully kept in memory. However, in such a narrow space it is not possible to tell whether it is a portion of a sphere rather than of a spherical further, before I pass on to the other two tunics, more has to be said of this exterior tunic. For even though it is the outer with respect to the other principal tunics, it is still not the outermost of the whole eye. For in the hidden hemisphere, it is completely covered with muscles on all parts, both those on the globe and those on the optical process; and these muscles also send tendinous ends on the iris parts out even to that part of the tunic of the sclera that is in the white of the eye, and these enfold it everywhere. Not only this: there is also a certain membrane that arises from the peristernum, called the side-swimming (adnata), adhering, or entamedos, 14 to which the globe of the eye is attached at that circle by which the hidden hemisphere is divided from the apparent one. This membrane is made doubled there: on the one side it surrounds the eyelids, and on the other it covers the tunic of the sclera and its overlying tendinous muscle ends; it is itself transparent, so that the white tunic of the sclera may nonetheless be seen through it by those who examine it. If I understand it correctly, its sense is that not just the white of the eye, but the iris itself as well, is covered by this pellucid and extremely thin membrane. This seems in fact to require a mode of attachment, since it is above all by this bond that the eye is kept in its socket. However that may be, Platter openly asserts that the corneal tunic, which is part of the sclerodes, stretched over the iris, is bare, chiefly because those little reddish veins that are seen to run about over the white of the eye for the benefit of this adhering membrane, do not go in over the iris or the cornea. These things are added for the sake of distinguishing the parts, although they do not contribute to the forming or perceiving of the likenesses of objects, and only pertain to motion and fastening, perhaps also to the nutrition of the eye. The whiteness of the sclerodes and the different colors of what is called the "iris" [rainbow], might give occasion for suspicion that the exterior parts contribute in

13 Evidently the term "sclera", when we use today, was not then common. In the Platter diagram below (p. 186), it is called the "cornea tunic" (tunica tunic). I have therefore kept Kepler's Greek term to the translation.
14 "Clipping close". This term was used by Galen to denote the membrane we call the conjunctiva, which covers the innermost surface of the cornea and the inner surface of the eyelid. Cf. De symptomatum causis liber III, in 0500 VII p. 101.
a way to vision, since the inner parts are perfectly black, while the outside is white to the same degree. However that may be, I do not understand how they could carry anything inside, since those parts beneath which they are carried do not allow them a passage within, because of their darkest black color and opacity. And so if the cause of these colors of the eye is worthing higher than the necessity of a material that is nearly cartilaginous for supporting the round chamber of the eye, it is not discordant with the truth that the reason why what is called the "sun" [i.e., the iris], surrounded by the white, and black in the center, is itself of one color, while the other parts are of another, is so that the sun or the iris in the white, and the black in the iris, might be all the better seen from without, and so that someone might be able to judge at a glance whether someone else were closing his eyes upon him. But consider also Prop. 38 of the first chapter, whether the reason for the globe of the eye's being surrounded by a white tunic is so that it may be more mildly struck by the sun's light, and that it not be destroyed, had it been made black.

Now the second tunic of the eye, which lies beneath the sclerodes, arising from the thin meninx of the cerebrum, is in its posterior part called the choroid, in the anterior, the uvea. These parts do not differ only in location, but also in thickness: the uvea is regarded as twice as thick as the choroid. Finally, they differ also in the application to the exterior tunic of the sclerodes, for where it is called the choroid it adheres to the sclerodes by fibers spread out every-
where, and originates in the same circle by which the sclerodes goes over into the cornea. But the uvea that arises thence does not originate in the remaining part of the sclerodes, which is called the cornea, but diverges from it toward the interior of the eye, the aqueous humor filling up the gap. And while the sclerodes with its cornea constituted an integral tunic, here the uvea is perfec-
trated in the middle, the aqueous humor filling the opening, and also wringing it from the inside, to the extent that this whole part that is called the uvea floats in the aqueous humor, with the sole exception of that circle by which I said it was bonded to the sclerodes, and its posterior part, which is called the choroid. The inner part of the choroid is stated to be black—not like the black of the uvea, but more blue. The inner part of the uvea is rough and very black; the outer part, by which it displays what is called the "sun" [the iris in the eye through the translucent cornea, is light, and along with the exterior surface of the choroid is of various colors, now dark, now gray, now light brown, now brilliant. Thus displaying a broad circle [in the middle of which is the black of the eye or the opening], it is called the Iris. It is by the rubbing of this, since it is light and rigid, that those sparklings which we perceive from time to time in the eyes, appear to be aroused, exactly like the sparklings of the furry backs of cats, and also of the recently cut hair of humans, when rubbed. In the humors themselves, there can be no light without the vision at the same time being impaired.

However, ils persistent lights that are in the eyes of cats, not the transitory ones, like the sparklings, are thought by Ieseneus to reside in this iris.

The third quasi-tunic, arising from the nerve spreading itself out into the hollow hemispheres, is likened, in Platter's opinion (not in the configuration of its substance, but of its shape), to a net bounded by a sna, or better, to a funnel.
Hence, it has received the name "reiform" (net-shaped). It is said to resemble the substance of the cerebrum, but to be more opaque and reddish (bluish, according to Jessenius), whereby one concludes that it seems to be above all a dilated white tinged with redness or blueness. Jessenius also divides this into the substance of the nerve and the tunic.

Here, Jessenius, along with Wielo, and Plater go off in different directions. Jessenius, from Wielo, 17 says that the reiform, where it has embraced the vitreous humor, having progressed, is attached to the greatest circle of the crystalline humor, which is the middle of the humerus; and that as a benefit of Zeus it happens that the crystalline is established at the middle in the vitreous humor, Platen. 18 on the other hand, says that beneath that circle of the concave surface, by which circle the second tunic is pinned to the first, and begins to be called the "vitreum" — that is, from the vitreous, and not at all from the retina — there arise certain processes all around, which by their resemblance to hairs are called "cilium"; that end in the greatest circle of the crystalline. Thus they join the crystalline to the vitreus rather than to the reiform. Those processes are black, and are divided like the teeth of a comb; the stature, moreover, that the retina ends below those ciliary processes, nor is it entirely of the same origin as the choroid of the vitreous humor, which is surrounded, or with these ciliary processes. Thus, in fact, when the vitreous humor is poured out (by whose stuffing alone the retina is brought into contact with the choroid), the retina wrinkles up and does not adhere to anything except its own nerve. I tend to agree with Plater, seeing that he also regarded these things in greater detail. For so far as it appears, Wielo ascribes the faculty of discerning visible things to the crystalline humor, with the result that he sought a passage for this faculty from a nerve for the retina, offspring of a nerve, into the crystalline through these ciliary processes. Plater, on the other hand, leaves the faculty of discernment in the retina, which is in agreement with the truth. The crystalline, however, he includes in the listing of eyestrains, concerned which see below. Further evidence for this is that the ciliary processes, by which the greatest circle of the crystalline is held in place, resemble in their blackness not the retina, but the vitreous.

Now for the humor: there are three in the eye, distinguished by their respective seats. The vitreous is in the posterior part of the eye, the aqueous is in the anterior, and the crystalline is in their middle. 17 And the ciliary processes making a wall between the vitreous and the aqueous. And the vitreous has its very thin andophilic skin, called the hyaloid tunic, of the same name as the humor; and it contains the vitreous, even when this humor has been freed from the tunic of the steroidea, choroid, and retina. The crystalline, in turn, also has its tunic called the arachnoid or spider's web because of its fineness, likewise pellucid, by which the crystalline is contained, also when freed from the surrounding humors. But the anatomists should consider the cause of this name. For...

17 This is, of course, the retina. I shall, however, follow Kepler here in calling it the "reiform," except where Kepler himself calls it the retina.
this name, spider's web, appears tenuously not just to the skin of the crystalline, but to the ciliary processes taken together. For truly, just as the spider stays suspended in the center of her web, so the crystalline humor is suspended in the center of the ciliary processes, by means of the aqueous humor, in turn has no skin, but is held in place before the cornea, behind the spider's web, by means of the ciliary processes of the uvea, and the underlying hyaloid tunic of the vitreous humor. It is, however, ensconced in the middle, and nearly divided in two by the uvea, which it bulges within and outside the opening. The vitreous has the greatest mass, the crystalline the least, its axis being hardly the fourth part of the axis of the vitreous.

The vitreous has most nearly the shape of more than half of a sphere. The aqueous does consist of spherical surfaces, but it has been driven back in the middle, so I have said, into a narrow neck. The crystalline, or that face that is bounded by the ciliary processes and immersed in the aqueous, acquires either a spherical figure, or a portion of a lenticular sphere, a roated ellipse, divided by the axis, the side remaining straight. On the posterior side, which is bounded by the same ciliary processes, it is immersed in the vitreous. Its figure is a hyperbolic conoid, a hyperbola rotated around the axis. For Jerosimus thus relates, that it is not spherical, as Platter said, but that it protrudes markedly, and is nude oblong, stretching up almost into a cone, and that on its anterior face it is of a flattened roundness, the anterior roundness of the cornea and the crystalline humor being perceptibly similar, their dimensions being in the ratio of 4 to 3. The aqueous is least dense, the crystalline densest, both are perfectly transparent. The vitreous is slightly more tinted. If an axis is drawn through the center of the opening of the uvea to the optic nerve, the center of the circles of the crystalline lies upon it. But the crystalline is rightly said to be fixed in the center of the eye, if you understand the eye to be cut by a plane perpendicular to the axis, and the center of the eye to be the center of that circle. But as regards the overall globular shape of the eye, the crystalline serves more to the front of the axis described. The crystalline is at the greatest distance from the end of the optic nerve, less from the opening of the uvea, and the distance of the opening of the uvea from the surface of the cornea is least.

2. The Means of Vision

Although the brow, eyebrows, eyelids, socket, also themselves contribute to the means of vision, they do not more with the chief tasks. Therefore, I have now prefixed a description of their use, and joined it with an account of them. But the parts that I have appended, relating to the actual globe of the eye, all contribute to the principal function. Therefore, in order that the use of each eye be apparent, I shall describe the means of vision, which no one at all to my knowledge has yet examined and understood in such detail. I therefore beg the mathematicians to consider these carefully, so that thereby at least there might exist in philosophy something certain concerning this most noble function.

5 Acanth, which can mean either spider or spider's web.
I say that vision occurs when an image of the whole hemisphere of the world that is before the eye, and a little more, is set up at the white walls, tinged with red, of the concave surface of the retina. How this image or picture is joined together with the visual spirits that reside in the retina and in the nerve, and whether it is assigned within by the spirits into the cavities of the cerebrum to the tribunal of the soul or of the visual faculty, whether the visual faculty, like a magistrate given by the soul, descending from the headquarters of the cerebrum outside to the visual nerve itself and the retina, as a lower court, might go forth to meet this image—this, I say, I leave to the natural philosophers to argue about. For the arsenal of the optical writers, does not extend beyond this opaque wall, which in fact occurs first in the eye. I do not think that Witelo should be heeded in regard to Boet. 3 Prop. 20, in thinking that this image of light originates further along through the nerve, as far as where in mid course the nerves of the two eyes come together in a kind of joint, and again diverge into their individual cavities of the cerebrum. For what can be pronounced by optical laws about this hidden conjunction, which, since it goes through opaque, and therefore dark, parts, and is administered by spirits, which differ entirely in kind from the visible and other transparent objects has already completely removed itself from optical laws. And so, when Witelo argues, in Book 3 prop. 20, that the images need to be united in prop. 21, that refraction therefore occurs at the back of the vitreous humor, and, in prop. 22, that spirits are transparent; I invert the sequence: spirits are not an optical body, and the narrow cavum of the nerve is not optically straight, and if it were so it would immediately become bent when the eyes are turned, and it would place in opaque parts against a very slight opening or mouth of a passageway. For this reason, light does not pass through the back surface of the vitreous humor, nor is it refracted there, but is driven against it. And how can an image be refracted if it enters perpendicularly? It is remarkable that Witelo was not struck by this in writing proposition 31 of the third book. As a result, he was caught in difficulties that were not trivial in prop. 73 of the third book, on account of this conjonction of images at the conjunction of the nerves. If anything is to be said about this conjoncation of the nerves in mid course, is should be according to physical principles. For it is more certain than certain that no optical image enters through this far. It appears, accordingly, that if each nerve had carried, through freely and directly to its seat in the cerebrum, the result would have been that when we used two eyes we would that we were seeing two objects instead of one. Or has conjonction been brought about so that when one eye is

---

18 Idem.
19 Physics
20 Species. For propositions from Witelo see vol. II of the Theaemata.
21 Witelo ill prop. 31: "When a single gaze of the object seen strikes perpendicularly upon the surfaces of the two eyes, it is necessary, that the radial axes be reflected angularly, in the centers of the openings of reflection of the concave nerves." Theaemata II p. 99.
22 imag.
closed, that hollows or the cornebras might not also cease its function of judging. Or perhaps that doubling of the hollows is not only for the sake of the eyes, but in itself facilitates the making of correct decisions, just as the association of the eyes goes for distances. Accordingly, in order correctly to distinguish visibly as well those that are perceived with one eye as well as with two, this confidence of the passageways had to come to pass. This one thing must be said here from the conclusion of chapter 1, from an optical perspective, that spirits are acted upon by colors and lights, and this action is, so to speak, a coloring and an illumination. For the images of strong colors remain in the vision after perception of them has occurred, and mingle with colors impressed by a new perception, and a confusion of the two colors occurs. This image, which has an existence separable from the presence of the object seen, is in the histories or the images, as was proved above. Therefore, it is in the spirits; and vision occurs through this impression of images upon the spirits. However, the impression itself is not optical, but physical and mysterious. But this is a digression.22 I return to explaining the mean of vision.

Vision occurs through a picture of the visible object at the white of the retina and the concave wall; and those things that are on the right outside, are depicted at the left side of the wall, the left at the right, the top at the bottom. Further, green things are depicted in the color green, and in general any object whatever is pictured in its own color within. The result of this is that if it were possible for this picture on the retina to remain while the retina was taken out into the light, while those things out in front that were giving it form were removed, and if some person were to possess sufficient keenness of vision, that person would recognize the exact configuration of the hemispheres in the compass of the retina, still as it is. Moreover, that proportion is preserved, such that when straight lines are drawn from the individual points of objects that can be viewed at quite particulate point within the compass of the eye, the individual points within are depicted in very nearly the same angle at which the lines will have come together, so much so that not even the smallest points are

22 Here Kepler inserts an aserisk referring the reader to the following endnote.

To p. 170. It is not worthy of consideration, what Master D. Mathaue Wagner, the Emperor's Councillor of the Imperial Hall, told me once happened at Heidelberg. Someone who had lost one eye and who covered the other completely with the pith of his hand observed that, if something strong were to be set beneath his nostrils, he would be aware of its dinting and would also to some extent disorganize it; it is therefore in doubt whether some opening might have been set up in the empty eye socket through which the nose was the head and to the end of the visual nerve, the seat of the spirits; or whether the same spirit might also be poured forth into the instruments of smell.

23 What is chiefly confirmed by this example is that the spirits are so up to have a capacity for light and color.
left out. And so much as the fineness of this picture within the eye of any person you please be as great as the acuteness of vision in that person.

Now, in order to approach closer to the way this picturing happens, and to prepare myself gradually for the demonstration, I say that the picture consists of as many pairs of cones as there are points in the object seen. The pairs always being on the same base, the breadth of the crystalline humor, or making use of a small part of it, so that one of the cones is set up with its vertex at the point seen and its base at the crystalline though it is altered somewhat by refraction in entering the cornea, the other, with base at the crystalline, common with the former, the vertex at some point of the picture, reaches to the surface of the retina, this too undergoing refraction in departing from the crystalline. And all the outides of the cones come together at the opening of the uvea,25 at which space the intersection of the cones takes place, and rays begins left.

So that these things may better be grasped, I shall repeat a step at a time.

Let there be some visible point, directly opposite the eye, so that a line through the orifice of the nerve and the center of the opening of the uvea falls upon this visible point. Since any point you please radiates in an orb, it will therefore also radiate to the parts of the orb, with the result that it will radiate to the whole of the small portion of the little sphere of the cornea, and will illuminate the iris and its black corona, of the opening of the uvea. And since the iris is opaque and black, it turns away and keeps out the rays that are all around the edges, admitting within only those in the middle, so far as it is spread out at the opening. And since the cornea, and the aqueous humor that is beneath it (both of which I take as the same medium with respect to density), are a medium denser than air, the rays sent down to the inclined surface of the little sphere are accordingly refracted towards the perpendicular. Thus those rays which previously were spreading out in their progress through the air, are gathered together now that they have entered in to the cornea, so much so that any great circle described by those rays upon the cornea, which in their descent touch the edges of the opening, is wider than the circle of the opening of the uvea; however, these rays, all the way to the opening of the uvea, are so strongly gathered together through such a small depth of the aqueous humor, that now the edge of that opening are trimmed off by the extremes, and by the descent they have made they illuminate a portion on the surface of the crystalline too, is smaller than the opening of the uvea. Now, having entered into the front surface of the crystalline humor, if indeed they all were first arisen from a point at a certain and proportionate distance (which is peculiar to each eye, and not just the same for all), they all fall approximately perpendiculary, because of the similar convexity of the cornea and the crystalline humor. As a result, hardly any new refraction of rays of a point directly in front of the vision and at a suitable distance occurs at the front surface of the crystalline, even though the medium of the crystalline is denser than that of the aqueous humor. Again, however, it here ascribe to the spider web and to its humor, i.e. the crystalline, the same density with regard to refraction, as I do below to the vitreous humor and to the hyaloid tunic. Thus these rays of one point, in the

25 The pupil.
amount that was admitted through the opening of the uvea, descend through the entire depth of the crystalline, always coming closer and closer together, until they come to the hyperbolic posterior surface of the crystalline. So if it were possible for one to make a likeness of this series of rays in a cross section of the eye, he would make one and the same conic surface from the heights of the cornea all the way to the last parts of the crystalline, whose breadth, is, of course, that of the opening of the uvea, and, that while its vertex would stop at some point beyond the eye. But now, when these rays go out through the conic posterior surface of the crystalline into the vitreous humor, whose medium is rarer than the medium of the crystalline, they are refracted from the perpendicularly drawn to the surface through the points of refraction. As a result, it happens that the refracted rays converge towards the axis, and so are bounded by a shorter and more oblique cone than that by which they had previously come through. Finally, then, all these rays from a single visible point come together in exactly one point, which is the very center and end point of the optic nerve, at the place where it is joined to the retina.\textsuperscript{20} For corresponding to this density of the crystalline and the magnitude that these refractions are going to have, Nature has measured out the space of the vitreous humor between the crystalline and the retina.

And this vision, finally, is the quose distinct, where all the light of the same point, however much it is spread over the breadth of the cone admitted through the opening of the uvea, is brought together by two refractions, one at the cornea, the other at the posterior surface of the crystalline humor, and diminishes most strongly a single point of the retina, namely, the surface of the nerve bearing the visual faculty or spirit; and no other rays from any other focal point can fall upon that place, because of the beneficial action of the blackness and opacity of the uvea, of the narrowing of the opening, of the ciliary processes, and of the reticulum, which will be described shortly.

For the rest, we may see how such a visible object was a point, not a body, and accordingly it had no parts, nor was right distinct from left, above from below, and it was not really visible, but was an element, or rather a boundary of a visible object; and thus the vision itself of the point that this has so far been explained must not be taken as the sum total of vision, but as a certain element of vision. For we, as there are many points in the whole object, so there are, as it were, many elements of the vision of that object. Wiels Io. 19 nonetheless says, that nothing is seen unless it has some proportionate magnitude.\textsuperscript{22} Therefore, let there be another point next to and directly opposite the previous one, inclining off to the right of it. This point will therefore also itself illuminate the cornea, and will face the underlying iris and its opening obliquely. Thus, the rays sent in through the circle of the opening will display the form of a scalene cone, which will cut the right cone of the previous point at the opening of the uvea, and after the intersection will pass over to the left side within the uvea, illuminating a part of the surface of the crystalline also illuminated by the previous one, but also a part not illuminated by the previous point, but more to the left, and

\textsuperscript{20} Reading "quo in loco" for "qua in loco."
\textsuperscript{22} Bucersius III pp. 93-4.
nearly the same thing happens that we proved in ch. 2 above happens in a closed chamber. For the pupil takes the place of the window, the crystalline takes the place of the panel opposite, except that because of the proximity of the pupil and the crystalline, the complete intersection has not yet been brought about here, with the result that everything is still confused. Next, when this cone contracted on the left strikes upon the anterior surface of the crystalline, it will be refracted towards the right cone, and nonetheless takes an oblique path through the crystalline, and to that extent strikes more directly upon the hyperbolic surface of the crystalline, where it will again be refracted towards the previous right cone, but slightly so, with the result that it departs less from the previous right cone in the vitreous humor than in the crystalline, but it does nevertheless depart, and thus will fall upon the left wall of the retina. But the ratio of the hemisphere might be perturbed if the points outside in the air, opposed each other from the center of the eye, were deflected from the opposition by the triple refraction made by the surfaces of the cornea and the crystalline, and were so slow into the depth of the eye as if from an angle, and were thus to gather themselves into a portion of the hemisphere that is less than the hemisphere. To keep this from happening, Natura has found an excellent proportion, by having established the center of the retina, not at the intersection of the axes of the cones which go all through the vitreous humor, but far within, and by having moved the edge of the retina from the sides, so that thereby the longer cones, which are more divergent, would intercept the straight (and therefore short) parts of the retina, but those that are shorter, and diverge less to the sides of the retina, would mark out large parts of the retina, presented obliquely, with a narrow angle; and those cones belonging to points radiating from opposite sides, though they are not opposite when refracted, nonetheless would fall upon opposite points of the restiform, and thus compensation would be made. And thus, finally, if straight lines were drawn from the points of the hemisphere through the center of the retina and of the vitreous humor, those lines would mark the points of their own respective picture on the retina opposite. If this were not so happen, the quantity of things seen individually from the side would always vary with the turning of the eye, as happens when lenses are moved towards the eyes. For these, even though they adhere to the eye without motion, if they are carried around with it, represent those things that are at rest with some motion, because of the variation of the apparent quantity of the hemisphere from the sides.

Further, it is worthwhile here to consider the difference between direct and lateral vision. First, the cone of direct vision is enclosed by the uvea alone, so that the whole of it is in the cornea; some oblique cones are left outside of the cornea itself. For they could be wider if proportion to the uvea, and so they measure out light gradually to the retina. The direct cone is circular or right; the oblique cones are flattened scalene. The axis of the direct cone is not refracted at the cornea, the axes of the oblique cones are refracted at it. All the lines of the direct cone are approximately perpendicular to the crystalline, none of those of the oblique cones are. The direct cone is cut equally by the anterior surface.
of the crystalline; the oblique cones are cut very unequally, because where the anterior surface of the crystalline is more inclined, it cuts the oblique cone more deep. The direct cone two the hyperbolic surface of the crystalline; or the boss, circularly and equally; the oblique cones cut it unequally. All the rays of the direct cone are gathered together at a single point of the retina, which is the chief thing in this process; the lines of the oblique cones cannot quite all be gathered together, because of the causes previously mentioned here, and as a result, the picture is more confused. The direct cone aims the middle ray at centre of the retina; the oblique cones aim the rays to the sides. The direct cone falls directly upon the retina; the ones at the sides, obliquely, because it was just now said that the center of the retina is below the intersection that the axes of the cone, which are in the vitreous humor, will have made. Finally, the sensory power, or spirit poured in through the nerve, is more concentrated and stronger with respect to both the source and the orb, in that place where the retina is presented to the direct cones; it is spread out from that point with the spherical surface itself of the retina, and withdraws from the source, on which account it is also weakened. And, as in a funnel, and in a fishing net with a small bag, which above were analogies for the retina, all the sides turn either the liquid or the fluid aside into the channel or the little bag, so the sides of the retina use their measures of sense not for its own sake, but whatever they can do they carry over to the perfection of the direct vision. That is, we see an object perfectly when at last we perceive it with all the surroundings of the hemisphere. On this account, oblique vision is least satisfying to the soul, but only invites one to turn the eyes thither, so that they may be seen directly. And so Wieso III Proposition 17 needs interpretation.29 For by the perpendicular alone nothing is perceived distinctly. But by all the radiations of the same point from which a perpendicular can be drawn through the centers of the opening and of the hument, gathered together at the center of the opening of the nerve (by Wieso III 29), vision that is completely direct and detailed, or distinct, occurs.30

Now The color of the retinas is not dark or black, so as not to ruin the colors of objects; nor is it quite as bright as possible, so as not to pour excrescent brightness into the vitreous humor. and so that the things represented as white and shining upon it might not obscure the more colored things in an immediate proportion. See the corollary to Prop. 30 and 31 of ch 1. The figure of the retina is more than a hemisphere. First, because it was turgid that it should be a hemisphere for receiving a picture that is proportioned to the objects, as has already been said. But the remaining edge is extended all the way the the cornea processes, so that the retina might be stretched out by the stuffing of the globe of the vitreous humor, the neck now being made narrower than the belly. For it was impossible

29 Wieso III 17 states, "Distinct vision occurs only following perpendicular lines from the points of the object seen extended to the surface of the eye. From this is evident that every shape seen is reflected on the surface of the eye just as is a oriented on the surface of the seen object." (Theatres II p. 92).

30 Wieso III 29: "Every point, belonging to a form, that is incident upon the surfaces of the eyes along the radial lines, must necessarily reach the center of the opening of rotation of the optic nerve." (Theatres II p. 99.)
to attach it anywhere because of the tenderness and fineness of the visual spirit, which required a channel in the nerve, which, contrary to the nature of the rest of the nerve, does not even support the substance of the nerve. For if the retina had not occupied more than a hemisphere, it could easily have been wrinkled up and have run back to the pia mater with the nerve. And further, the hollow had to be filled up by the hemispheres all the way to the ciliary processes, and thus this would have had to be accomplished either by the choroid or by the vitreous humor. This, however, is done more beautifully by the retina. But this edge also does not appear to cease its function of seeing. For even though none of the cones formed through the crystalline should touch it, the fissures of the ciliary processes nonetheless appear to be molded into this edge itself, so that light might to some extent come in from the sides through the ciliary processes and might be received by this extended edge of the retina. For a straight line drawn from the extremity of the cones through the nearest edge of the opening of the urea nearly falls upon the facies of the crystalline with the ciliary processes; drawn through the opposite edge of the opening, it nearly touches the origin of the ciliary processes from the urea. And it is by this device that nature seems to have secured our ability to perceive more than a hemisphere without moving our eyes, fully as much as is admitted through the cornea of the eye, with this eye turned lock ever so slightly. It is indeed little short of your being able to perceive the cars themselves with the neighboring eye, especially if they are either long, I have often seen, to my amusement, both the sun and my shadow, as if the two were out in front and not opposite each other. This precaution above seems to have provided for the guardianship of the eyes, so that things that were not turned away from the chamber, where it ought to be open, might in approaching some immediately into view, wherever the eye might be looking. For why should that which takes care of the whole animal, not also take good care of itself, and give this function to the care of no bystander? The vitreous humor, in turn, has acquired a 6vun, so that it might not weaken the nerve or the retina and make their grow lamp, by wetting them; and so this it not be mixed in with the aqueous humor on its angular side through the fissures of the ciliary processes. It has to differ in density from the crystalline, as well as from the aqueous, for the sake of refection. For unless it were rarer than the crystalline, the rays would not be gathered together from their perpendiculars to the axis of any of the cones. And if the vitreous humor is denser than the aqueous, the rays that come in through the ciliary fissures will be able to descend deeper in proportion to the density, and will approach by way of the outside cones that are given their form by the crystalline. Further, the crystalline has acquired a small tunic, so that it, being soft, should not become mixed together with the aqueous humor. It is, moreover, too small to touch the sides of the retina, in order that through this gap the rays of the cones may be gathered together at a point. Because it had to be both supported and connected, for the sake of nutrients, it was bound to the choroid by the ciliary processes. These are black all over, so as not to create glare when illuminated, and closely spaced, so that the lesser changes of the vitreous humor might be dark, as it should be, and so that the vitreous humor might not shine the
images by being illuminated. It is no doubt for this application that they i.e. the ciliary processes will acquire the motion of pulling themselves up, or becoming cloudy in bright light, and of becoming rare in bad light, like the vortex. And a hyperbola, or a figure akin to it, is at the posterior part of the crystalline, so that the rays of the same right cone that tend downwards within the hyperbola and converge towards the same point, should be gathered together at the same point, but at a shorter distance. That this cannot happen in another figure, will be demonstrated below. But the anterior surface of the crystalline is gibusous, so that the more obliquely the opening of the vortex is presented to the radiating point, the more on the slope this surface is cut by the scalene cone, and thus the equality of the viscusinity intercepted by the cone is preserved, as much as possible; and so that it might be perpendicularly bethall all rays refracted at the cornea (coming from the same particular point) and tending to the same point after refraction. I maintain that this anterior surface of the crystalline is entirely circular or sphericall.

The whole vortex with the ciliary processes is present only to produce darkness, so that the light not be excessively troublesome. It brings nothing to the forming of the image or picture, and if it had brought something, it never would complete it and bring it to perfection, because the opening is too wide in proportion to the narrowness of the eye. For, in fact, when it is established in darkness it is dilated three times more widely than in the light of the sun, so that in darkness it might uncover a greater part of the surface of the crystalline, so that thereby more of the light that is so weak, gathered at the same point through the crystalline [lens] (which gathering at the same point is accomplished by the crystalline without the help of the opening), may arrive the sense all the more obviously. In the light, on the other hand, it is narrower, so as to keep out more of the light, lest so strong a light injure the sense. Thus the location of this opening is the place where the rays intersect, existing as an effect of the crystalline itself. However, this intersection does not occur in a point, but is spread out into a very long cone, because of the circular surface of the crystalline. Therefore, the location of the opening becomes the base of this cone of intersections. For between this opening and the crystalline there is no intersection, and if something were set out there before the sense of vision, it would appear inverted and confused. On the inside, the area is rough, so as not to cause a reflection of a ray reflected at it from the surface of the crystalline, as it would if it were smooth. It is still black, even where it envelopes the aetna, with a likeness of substance. For the motion, by which the anterior parts had to be made black, has its passage through the choroid. Unless perhaps the aetna too has transparency, and this black is the boundary of that transparency.

The aqueous humor was needed to fill up the chambers, and to continue the refraction created at the cornea, so that it should be the same all the way to the crystalline.

31 See Prop. 24 of this chapter, p. 244.
32 Reading “constitutus” for “constituens”.
The tunic of the cornea also seems itself to be a small portion of a spheroid, so that the rays that fall perpendicularly upon the anterior surface of the crystalline might come together in one point. There is, however, nothing to prevent the cornea's being perfectly round, as will be said below.
In order to convey the reader for whom anatomical plates are not at hand, I thought it advisable to place here a copy of Pliny's 
Plate; I would that this plan had prevented itself to the same at the start of work, for I would then have accommodated the text to it. No explanation of the letters and names to be added other than those of Pliny himself; I have added only the references to my text.

There are 19 figures, for the law two (the organs of hearing) the engraver added unbidden.

1. Portrayal, through a line drawing, of the membranes and humors of the eye, in imitation of the real eye. In which A is the crystalline, B the vitreous, C the aqueous, humors; D the adaxia tunicae [i.e., the conjunctiva]; E the opaque part of the thick tunic (tunica vera); F the uvea, G the retinal, H the hyaloid, I the crystalline, tumes; K the ciliary processes of the uvea tunic, L the indentation of the uvea separating off from the thick tunic; M the corneal part of the thick tunic, whose protruding convexity, noted by others, is indicated by dots; N the muscles of the eye; O the visual nerve; P the thin membranes of the nerve; Q the thick membrane of the nerve. p. 163.

II. The whole eye with the muscles removed from the skull, separated only from the eyelids.

III. The anterior part of the globe of the eye. p. 163.

IV. The thick tunic, i.e., the sclera of the eye with a part of the optic nerve, pp. 161, 163.

V. The thick tunic [i.e., the sclera] of the eye, divided by a transverse section.

VI. The uvea tunic with a part of the optic nerve. pp. 165, 176.

VII. The vortex layer of the surface of the eye. pp. 165, 177.

VIII. The retinal tunic with the substance of the visual nerve. pp. 163, 166, 164, 175.

IX. The hyaloid tunic. p. 167, 176.

X. The ciliary processes, spread radially through the anterior [parts] of the hyaloid tunic. pp. 166, 167, 175, 176.


XII. The crystalline humor, still covered by the tunic.

XIII. The crystalline humor bare, placed so as to be seen from the side.

XIV. The anterior surface of the crystalline humor. p. 167.

XV. The three humors of the eye at the same time together: the aqueous, the vitreous, and the crystalline, only shadowed in incidentally. p. 167.

XVI. The vitreous humor containing the crystalline. p. 167.

These are the page numbers following each item in the list. They refer to Kepler's page numbers (the numbers in the margins of the Latin edition). The line, along with the page, were inverted between signatures Y and Z; on two unnumbered sheets.
The anterior seat of the vitreous humor alone.

The anterior seat of the aqueous humor alone.

The visual or optic nerve. p. 172.

The thin tunic surrounding the nerve.

The thick tunic, wrapping the nerve.

The muscles of the eyes of one side. pp. 161, 164.

Part of the adnata tunica [i.e., the conjunctival], stretched beneath the eyelids. p. 164.

Part of it expanded, with veins interwoven. p. 164.31

The dark or iris of the eye, which the white surrounds, pp. 165.

The root or center of the eye, in the middle of the iris, p. 165.

Note to it, IV. V

The arc as the letter G. delineated by dots, arising from the extrinsics of the iris, and a portion of a smaller circle than n, the globe of the eye, was added by me from the observation of others, and indicates the protrusion of the cornea, by which it protrudes from the white. p. 164, 177.

The tissue at the inner angle of the eye. p. 161.

The tear openings.

Ducts spread through the thick membrane.33 p. 164.

The fibers by which the uvea tunica is joined to the thick [i.e., the sclera]. p. 165.

The indentation of the uvea, where it separates off from the cornea. p. 165.

The opening of the uvea, or the pupil. pp. 165, 177.

The beginnings of the ciliary processes. p. 166.

The beginning of the uvea, expanded out of the thin tunic. p. 165.

The extent of the retinal tunic, stretched forward beyond the middle of the eye. pp. 166, 173, 175.

The hollow of the hyaloid tunic, supporting the crystalline humor.

The breadth of the crystalline tunic. p. 167, 168

The spherical posterior part of the crystalline humor (promoting in a cone, as some explain it; hyperbolic, as I explain it). pp. 167, 176.

31 See Kepler’s note on p. 175.
33 Cataract membrane, or cataract tunica, i.e., the sclera.
in XIV. XVI. The flattened anterior part of the same. pp. 167, 176.
V. XV. XVI. The extent of the vitreous humor. pp. 167, 169.
X. XV. XVIII. XII. Extent of the aqueous humor. p. 167.
Y. XV. The seat where the vitreous humor is distinguished from the aqueous by the interception of the hyaloid tunica. p. 167.
Z. XV. XVIII. The seat where the aquea floats upon the aqueous humor. p. 167.
& XVI. The cavity in the vitreous humor that remained when the crystalline humor was extracted.
§. XIX. The cavity in the aqueous humor, from the same cause.

3. Demonstration of those things that have been said about the crystalline in regard to the means of vision

Common experiences of nearly all the things that have so far been said of the crystalline may be perceived in crystalline balls and in urinary vessels filled with clear water. For if one were to stand with a crystalline or aqueous globe of this kind in some room next to a glazed window, and provide a white piece of paper behind the globe, distant from the edge of the globe by a semidiameter of the globe, the glazed window with the channels overlaid with wood and lead, enclosing the edges of the windows, are depicted with perfect clarity upon the paper, but in an inverted position. The rest of the objects do the same thing, if the place be darkened a little more, to such an extent that if the globe be brought onto the chamber which we described above in ch. 2 prop. 7, and set opposite the little window, whatever things are able to reach through the breadth of the little window or opening to the globe are all depicted with perfect clarity and most pleasingly through the crystalline upon the paper opposite. And while the picture appears at this distance uniquely (that is, a semidiameter from the globe to the paper), and nearer and farther there is confusion, nevertheless, exactly the opposite happens when the eye is applied. For if the eye be set at a semidiameter of the globe behind the glass, where formerly the picture was most distinct, there now appears the greatest confusion of the objects represented through the glass. For the glass appears either entirely bright, or entirely red, or entirely dark, and so on. If the eye comes to be nearer to the globe, it perceives the objects opposite even and large, where they are poured together on the paper, if it is on the other hand removed further from the globe than a semidiameter of the globe, it grasps the objects with distinct images, inverted in situation, and small, and clinging right to the nearest surface of the globe. Before, however, when the paper was placed there, the picture had entirely vanished. All these things happen with regard to an aqueous globe, because of the refractions and the shape, as a result of there being some convexity in the shape. And since the crystalline is made of convex surfaces, and is also denser than the surrounding humors, just as water in the glass is denser than air; therefore, whatever we shall have demonstrated concerning the aqueous globe in this way, and using these media, have also been proved concerning the crystalline, with privileges reserved to it because of the particular
convexity of shape, inconsistent with the convexity of the globe. Let us proceed, then, to the demonstration of those things that happen in relation to a crystalline or glass globe.

Proposition 1. Problem

To find the place of the image by the commonly known path, when the object is observed through a globe of a dense medium. Let $E$, $F$, $G$, $H$, $I$, $J$, $K$, $L$, $M$, $N$, $O$, $P$ be points on a globe of a dense medium. Let $A$, $B$, $C$, $D$, $E$, $F$, $G$, $H$, $I$, $J$, $K$, $L$, $M$, $N$, $O$, $P$, $Q$ be the points of the refractions towards the visible object $A$. First, if $E$ is the point of refraction, and $EF$ be the ray that is refracted from $AE$, then $A$, $E$, $F$ will be one surface, by ch. 3 definition 2. But by 16 of that chapter, the surface $AEF$ is perpendicular to the globe: that is, it passes through the center $I$. Therefore, $AEF$ are in one surface. Now if $I$ is the point to which $EF$ is refracted into $FC$, and through $I$ comes one eye at $C$, by the same proposition, $FCI$ will therefore be one plane surface. And since surfaces $AEF$ and $FCI$ have a common part, the surface $FCI$, therefore, $AEFCI$ are in one surface; and $AI$ extended will meet with $FC$ at the intersection $D$. In exactly the same way it will be proved that the other eye $B$ and $DHGAI$ are in one surface. Therefore, $D$ is on both surfaces, and consequently $BH$ and $CF$ cut each other mutually at $D$. Thus $D$ is the place at which the image of the object or of the viewed point $A$ is seen with the two eyes, by chapter 3 17.

Proposition 2

The sense of vision looks upon things that are very close with greater difficulty than upon things that are more distant. For it was said that in the viewing of nearby things the eyes have to be turned towards each other. Turning towards each other is variable with nature, which has given the eyes a parallel situation. Fatigue follows, as a result, and less fatigue from a lesser turning. Hence, those who are given to thought are easily distinguished from others, although no one understands the indication. It is just those that they relax the muscles of the eyes, so that they are less turned towards each other for nearby objects, for thus they return to their parallel situation.

Proposition 3

The sense of vision is drawn to the obvious, poorly attuned by the faint or ravishing. What experience testifies, the property of vision confirms. For it has been given in order to be moved by light, and it will therefore be moved strongly.
Chapter 5

by a strong light. But to be moved by light is to see. Therefore, one who had first been looking at a weak light will follow a stronger light when one such arises from the same direction, and will do so in exactly the same way.

Or, by chapter I, prop. 28, when strong lights illuminate the eye excessively, a weaker illumination will remain hidden beneath it. But the sense of vision follows the manner of illumination.

Proposition 4

Darkness accommodates an image, but when a stronger light arises from the direction of the image, the image passes away. For an image is in part a creature of intention, thus the work of the sense of vision. Thus, when the act of vision passes away, the image passes away. But the act of vision passes away with these conditions, by the preceding. On the other hand, when the place or matter is dark in the direction of the image, the stronger light of the image, by the preceding, consequently causes the act of vision.

Proposition 5

In front of an aqueous ball or globe there is no place for the image of an object hiding behind the ball. Experience confirms this. Causes therefore had to be sought. For this proposition is not only a means of correcting what J. B. Porta proposes in Book 17 ch. 13, “with a crystal ball, to make an image appear to be hanging in air,” but also detracts from proposition 1 of this chapter. So, since the place of the image, by 1 of this chapter, is nearer the eyes than is the aqueous globe, therefore, by 2 of this chapter, the eyes will be turned towards each other to the place of the image with more difficulty than to the globe. And by 3 of this chapter, the eyes will be drawn to the globe, whose light is stronger and more obvious than the rays of the object reaching through the ball to the point or place of the image. Therefore, by 4 of this chapter, the light from the aqueous globe, illuminating the intermediate place of the image, drives away the image, when a new act of its seeing is aroused. Hence Porta, vacillating, says, “If a visible be of the greatest visibility, such as a tree or a candle, the object will be seen without difficulty, and more clearly.” Therefore, those that are not fire, by Porta’s admission, are perceived with greatest difficulty in the image’s usual place. What need of more? These are the conclusions of the optical writers originating from an insufficient and non-universal demonstration, concerning the place of the image. Pertinent to this is what Porta had taught in chapter 10 preceding, “with a convex

35 Intentionale erga.
36 Here Kepler refers to the following endnote:
   37 To p. 190, prop. 4. This is the reason why a bluish and nearly colorless material is laid beneath glass mirrors, while metallic ones are mostly made of iron or another black material. Hence also, when darkness outside surrounds glass windows, i.e., at night by candlelight, the windows appear to be as mirrors.
38 J. B. Porta, Magia naturalis (Naples, 1589) p. 270.
39 Porta, Magia naturalis, p. 271.
crystalline lens, to see an image hanging in air. For the reason is very nearly the same. For this reason, he adds, "If you will place a piece of paper in the way, you will see clearly that a lighted candle appears to be burning upon the paper." That is, the image will be seen weakly and hardly at all in the bare air itself, by Porta's admission. But if you put a piece of paper in the way—if, I say, you interpose a piece of paper between the lens and the sense of vision (for, with me, Porta here is still speaking about the image, not yet about the picture, of which this is true, as will be clear below), the image will now appear, not hanging in air, but fixed on the paper. For the paper, striking the eyes more obviously, steadies them on the place of the image, so that they may be turned towards each other in that direction. And nonetheless, because the paper is then brighter than the image, the paper will be seen primarily, the image secondarily. For it is not mathematical dimensions alone that create the image, but also, and much more, the colors and lights and physical causes, with which prop. 2, 3, and 4 of this chapter are concerned. If you should focus the eyesight upon one place, namely, upon the place of the image previously investigated, as it has been described in prop. 1 of this chapter, when a clearly visible object is placed nearby, then the eyes, coming together upon this object, will also see the esquired image secondarily. Another more tricky caution, from 4 preceding, can be applied, an experiment of which I saw at Dresden in the Elector's theater of attrilbes, but what I, steeped in demonstrations, stated that I had seen, the others denied. I therefore attribute it, not to the overseer's intent, but to chance. A disk thicker in the middle, or a crystalline lens, a foot in diameter, was standing at the entrance of a closed chamber against a little window, which was the only thing that was open, slanted a little to the right. Thus when the eyesight traveled through the dark emptiness, it also, fortuitously, hit upon the place of the image, nearer, in fact, than the lens. And so since the lens was weakly illuminative, it did not particularly attract the eyes. But the walls were also not particularly conspicuous through the lens, because they were in deep darkness. But the little window and the objects standing about it, which had the benefit of much light, lying hidden beyond the lens, set up a bright image of themselves in the air (between me and the lens). And so at first glance I perceived this aerial image, but with repeated gazing, gradually less and less. The games can be made more elaborate. We will here propose things that are more obvious and ready at hand, that is, suited to our purpose.

Proposition 6

Images of objects seen through an aqueous globe with one eye adhered to the nearer surface of the aqueous globe. Since this is confirmed by experience, it strongly confirms the truthworthiness of my demonstration of the place of the image, proposed in ch. 3 above. For vision is deprived of the association of the eyes when only one is used. Therefore, the place on the perpendicular will result entirely from this association of the eyes. Here I prove what is proposed to be proved from 5 and 3, preceding. For since between the eye and the globe there

11 Porta, Magia naturalis, p. 269.
occurs so place suitable for the image: the globe is the first thing that holds the eye by its light, and it consists of the same light of which the image consists. Consequently, by 3 preceding, both the globe and the image will be seen in the same act of seeing.

Proposition 7

Images of objects seen through an aqueous globe with both eyes meet often appear confused and double. This would never cease to be confirmed by experience, if the experiments of Persius introduced were to stand without further caution, or if Wittelo 10.47 were true without any restriction. But I prove what is proposed thus. By 5, the usual place of the image in relation to the use of both eyes is most often made inapplicable. Therefore, by 6, for either eye the image appears to inhabit in the surface of the globe. Now if the surface of the globe were the usual place of the image for the two eyes (as can happen in a number of media), then one and the same image would be perceived by both eyes. But here it is supposed that the place in which the same image is perceived with the two eyes is nearer to the sense of vision than the globe is. It is therefore necessary that the image be double, and the vision be confused, for the reason that in the same glance it perceived the same glass directly, and at the same time and in the same sight in which it perceives the globe, it also perceives the image directly with one eye, indistinctly with the other. In the above diagram, while D ought to have been the place of the image of point A when the two eyes B, C are observing, it is also impossible for the vision to be fixed at D because the object FB is more obvious. It therefore happens that when the vision is carried over to F, the other point of refraction, the point F is indeed seen properly, but with two lights, one coming from A to the eye C along the lines AE, EF, FC, the other to the eye B along the lines BK, KB, FB. For that reason, the sense of vision, unaccustomed to seeing two different objects A, F directly (by refraction), when the two eyes are turned towards F, is indeed correctly informed about F, but is confused about the images AL. For if the vision remains the same, and the eye B is directed along BF, the point A comes into the eye B laterally by the lines AG, GH, HB, while it had come directly into the eye C. Therefore, the sense of vision seeing the point A with one eye directly, with the other indirectly, rightly considers that it sees two objects. If the vision be directed neither towards F nor towards H, but towards some intermediate place, then this place of the globe will be seen directly, but some image on the two sides indirectly.

11 Wittelo 10.47: "A single refracted image occurs through the same observer's seeing with both eyes." (Thesaurus p. 443).
Here, because the diagram provides the opportunity, although it is off the track, I append a demonstration of what Aristotle seeks in section 31 problem 11 and 17. I say that, although each eye is brought into use by the same principle, there is still in the eyes a faculty for taking one object as double, even outside the consideration of the aqueous globe or cornea mirror? I reply that the facility of the eyes, and the distinct vision from opaque or of oblique visions from each other through different parts of a surface, are in agreement. In the preceding figure, let $FH$ be the wall, the eyes $B, C$, whose powers of sight are directed to one point; let it be $F$. Let the visible object, whose form is to be doubled, be located in an intermediate place: let it be $D$. Now, since $FC$ is the axis of the eye $C$ extended, $D$ will appear to the eye $C$ on the line of the axis. Since $FB$ is the axis of the eye $B$ extended, while $BD$ is a line other than the axis, $D$ will appear to the eye $B$ outside the line of the axis at another part of the surface. Thus, according to different locations in the eyes, different forms will appear. Therefore, this happens to the inbred and the sick, and to boys and to the old, and those whose voluntary motions are impaired.

Proposition 8

Rays from some point at a distance beyond comparison that flow to any points of an aqueous globe, meet the axis after a double refraction, that is, they meet the line that is drawn from the radiating point through the center of the globe. Let $A$ be the center of the aqueous globe $BC$, $DAF$ the axis. Let the radiating point be towards $D$, at a distance beyond comparison from the globe. Therefore, the lines of a ray drawn from that point to any point of the sphere will differ imperceptibly from lines equidistant from the axis $DAF$. Let points $B, G$ be lines parallel to the axis $FB, LG$, say that they will meet the axis $DAF$. Because $KB, LG$ strike obliquely upon a denser sphere, they will therefore be refracted towards the perpendiculars $BA, GA$, and $KB, LGH$ become a pair of lines making angles at $B, G$. Therefore, $CG$ cuts $BK$, and consequently also when extended, cuts its parallel $DA$. Likewise, $FH$ cuts $GL$, and consequently also its parallel $DA$. And because $BC$, $GH$, which are going to meet $AF$, go out into a rarer medium, they will therefore be refracted from the perpendiculars $AC, AH$ in the opposite way, but with the same angles, so that $KB, BC$ $FA$ are equal, and also $LG, GH$, because the angles of incidence $B$ and $C$ are equal, as are

53 For a literal translation of the Greek ἐξωθέν, see, e.g., Aristophanes, Frogs 949. Olive trees stood, at the end of the Athenian racecourse, hence, to go "exωθέν" is to go outside the racecourse.

When straight lines are drawn from equal arcs of a circle to a point outside the circle, the angles subtended by the equal arcs at the point are unequal, and those whose arcs are closer to the diameter through the point are greater.

Let \( AB, BC, CD \) be equal arcs of a circle about center \( F \). And on diameter \( AF \) let \( E \) be a point outside the circumference of the circle. Let the ends \( A, B, C, D \) be connected with \( E \) and with the center \( F \), and with point \( G \) of the surface; and let the neighboring ends be connected among themselves, by the lines \( AB, BC, CD \).

I say that \( AEB \) is greater than \( BEC \), and that this is greater than \( CED \).

What is proposed is readily apparent. For at \( F \) all the angles are equal: from there towards \( G \), those that are farther from \( EA \) become greater; until at \( G \) they again become equal, by Euclid 3, 21. Therefore, beyond \( G \) towards \( E \) the ones that are farther from \( EA \) become smaller. The cause is the inclination of the equal lines \( AB, BC, CD \), greater in the two distant ones.

For let \( EC \) be extended to \( H \), and let perpendiculars \( BH, BR \) fall from \( B \) to \( EA, EH \). \( CBR \) will be right triangles upon equal bases \( CB, BA \). I say first that \( EAB \) is greater than \( HCB \). I prove it thus. \( GAB \) and \( GCB \) make the sum of two right angles, by Euclid 3, 22. Therefore, \( GAB \) and the supplement of \( GCB \) are equal.
But \( ECB \) is greater than the part \( GCB \). Therefore, \( HCR \), the supplement of \( ECB \), is less than the supplement of \( GCB \), that is, less than \( EAB \). Therefore, since \( EAB \) is greater than \( HCR \), while the bases of the right triangles are equal, and the right angles to them, \( B1 \) will be greater than \( BH \). Therefore, the right triangles \( B1E \), \( BHE \), upon the common base \( BE \), will again fit into the same semicircle, and therefore \( BEA \) will be greater than \( BEH \) or \( BEC \), because for the former the longer line \( B1 \) is the subtense, for the latter the shorter \( BH \). In the same way \( DEC \) will also be proved to be less than \( CER \), provided that you keep \( GBE \) much smaller than \( GCE \), because \( GE \) is more directly, and more closely, presented to the latter than to the former.

**Proposition 9**

Parallels to rays that have entered an opaque sphere move the axis sooner according as they are farther from the axis. In the preceding diagram, let lines parallel to the axis \( AE \) fall at points \( A, B, C, D \), and let them be \( EA, MB, NC, OD \). And first let the refractions be proportional to the inclinations. The inclinations here are \( EAF, MFB, NFC, \) and \( ODE \) horizontal or maximum. And let \( MB \), by one refraction occurring at \( B \), have met with the axis at \( E \) beyond the globe, the second refraction which occurs in the back part of the globe's surface. Being neglected for now. Now, since by the preceding lemma, the angles at \( E \) are unequal, while the excesses of the inclinations are equal, the angles at \( E \) will not be able to correspond to the angles of incidence proportionally. That is, if \( BEA \) (the supplement of \( MBE \)) is the correct angle of refraction of \( MB \), then \( CEA \) will not be able to be the angle of refraction of \( NC \), because the inclination \( NFC \) is twice \( MFB \); while \( CEA \) is less than twice \( BEA \). So if the angle of refraction of \( NC \) should also be twice that of \( MB \), it should be increased. But it will be increased by drawing \( CP \) so that it cuts the axis \( EA \) before \( E \) at \( P \). For the interior angle \( CPA \) is equal to the exterior angles \( CEP, ECP \). And this is greater than \( CEP \) or \( CEA \). Therefore it is evident, if the angles of refraction were to be proportional to the angles of

---

26 In ch. 3, sect. 6, prop. 6 and 8, Kepler establishes that the angle of refraction — that is, the angle through which the ray is deflected in passing through the refracting surface — increases in a composite ratio which is greater than the ratio of increase of the inclination. Here, however, the angle at \( E \) increase is in a ratio that is less than than of the inclinations. Therefore, the angle at \( E \) cannot represent the angles of refraction.

27 If this is intended to apply to the triangle \( CPE \), using Euclid I.32, angle \( CPA \) is the exterior angle and \( CEP, ECP \) an interior angles.
inclination after one refraction, then, of parallel rays, those that are nearer the perpendicular are going to meet with it farther on, while those that are farther from the perpendicular are going to make an intersection nearer the sphere.

Now, however, if the refraction should happen twice—that is, if another one should be added at the opposite part of the surface, which happens in those media whose density is so adjusted that all the radiations finally meet with the axis beyond the sphere—then this ratio is doubled, since, by Witelo's rule, the angles of refraction are equal whether the light passes from water to air, or from air to water by the same line.

Finally, because the refractions are not proportional to the angles of incidence, but the angles of refraction are much greater in rays that are more inclined than the size of the angles of incidence bears, therefore, a new cause is again added, to bring the radiations that are more distant from the perpendicular together with the axis closer than before, while making the radiations that are nearer the axis come together with the axis farther from the sphere than before.

**Proposition 10. Problem**

At a given point of a sphere to apply a ray of some point of the solid body so that when refracted it should not meet with the axis drawn from the radiant point through the sphere. Let the point H, in the next to last diagram (Prop. 8), be taken, and from H let any straight line be drawn, not through the center A, cutting the sphere, and let it be HG, to which let H; be inclined at the correct angle so that the angle of incidence of the refracted ray GH requires at the surface of the sphere, by prop. 9 ch. 4. Let I be IHG, and let HGL be made equal to it, and parallel to OL, let there be drawn through the center A, the straight line DL cutting H1 at 1. Let 1 project the ray to the given point H, that is, HH, which, refracted at H and G, does not meet AD. The demonstration is evident from the preceding, of which this is its converse.

---

25 See the note on this proposition above.
26 Here Kepler refers to the following enunciation:

The demonstration of this is very easy. For GL, AD (reading AD for AK) are parallels by construction. They therefore do not meet, by the definition of parallels. And so there is no point in the whole arc, NH, that does not have its own non-cutting line [i.e., no cutting line] $\sin (\theta + \pi) = \cos \theta$. This is the meaning of line 12 p. 188 [i.e., the statement, "such an arrangement is possible at any point" near the end of Kepler's p. 188 to the end].
Proposition 11

It is impossible that the same ray, refracted in a medium not perfectly dense, should descend from many conjoint radiations out of a rarer medium coinciding at the same point of the surface of the denser medium: for by Prop. 8 of the fourth chapter, the refracted angles at all inclinations are established as follows: first, some portion of the angle of refraction increases with the inclinations; then, when this portion is subtracted from the inclination, the secant of the remainder multiplies the portion again. Therefore, since the elements differ quantitatively at different inclinations, the composites, or the angles of refraction, must necessarily differ also. As a result, those of different inclinations, that is, those of different radiations to the same point of the surface of the denser medium, are different: but the refracted rays are by no means the same.

Proposition 12

Rays coming from different directions to the same point of the surface of a denser medium on each other at that point, and the refracted ray of the higher radiation becomes lower. If not, let the radiation OH be higher than IH, and its refracted ray be HP, which cannot coincide with HG by the preceding. Accordingly, let HP also be higher than HG; so that IP is refracted into HG and OH into HP. Let the perpendicular also be extended to some point Q. For it is certain that QH, when it goes beneath the water, is not refracted, but proceeds along HA. Therefore, when QH is made to incline and arrives at OH, HA, which is beneath the water, arrives at HP. Therefore, when the former lies somewhere between Q and O, the latter will lie between A and P, and will coincide with HP. Previously, however, HG was the refracted ray of IH, lower than OH; while now it is that of some ray higher than OH, which is impossible, by the preceding. Therefore, OH is not refracted into HP above HG, but into some line below HG, and the intersection takes place at point H. Alternately, if OH does not intersect HG and descend below it, then all the higher ones will have refracted rays that are higher than IH, because the ratio

50) Here Kepler refers to the following endnote:

To Prop. 11, p. 187 (in a medium that is not perfectly dense: For in a perfectly dense medium, all the radiations from the whole hemisphere entering at the same point are refracted to the center along a single line of regression, as demonstrated in prop. 7 p. 113, and as you will find in use below in ch. 6 p. 225.)
of all is the same, since it has been proved that the refractions increase with the inclinations; and where any one is higher, its refracted ray will also thereby be higher. Let GH be extended some distance to R. Now, since RH is higher than OH, its refracted ray will be higher than HG. Let it be HP. Now, since RHG is one straight line, RH, upon entering the denser medium at H, will be refracted away from the perpendicular HA into HP, which is absurd; for it should have been refracted towards the perpendicular in the denser medium. Therefore, the refracted rays of rays having a higher angle of incidence are not higher. But, by the preceding, they are also not the same. Therefore, they are lower. Accordingly, an intersection occurs at H, which is what was to have been proved.

Proposition 13. Problem

To find a point on the axis outside the sphere which forms the nearest boundary of the radiations into the sphere, meeting with the axis. In the diagram of prop. 8, let FA9, through the center A, be the axis of the sphere BC, to which let AG be erected at right angles from A, and let GL be tangent to the circle at G. This incidence will be “horizontal”, as we named it above in ch. 4; thus, LG would be refracted with the angle of the horizontal refraction, in the amount that the density of the medium requires. And let LGH be the correct angle. Now let GHI be set equal to it; that is, let another line be tangent to the circle at H, and let this be HI, cutting the axis AF at I. I say that this is the boundary of the radiations meeting the axis; that is, that I projects no further ray upon the sphere which after the refractions meets the axis; much less, if the point which is taken is nearer to A than I is. But if a point is taken farther from A than I is, those radiations upon the sphere certainly do meet the axis. First, because IHI, GLI are tangent to the circle, IH, GL are accordingly the horizontal radiation, and there cannot go forth from I to the sphere a line that is more inclined. And because GL is parallel to FA, this last radiation of the point I will therefore not meet the axis. Now let another line be drawn out from I to a nearer point of the circumference let it be C1; and by the converse of 10 of this chapter,13 let the straight line CB cut the sphere at C in such a way that the angles of refraction in both places, or their supplements C5K, BCF, are equal to this standard of measure of the inclination CBA.14 make BK go forth parallel to I AD. For such an arrangement

51 Here Kepler refers to a long endnote, which has been included below as an appendix to this proposition.
52 Here Kepler refers to the following endnote:
53 To 188. The axis is always taken as in prop. 8, to mean the straight line drawn through the center and the radiating point in prop. 17 and 18 below, through the point of the eye above.
54 In Prop. 10, we are given a point, a ray is chosen arbitrarily, and the axis is constructed. Here, the axis and the point are given, and the final direction of the ray (BK) must be parallel to the axis. These conditions will determine the position of the line CB.
55 The radius HA is the normal to the surface of the sphere, while CB is the path of the ray. Thus angle CBA is the angle of incidence, or the angle of the refracted ray (depending upon which direction the light is considered to be travelling). This angle
possible at any point. Let there be, I say, such an arrangement. Therefore, by 9 preceding, \( CF \) will cut \( DF \) at \( F \), more remote than \( HJ \), for the reason that we have supposed \( C \) to be nearer the perpendicular \( FA \) than \( H \), whence \( B \) was also nearer than \( G \). Therefore, from two points in \( A \), which are \( F \) and \( J \), there exist two radiations \( FC \) and \( IC \) to the same point of the aequansphere, with the result that, by 12 preceding, a cutting of the rays occurs at \( C \). And \( IC \), which was inside, after the refraction and the cutting at \( C \), becomes outside. Moreover, since \( CB \), \( BK \) have just ceased to meet \( FAD \) on that side, much less will \( IC \) meet after making two refractions.

It has therefore been demonstrated that no radiation made from \( J \) misses \( IAD \),

But also, no moving radiation will proceed from any point nearer than \( I \). For again, that which is farther in than \( IC \), or any whatever from \( I \), after making an intersection at \( C \), will stay farther out than \( IC \). And since the latter does not meet, much less will the former meet.

Now let some point be taken that is higher than \( I \). And let it be \( O \). I say that some radiations from \( O \) do in fact meet the axis. For again, let the ray \( OH \) be drawn to the point \( H \), which is the boundary of the meeting point of the radiation from any point nearer than itself. Therefore, an intersection occurs at \( H \). And since \( HHGL \) is the boundary of the meeting point from \( I \), and \( GL \) is parallel to \( AD \). when \( OH \) is made to go farther in, it will by all means be inclined, and as a consequence will meet the axis. This will happen to all radiations from \( O \) from the radiation tangent to the sphere to that radiation which is drawn from \( O \) following the rules of 10 above. However, those which fall within \( CN \) from the point \( F \) do not meet beyond the globe. Or, what is the same thing, no parallel between \( KB \) and \( DA \) can meet with \( AF \) this side of \( F \). Hence it appears that Wicelo X 43 is not universal. Which he indeed does not conceal in concluding as follows: “But there is great variety in these, which we leave to the study of the curious.” But these studies languished after your time, Wicelo, and those who are trying to stir them up are nearly wasting their time. For I fear that you, and now I as well, have used up more time and effort in plowing up, sowing, and carving out those propositions, than everyone else together are going to use up in reading them.

"Kepler’s endnote to this proposition:

[The note begins near the top of p. 440 of the 1604 edition.]

To Prop. 13 p. 188–189. For the sake of clarity, let me place before the eyes all the steps of this procedure even though the proposition says a certain amount

is equal to \( BCA \), and so the refractions are equal at \( C \) and \( B \), as Kepler noted in Proposition 8.

55 Here Kepler refers to the following endnote.

To 189. The words: “nearer” and “farther” are with respect to the globe.

56 Wicelo X 43 (Thesaurus II pp. 440–41) argues that the image of something seen through a diaphanous sphere “is seen as being in the shape of an egg, much greater than the seen object.”
about some things, while 16, following, speaks of the rest), and all the more so in that the demonstration on p. 189 line 30 itself also proceeds rather long.

There are two points on the axis $AN$ extended, one $I$ and the other a little above $F$, which divide the radiations into three classes. The first is that of the points between $I$ and $N$, the second that of the points between $F$ and the other point slightly above $F$. The third is that of all the remaining points above that point at $F$ along the infinite line.

The conditions of the first and third class are contrary. For the radiations from the line $IN$ not only do not meet with $AD$, but all, without exception, also diverge from it. But the radiations above the sign at $F$ on to infinity all (without exception) meet the axis $AD$.

So the conditions of the bounds of the first and third class are again contrary. For the radiations from $I$ is less than those from the line $IN$ all diverge from the axis, except, obviously, for the last $IH$ ending at a lesser circle of the globe, at which circle those radiations are tangent to the globe. For these are the first and the only ones for $IN$ that neither diverge nor meet, but become parallel to the axis $AD$, as GL, going forth from $I$.

Further, the radiations from the sign at $F$ all meet the axis, all the way to the innermost $FN$ which coincides with the axis. All the rest above $F$ do this same thing, but this is the first among them, of which all the rays meet the axis.

But the condition of the second class, that is, of points between $I$ and $F$, is intermediate. For the radiations of (for example) point $O$, which are close to the axis $OA$, diverge from the axis, all the way to some point of the surface at which the laws of proposition 10 are expressed. For at the point, or rather, at the circle of the sphere about the pole $N$, the radiations become parallel to the axis. And from that point all the way to the one at which the radiation from the point $O$ is tangent to the sphere, all the interposed radiations coming out from $O$ meet the axis. And thus there is a broad limb or belt in the globe, situated uniformly about the axis, at whose points all the incident radiations from $O$ meet; within that limb the surface is like a cap or like the arctic zone, or the polar land on the sphere of the world, the radiations of whose points do not meet the axis.

All the radiations of the first class remain within the cap $NH$, whose edges form the circle $H$ that divides the belt from the cap, like the arctic circle; and they have this circle, lacking breadth, in place of a belt. The radiations of the second kind now obtain a belt, broader in proportion to their remoteness from $I$, and they advance the boundaries of that belt on the near and far sides of $H$. For the boundary of that belt to which the point $F$ belongs (which point is a little lower
than the boundary of the second and third class, as has just been said), closest to $N$, is most distant from $C$ in the place where a line drawn from $F$ is tangent to the sphere $CM$.

The last boundary of the intersections of the parallels, or the point above $F$, is the first that leaves no cap about $N$, but makes a certain sort of cap of the belt itself.

Therefore, the points of the third class, from the sign above $F$ through the infinite line from this improperly named "belt" more and more make a hemisphere, to the extent that if there were given a point of infinite distance, its radiations would be tangent to the sphere at the opposite points $M, G$, and would be completing the hemisphere.

Omitted here is the demonstration that the radiations from $O$ are still going to meet the axis when they shall have fallen upon points more distant from $N$ than is $H$, in which proposition 10 cannot hold while $AN$ remains the axis. But it is easily demonstrated, and this is done through prop. 9 and the present diagram. For because the most distant parallel $LG$ comes through to $H$ under refraction, all the remaining parallels, such as $KB$, fall upon a point closer to the axis or pole $N$ than is $H$, in order that as a result at $G$ none of the parallels will fall beyond $H$ towards $G$ or $M$. If, therefore one of them falls at a point beyond $H$, it will not be parallel to $AD$, diverging much less from $AD$ in the direction of $D$, for all those fall within $NH$. It therefore remains that they meet with $AD$ on the side of $D$. And that was to be demonstrated. Use this same demonstration also for the full demonstration of prop. 16 following. [end of Kepler's note]

**Proposition 14. Problem**

*In an aqueous globe, to determine the places of intersections of any radiations parallel to the axis.* In the diagram of Prop. 8, just now repeated, since $LG$ is tangent to the sphere at $G$, the horizontal refraction is $36\frac{1}{2}$. By chapter 4, prop. 8, with the result that the complement $AGH$ is $53\frac{1}{2}$. But twice this $AGH$, or $MGH$ at the surface, is $MAH$, and as a result, its measure, the arc $MCN$, is $107\frac{1}{2}$. And the supplement, the arc $NG$, is $73\frac{1}{2}$, while the excess $NH$ over the quadrant $MH$ is $17\frac{3}{2}$. And because $NH$ is tangent to the sphere, $IA$ is the

---

55 Sothe Latin reads. But $NH$ is actually the excess of $MH$ over the quadrant.
seanet of the arc NH, or 164,569, and NI is 459; about the twentieth part of the semidiameter AN.

Now let KB be the incidence, or the inclination of 0 degrees, the angle of refraction from Witelo being 2 15'. Thus since BAD is 10, CBA will be 7 45'. And ACH is the same amount. Therefore, the arc BMC is 164 30', and when BAD is added, 174 30'. The remainder, CN, is 5 30'. And in fact if you should double the refraction of 2 15', CFN would be set at 4 30'. And CFN is nearly right. Therefore, as the sine of the angle CFN is 5 30', or its subdual, 9,596, so is the sine of FCN, 85 30', to NF. As a result, just as CFN is slightly less than the arc CN, so the semidiameter AN comes out slightly less than the distance of the cutting NF, which is shown by an inclination of 10 degrees.

Proposition 15

The last limit of the cuttings made by parallels with the axis is not greatly distant from the cutting of the reflection that maintains an obliquity of 10 degrees. I have disposed of defining geometrically the exact point at which the last intersection occurs. I beg you, reader, help me here. It certainly cannot be infinitely distant. For by 8 of this chapter, I have proved that all rays parallel to the axis meet the axis after a pair of refractions. But the reason why these points of meeting are not scattered further, but are gathered at the end, and in fact a near union is made of most of the radiations close to the axis.

I shall prove in some manner from Proposition 9 preceding. For test, as was demonstrated in Chapter 4 Prop. 8, it is quite imperceptible that the refractions

56 Here Kepler refers to the following endnote.

To 190. For (BA is) refracted, the declination from the perpendicular of the part of it that is set up in the air is 10; the refraction 2 15'. When this arc is subtracted, the remainder is the inclination of the refracted ray, 7 45'. Thus in isosceles triangle CBA, the sum of the angles at the base is 5 30', which, subtracted from 180 degrees, leaves CAB at the center: A, 164 30'. And since KB is inclined by 10 to the perpendicular AB, DA, which is parallel to it, will be inclined to the same by the same amount, with the result that BAD is also 10. Further, CN is generated through the doubling of the refraction, because KB is refracted twice by 2 15' towards the parallel DA, once at B and again at C. It is demonstrated by parallels to the axis drawn through B, C.

57 As will be seen, there are two "limits," outer and inner. The inner limit, point J, has already been found (in Prop. 13); here Kepler is seeking the outer or "last" ray.

60 Like the marginal note to Ch. 4 prop. 6 p 126, this note alludes to Vergil, Eclogue 5, 1. 304: "Luc, quaebis in terris et oris mith magnum, Apollo" (Tell in which lands, and you will be the great Apollo to me).
are proportional to the incidences near the perpendicular at a small inclination. As a result, the refraction of one more remote from the axis (which is, however, close in itself) is imperceptibly greater than the refraction of a nearer one, with respect to its own angle of incidence. But in the Lemma to prop. 9, CEB is an imperceptibly smaller than BEA. Therefore, in the case of great nearness to the axis, the refracted rays are very closely represented by the lines AE, BE, meeting at almost the same point, and CP (closest to the axis AE) will cut AE imperceptibly higher than BE, so that P, E are very nearly the same point.

Proposition 16

All the radiations of a point that is more distant from the sphere than the last boundary of the intersections meet with the axis, after making a pair of refractions. This is demonstrated in the manner of 13 above, and from the preceding; there is no need of words.

Hence arise these corollaries.

1. When the radiating point is infinitely distant, the boundaries of the intersections are nearest both to the globe and to each other.

2. When the radiating point has approached the nearly last boundary of the intersections (determined by parallels, through 15 preceding), the boundaries of the intersections made by these new transversals are of the point distant as far as possible both from the globe and from each other. Parallel rays striking a globe of a denser medium are refracted on both sides, and meet with the axis, with boundaries of intersections at equal distances from the globe on both sides.

Kepier’s endnote to these corollaries:

[The note begins in the middle of p. 442 of the 1604 edition.]

To 191 Prop. 15, first corollary. If you desire a demonstration of the corollary, see the diagram in Prop. 13. Let the radiating point on AD towards D be infinitely distant within the limits of sense perception. It will therefore straddle approximately the whole hemisphere MBG, with, moreover, radiations KB, LG, approximately parallel, by the postulate of the second chapter. Therefore, by 13 of this chapter, J is the nearest possible intersection of all to the globe.

But the other boundary of the intersections, F, is also nearest of all. For direct your attention to the upper part of the diagram, and suppose the radiating point to approach from an infinite distance: its rays will ever and ever all meet with the axis, until it gets close to F, by 13. Now from F certain ones no longer

61 This is the boundary established in Prop. 15, where the obliquity of FC to the reflexing surface of the globe is 10.

62 The nearest boundary is the point where the ray from F tangent to the globe intersects the axis; the farthest boundary is where the ray from F nearly parallel to the axis intersects the axis.

63 That is, F is the nearest boundary of all the outer intersections of the rays with the axis.
meet. And since they become parallels from cutting lines, the intersection runs out to infinity, before it devolves into parallels. Therefore, when the radiating point approaches the globe all the way to the closeness of $F$, the last intersection runs off to infinity. But before, when the radiating point was infinitely distant, the last intersection above $F$ was distance by a little more than the semidiameter of the globe, no farther. Therefore, since the cause continually increase from the infinite distance of the radiating [point] to the shortest distance, the effect, or the closeness of the last intersection, also ought to grow continually from the shortest to the infinite, but without interruption. Thus, therefore, it will in the beginning be closest of all to the globe, which was to be demonstrated.

The second corollary has just partly been stated, as far as concerns the last boundary of the intersections. The rest is perceived most clearly in the diagram of p. 192. For the more $\alpha$ approaches, the more $v$ retreats. And in the diagram on p. 199, when the radiating point at the boundary is placed at $F$, all the radiations meet when it descends below $F$, only the outer ones meet, namely those which by their intersection mark the innermost boundary beyond the globe. Finally, when the radiating point is placed at $I$, the last rays also now cease to intersect, and the intersection returns to infinity, but if the radiating point be infinitely distant on the other side of the globe, the innermost boundary is close to the globe at $I$. So again, when the radiating point comes forth from the shortest distance to an infinite one, the nearer boundary here shrinks from an infinite distance to the shortest. Further, it cannot happen that as the cause continually increase, the effect increases for a while and then again decreases. Therefore, it should be least off all at the beginning, and greatest at the end. Accordingly, when the radiating point is placed near $F$, there is indeed some innermost boundary, and it is far away from the globe, but farther yet, indeed infinitely distant from the last boundary, which is now not a boundary, since it does not limit because the intersections are
running off to infinity. The meaning of the corollary is not, however, that when the radiating point is at \( F \) the internmost boundary is more distant than when the radiating point is at \( F \). For at \( F \) both the internmost and the last boundaries go off to infinity, at \( F \); when the last boundary is now vanishing, the internmost point nevertheless remains at a determined distance. The corollary therefore compares the point \( F \) here not with points \( F, I \), but with these radiations of the third class above itself.\(^{87}\)

The third corollary comes from the preceding, and can be stated orally in the diagram of p. 192 [i.e., the diagram to prop. 17]. If \( \theta \Gamma \) and \( \zeta \zeta \) are parallel, \( \lambda \theta, \lambda \zeta \) will be equal, because the bases \( \theta \epsilon, \zeta \epsilon \), of the isosceles triangles \( \Delta \epsilon \theta \), \( \Delta \epsilon \zeta \) are equal, as are the angles \( \alpha, \gamma \), because of the same refractions at \( \delta, \epsilon \), and \( \eta, \zeta \). Then, as a consequence, \( \lambda \gamma \) will also be equal to \( \lambda \beta \) (if \( \gamma, \delta \) are the nearest boundaries), and \( \lambda \beta \) to \( \lambda \mu \) will be equal to \( \beta, \mu \) are the last boundaries), and finally, if \( \beta, \mu \) are the last boundaries), and finally, \( \theta \beta, \theta \mu \) will be equal.

[End of Kepler's note]

Proposition 17

If the eye be removed from the globe beyond the boundary of the intersections of Proposition 15, whatever is placed after the globe beyond the last boundary of the intersections, which there nonparallel radiations of the eye make, its image appears in an inverted situation on the surface of the globe. Let \( \Delta \Gamma \theta \zeta \theta \) be the globe with center \( \lambda \), axis \( \lambda \zeta \theta \), upon which \( \beta, \gamma \) are boundaries of the intersections of parallels. \( \theta, \mu \) are boundaries of the intersections, not of parallels, but of those radiations that come from \( \alpha \), and let \( \theta \beta \) be the visible object, the former more distant than \( \beta \); the latter, than \( \gamma \). I say that \( \theta \beta \) is going to be seen inverted on the surface \( \theta \epsilon \). For since the object is beyond the intersection, no rays will come from \( \theta \) to the end \( \theta \gamma \) which are not intersected between \( \theta, \gamma \). Let the rays that include the ends intersect each other at \( \nu \), and let \( \Delta \theta \nu \) be \( \theta \beta \). \( \nu \gamma \) will be the point to be attached to \( \epsilon \), and \( \kappa \) at \( \delta \), and thus the ends appear with their situations exchanged. Further, because the

\(^{87}\) The radiations of the third class are those that originate from a point above \( F \). All of them intersect the axis, some of them tangential to the globe, or nearly so, farther near the globe, and others close to the axis further away.

\(^{88}\) Here Kepler refers to the following enucleate:

To p. 193 Prop. 17. The subject matter itself leads me to distinguish between the intersections of nonparallel and the intersections of parallels, which I state from Prop. 15, or the boundaries of the two. Therefore, correctly take note of \( \alpha \). The boundaries of the intersections of parallels always have the same situation with respect to the globe; the boundaries of nonparallel wander this side and the other side all the way to infinity.
parts which are interposed between them also removed beyond \( y \), they will be seen by rays intermediated between \( y \), \( x \), and, by \( 9 \) above, will intersect each other beyond \( x \), but inside the boundary \( y \). Intersecting each other thus, they also intersect all the interior parts. As a result, the whole object appears in an inverted situation. 50

**Corollaries**

1. Hence it is evident that if the eye is also at a greater distance than \( y \), and if nevertheless \( x \) be between \( y \), it is going to be seen partly in an inverted situation (imagine, at the edges, \( 90^\circ \) and in the midpoints) in an erect situation. And the same is also going to be seen as circular, according to Props. 43 of the tenth book of Vitruvius, and thus confounded.

2. If \( x \) be also the innermost boundary of the intersection \( q \), the eye remaining the same, the whole object will appear erect.

3. But if the eye be between \( y \), the intersection, running off to infinity and certain radiations from the eye being refracted parallel, and if the object were then situated on the axis, and less than the distance of the parallax, it will appear erect and inverted at the same time. If, indeed, it be more distant than the nearest intersection \( y \), on the other hand, it be assumed, it will appear erect only.

4. Further, if it should exceed the convexes of parallels, beyond the boundary, the whole will appear inverted, the middle erect, and partly circular.

5. Finally, the eye and the object standing on the near side of the boundaries of the intersections, the former that of the parallels, the latter that of the radiations of the eye, the object will appear erect and of the greatest quantity.

* Kepler’s endnote to the corollaries *

To Corollary 1, p. 192. Whichever is perceived by radiations from the cap (which we introduced in the notes so Prop. 13) will appear erect; whatever is perceived through the edge on the belt will appear inverted. Whichever is perceived through the circle which is the boundary of \( y, \) e.g., between the edge and the cap will appear circular only that is, one point—the pupil of the eye, for example—drawn into a circular border.

To Corollary 3. Geometrically it is the same whether the eye radiates to the object, or the latter to the eye; in both cases the interposed globe does the same thing. However, when the eye is located between \( y \), the radiations are divided into polar or cap-shaped parts: the circle and the belt. The radiation of the belt, such as \( x, x, x \), etc., cut each other between \( y \). An object placed here is therefore set in their way, and it flows into the eye inverted, if it be between \( y \), but at the same time erect, because it also encounters the polar radiations that do not cut each other. And the whole will indeed be seen erect, because it is smaller.

50 Here Kepler refers to the following endnote: Top. 192.1. 20. Supply the following corollary immediately here.

51 Theorems II pp. 440.1. A linear segment of the axis from \( 0 \) to \( 0 \) is refracted into a ring on the surface of the sphere, and thus the parts are distorted in the image. Büschel presents both a mathematical and an experimental account of the phenomenon.
than as to encounter the radiations of the circle. However, the images differ in situation. For is it is considered great when erect, so beyond the globe it will be considered at a distance, and will interfere in the actual convex surface of the globe.

Corollary 4: When the visible object is so great that it also encounters the radiations of the circle; let it be evaluated by 1 and 3.

Proposition 18

With everything the same as in Prop. 17 preceding, the middle parts of the object will appear greater than according to their proposition, and curved. For the radiations from it near the axis fall with a very slight obliquity contained within a large angle, and then suddenly fall more obliquely. Accordingly, in the middle for much space the refraction increases little; at the sides suddenly, over a narrow part of the surface. And so the extent that the refractions increase, the lines splay out and thus comprehend more of the seen object, and they set it upon a narrow part of the surface at the side. Further, the objects are considered curved, because they appear to inhere in a surface which is curved.

Definition

Since hitherto an Image has been a Being of the reason, now let the figures of objects that really exist on paper or upon another surface be called pictures.\(^3^1\)

Proposition 19

It has been said what occurs to the eye in the neighborhood of an aqueous sphere. I shall now show how exactly the opposite happens on paper. For first, it is deduced from what has been said, why, if the paper is almost touching the globe, the figure of the globe (if it be a spherical vessel with a long neck) is projected upon the paper with a very bright edge by the sun casting its rays upon the sphere.

For whatever rays are within this envelope have not yet been intersected by the axis. Since the paper touches the globe, all those here are ones that have been able to pass through the globe. The edge is then brighter because the rays are gathered together to the greatest degree, and intersect each other circularly (but not yet on the axis), as appears in the adjacent figure at \(\alpha\), \(\beta\), \(\gamma\), \(\delta\), as the reasoning of the preceding demonstrations affords. Now where the paper is removed from the glass by the twentieth part of the diameter (if it be the case that the illumination come from the heavens), certain rays now begin to intersect each other with the axis at the middle of the figure, at the latter end; hence, the middle of the figure also becomes bright. The figure diminishes, however, with the removal of the paper, not because the cone of the same lines moves rapidly into a top-shape, for it does not decrease proportionally, but suddenly at the beginning, slowly at the end. The true cause, however, is the succession of rays. For after intersecting with the axis and dispersing, those had

---

Proposition 20

Through a globe of a denser medium, any point more remote than the intersection of parallels strongly depicts itself upon paper, located at the last boundary of the intersection of its rabbitines, not before and not after this point: and the picture comprising all the points is seen inverted. The cause of its strongly depicting itself is that the globe gathers many point many rays close to the perpendicular coming from the emitting point, by what was demonstrated above.

Further, the cause of this not happening at the boundary of the nearest intersection is that the rays near the falling from the sides, weekly, because they are in a great refraction, make the nearest intersection in while the rest of the rays (of the same point shining through the globe), which have not yet intersected, but are none the less gathered together and bright, also occupy the place of intersection in on the paper, and so obliterate those gathered at the intersection in. Finally, the comets coming from other points of the luminous object are still more to the sides, such as 1. Where, and one occupies part of the place of another, and give birth to confusion.

In contrast, at the last intersection in many rays from the same luminous point, standing close by the strongest perpendicular you, come together into a single point by 15 above, and because the cones are thinned out into needles,

15 Pyhês pyrûm, CE. Di Curpte, Glosarium VI p. 508 and Partington, A History of Great Fire and Gunpowder pp. 130 and 162.
they occupy no foreign region, with the result that individual points are imaged separately and distinctly. Only certain rays of other points on the visible object come down, but they are now intersected and weaker, as if another cone were to be bounded at its. Consequently, the vertex of the new cone and the intersected line \pi_3 of the old cone would be in the same place, and if there is any confusion in the formation of this image, these give it birth. The image, however, is inverted, not because the boundaries of the intersections have crossed before the glass, as previously explained, but because any point of the visible object, radiating perpendicularly upon the sphere, makes an image of itself beyond the globe with a very strong cone, whence it happens that the apex of the cones you intersect each other at the center of the sphere without having undergone refraction themselves. This is the genuine cause of the inversion of the picture that results from a denser sphere, when it is not masked by something solid, with an opening left in it.

**Proposition 21**

*The place of the picture of nearby things is farther from the glass; and of distant things, nearer. This is evident through Corollary 2 of Prop. 16 and by 20, above. Some inventor will take note of this, like many other things in this book. You can also prove it in the daytime, by lighting a candle, which will appear to burn upon the paper, if the candle be properly located. Above, from Porta, I said that this was true, in this matter of the picture; not, however, in the matter of the image, nor in air, but upon the wax. 73*

**Proposition 22**

*This picture retraces the proportion of its distance. For \( \ell \) is traced out above all by the unrefracted perpendicular rays coming together in the center. By 20 above. This is evident from experience. When the candle is moved closer and the place of the picture recedes, the picture grows, and vice versa.*

**Proposition 23**

*When a tablet, and an open slit, is placed opposite a globe between the boundaries of the sections of the parallels, and the slit is narrower than the globe, a picture of the greatest part of the hemispheres is projected upon a paper which \( \ell \) is placed beyond the globe at the boundary of the last intersection of the rays of the luminous point. The picture is inverted, but is most complete and distinct in the middle. The variety of these things is so great, and constantly something new, that unless one is most attentive he is easily confused. So much so that I had was struck for a very long time during which I persuade myself that the account of the different things was the same. About censer \( \ell \) let \( \mathcal{RC} \) be a globe of water, opposite which let there be an opaque tablet \( \mathcal{DB} \) open in a small slit \( \mathcal{EF} \), narrower than the globe. Let the visible object be \( \mathcal{HH} \); the paper at \( \mathcal{K} \), where the last intersection of the radiances from \( \mathcal{H} \) is. If the tablet were absent, by 20 above, I would form an image of itself with the last intersection of its radiances.*

73 See Prop. 5, p. 191, above.
at L, and the point H at the point K. But they would project the lateral intersected rays of the ones, the one in the places of intersections of the other. Now, with the tablet interposed, no more rays flow down from H to the sphere than the amount that can flow through E.F, and the amount, approximately, that flows together at K.24 and that one, as it by a kind of reckoning, is diminished, so that no intersected rays can be cast upon L, or no confusion could appear there. In turn, the core of radiations from I that is to be bounded at L is quite castrated by the opposition of the tablet D.E, in its narrowest part, that is, the radiation through the center A, and at its brightest spot L is too far from K for anything it scatters there to create confusion. Further, there is left to the radiations from I no more than what enters through E.F. That portion, however, whatever radiations it consists of, successively intersects itself through the breadth MN, by 19 above, and now, after the intersection, passes through L, and falls upon the paper placed at K, nearer than is L, and not at a point, but spread out, because the intersection has already occurred at MN. For that reason, the picture is dark and confounded at the sides. If you should move the paper nearer at the side, the sides would indeed be more correctly pictured, but never in every detail, because the intersections are spread out, not only in the depth MN (which hardly matters), but also in breadth.

And since the intersection of I.E and I.F with HK occurs at E.R, but the rays are bounded25 at MN before they cross AL, by 19 above, the right rays I become the left MN, and it is impossible for them to come out on the right through a new cutting.

The slit E.F, moreover, must be narrow, lest it fail short of its purpose if it be made wider, and must be close to the globe, lest it radiate too little and indeed too confusedly from the hemisphere within.

**Corollary**

From this, the use of the opening of the eye26 in the eye is in part apparent:

---

24 Here Kepler refers to the following endnote:
To 196. The amount that elsewhere too without the obstacle of a perforated tablet, is accumulated very near the boundary K of the last intersection, by 15 above.

25 Here Kepler refers to the following endnote:
To 197. They are bounded by the opposition of the paper at KN, the place that the ray runs together.

26 That is, the pupil.
Proposition 24

Rays converging 77 towards some single point within a denser medium are gathered by the hyperbolic cuspial cone bounding the medium to one single point, closer than the former point.

About center α let αβ be a plane of a denser medium, in which γβ, χκ, δε, ελ, αβ converge at κ. And let δτ be the axis. Therefore, by the above, γα will be refracted into τα, and ετ into ετ, and δτ into δτ, and ακ into ακ. I wish all rays to gather together in a single point. I will therefore need there to be a greater refraction at τα, so that ακ to come together with τα at a closer distance, and I will need there to be a lesser refraction at δτ, so that ετ, ετ come together with ετ further away, and thus ακ should become a single point. Moreover, the refraction will be greater at τα if τε falls more obliquely, and less at δτ if αβ falls more directly, upon the surface. But because τα, ακ stay the same, the surface τακδε accordingly needs to be changed so that it not be so inclined to τακ at δτ, and so that it be more inclined to τε at κ. These two things, however, are accomplished, not by one or another circle, but by conic sections. For any of them is able to cut some circle at 4 points. See Apollonius Book 4 Prop. 25. Consequently, twice in this semicircle, twice in the other one. When it thus cuts at δτ so that τ enters the circle there, it will be presented more directly to τε, and when it cuts the same circle at κ, cutting again, it will be presented to τε more obliquely. Further, of the conic sections, only the hyperbola or some line very close to it, is the measure of refractions, as was shown in Sect. 5 of chapter 4.78 Indeed, this very thing was demonstrated here: that the surface making all the rays outside the denser medium parallel is a conoid that does not differ from hyperbolic.

Corollary

Hence is evident nature's plan concerning the posterior surface of the crystalline humor in the eye. That is, she wished to gather all the radiations of any visible object entering the aperture of the eye into a single point of the retina, in order both that the painting of the picture might be all the more evident, and that the rest of the points of the picture might not be confused by extraneous rays, whether straight or gathered together.

It is also evident that nothing is sought in the widening of the opening of the area, other than that which I said above, nor that the picture be confused thereby, namely, that it only become brighter.

77 Here Kepler refers to the following enunciates:

To 98, were the words "converge," "diverge," and so on, for the sake of brevity, of straight lines which, when produced in all directions, all converge at the same point.

78 See above, pp. 120-123.
Chapter 5

Proposition 25

The manner in which rays converging to the same point are led to the right point has now been explained. However, we must take pains to treat how this too happens: that the axis which are within the denser medium converge to the same point, which nevertheless originally had flowed from the same point. As, if in the previous diagram the rays $\alpha_1$, $\alpha_2$, $\beta_1$, $\beta_2$, coming from the same point, were to be gathered at the same point. Moreover, since, as has often been said, there is no difference of refractions whether we say the rays go in or come out of the denser medium, the demonstration will be the same, with this sole exception, that the surface which is going to be gathering the rays on the outside that are diverging within the medium, should be more acute that the one we have investigated in Ch. 4 Sect. 2, because the latter gathers parallels, that is, neither converging nor diverging. Or, the contrary, therefore, that surface which is going to gather more quickly on the outside the rays that are already converging and inclining towards each other within the denser medium, will be more obuse than that of Chapter 4. Besides, in such a narrow space these surfaces cannot be distinguished from each other temporarily if they are near the bottom or heel of the hyperboli or ellipses, nor does it even have any perceptible effect if neither of them departs from perfect roundness. Therefore, even if there is another central in the eye, it is necessary that this be in the cornea; but the surface of the Crystalline must be perfectly round, so that it might take in the incoming converging rays to the same point directly, and it is right that this roundness be somewhat depressed, so that its center shall come out to be far behind the eye.

Proposition 26

The thumb in contact with the chamber of the eye appears greater than is correct, and confused. For because vision occurs through a picture, when the picture is disturbed, vision is disturbed. But by Corollary 2 of 16 above, when the radiating point approaches (as the thumb does here), the intersection of its radiations recedes. Therefore, the eye, when it approaches the eye, the intersection (which is the point of this picture), by 20 preceding inhabitants itself right into the head, and the retina casts the image inside the vertex. Therefore, the cause point is drawn outlines that are not clear and distinct, and broadens the edges, drawing a surface instead of a line.

Proposition 27

When a movable object is located beyond the point that is given near an eye by nature, from which all radiations might be gathered into a point, that object appears confused. This is evident through contrary reasoning. For by Coroll. 2 of 16 above, the point to which the radiations of a distant object and to which those of a nearby object are gathered, is not the same. However, in one person, the place of the retina with respect to the borders is always one and the same. And it was not useful to obtain for the person the most distinct vision of the most distant objects, if as a result present objects should have escaped him. But neither ought he to see distinctly only the closest things, so that distant things should be excessively confused for him. Thus those were used of a balance.
Further, since it has its breadth, it varies with individuals. And it may be thar the position of the csatite line with respect to the retina is the same in all, but the density of the humors is not the same. Therefore, when a point of the visible object is exceedingly remote, its radiating cone is ended before it reaches the retina, by the above corollary. And thus when the insurrection occurs, it strikes upon the retina as it is now once again expanding itself. Hence it is that those who labor under this defect see a doubled or tripled object in place of a single narrow and far distant one. Hence, in place of a single moon, ten or more are presented to me.75

And indeed, in order that confusion also follow from this cutting of the cone, the tenderness of vision should be added, which is moved powerfully by strong radiations both at the point of the retina where the vision is distinct, and round about, through the breadth of the cone of rays (arising as a result of a defect). As a result, the cones of celestial radiation are spread out for everyone, but not everyone sees confusedly as a result.

Proposition 28

Those who see distant things distinctly, nearby things confusedly, are helped by convex lenses. On the other hand, those who see distant things confusedly, nearby things distinctly, are aided by concave lenses. It is a matter of great wonder, the use, so widespread, of so great a thing, whose cause is nonetheless hitherto unknown, so that, upon discovering the clearest demonstrations, I proclaim it furthermore. One Baptista Porta professed to have given an account in optics, which I have hitherto sought in vain from books and letters.80 More than once I torment myself in order that I might find the cause, in vain, while the means of vision lay hidden from me. And with greatest deserving I make most honest mention of Mr. Ludwig L. B. von Dietrichstein, prince of my acquaintance,81 who has for three years now kept me busy with this question.82 Accept at last, O man most fond of letters, arts, and professors, and also of me among the last, and on top of that most wise (let the princes of Austria, and the most ample Provinces praise the

74 This condition is now known as pellagra monophthalmica, and results from an irregular inclination. Kepler had nearly died of smallpox as an infant in 1573, and this visual defect was evidently a result of the disease. Cf. Caspar, Kepler, p. 36. Kepler was also overweighted, as is noted in Proa, 28, 28.

75 J. B. Porta makes this claim in Magicae naturae, Book 17, ch. 10 (ed. 1589 p. 269). Porta had earlier published a work De refractionibus, opticae pars, libri IX (Naples, 1573), which Kepler evidently was not able to find.

76 Patroclus, Messeres, was Horace's principal patron, praised in the invocation of his first Ode (Carmen 1.1.1-2).

80 None of the surviving letters to Kepler from Baron Ludwig von Dietrichstein, whom Kepler knew in 408 and who was his daughter Susanna's godfather, touches upon optics. The remark is thus probably to be seen as a grateful compliment, though Kepler recalls a conversation below. Von Dietrichstein remarked in his letter of thanks for the gift of the Optics (20 October, 1604, in JGGV XV, no. 295, pp. 56-7), "That you thought of my person in your little book, now completed and in print, leads me to conceive friendly thoughts, although I can only presume that it has as its object your bearing good affection for me rather than my person in reality."
rest [of your qualities]—accept, I say, the reply: if it be not sufficiently clear and indubitable, it is surely false enough! The cause is not located in this, that nearby things are rendered larger by convex mirrors, which was indeed once the only answer I had. Porta, having also followed this line, says in vain, "nearby things are made greater, but confused, by convex glasses." 65 In fact, they are made greater for all, but are not confused for all, just as in turn the concave glasses do represent objects smaller for all, but, for those who view distant things rightly, confused, and for those who view them confusedly. If it were the case that the greater things are, the more distinctly they are perceived, no one ought to have been helped by concave glasses. For they diminish objects. You, Most Noble Sir, recognize your valid objection. Others base the cause of corrected vision in the multiplicity of extractions alone. In vain: for all would be helped equally, and no one would complain that glasses blur things. Thus we should not look to entire objects, but to individual points of objects, as we always have hitherto. Therefore, those endowed with a point of distinct vision that is rather distant, when they use convex glasses, they alter the cone of radiations of the same nearby point, so as to appear to arrive as if from some distance, and so enter the eye. The cone is thus corrected so as to be bounded at the retina, which, unless it be set right by lenses, what we have described above in Prop. 26 happens: because of the nearness of the radiating point, the cone, which is to be bounded behind the retina, is cut by the retina, and thus the cones of the radiating points, having come allotted a certain amount of breadth, each create confusion for the other.

On the other hand, those endowed by nature with a point or distance of distinct vision that is rather short and close, by using concave glasses, after the cone of radiations coming from the same distant point, so that it may seem as if it originates and enters the eye from nearby. For unless they use lenses, what is described in Prop. 27 will happen to them: that is, the cone of such a remote point will be broadened before it reaches the retina. and, proceeding farther, will again spread itself out, this falling upon the retina will breadth, and the cones will disturb and confuse each other. Experience gives the remarkable confirmation.

I know two men, not of humble station, one of whom reads the finest line print, but brings his eye so close that he cannot use both eyes at once. The same man cannot read with clear vision as far as ten paces, beholding nothing but clouds. Nevertheless, lenses of deep concavity help him to see more distant things, although these same lenses completely confuse my vision, even though I also use ones that are correct for near moderate. The other man, who died some time ago, was nearly blind for nearby things, but was a Lycurgus 64 for distant things, so much so that he prided himself in being able to count, on a

65. Porta, Vinculo naturali, XVII, 10, ed. 1588, p. 268. Kepler alters Porta's words somewhat: "It is true that convex glasses magnify, but blurred. . . ." while Kepler wrote, "convexae propigna minores reddi sed vacues."
hoose at a distance of several stadia, the new roof tiles mixed in with the old. Using convex glasses, and a paper unfolded at a distance, as far as could be done with the arm, he used to read not badly. Anyone who brings less trust to my experiences should heed Aristotle. In Section 31, Problems 4, 15, 16, 25, he seeks precisely this, that it is that makes the near-sighted and the elderly suffer opposite afflictions, the former bringing nearer, that latter moving away, those things that they are going to look at; and the latter, though being weak-sighted, nonetheless write with the nearest of letters.

I also add diagrams, so that nothing may be lacking. Let $\delta y$ be a concave glass, and let the luminous point be so distant that the radiations are approximately parallel. Let, i say, the radiations $\theta x$, $\theta y$ come from the same luminous point. Therefore, because $\delta y$ is the concave surface of the denser medium, whose center $\delta$ is between $\theta$ and $\gamma$, therefore $\theta x$ will be refracted towards the perpendicular $\lambda \delta \alpha$ extended, and hence into $\gamma \delta \xi$ towards the outside, and $\gamma y$ will be refracted towards $\alpha \delta \beta$ extended, and again towards the outside into $\delta \beta$, and thus $\theta x$, $\theta y$, previously parallel, now diverge. And because $\delta y$ is the concave surface of the denser medium, or the convex surface of the rarer, with center $\delta$, therefore $\gamma y$ will be refracted from $\gamma \delta$ into $\gamma \delta$, and $\delta \beta$ from $\delta \beta$ into $\delta \beta$, and therefore $\gamma \delta$ and $\delta \beta$ will diverge more, so the extent that, extended in the imagination, they would meet at $\epsilon$. And so the radiations of the most remote point will flow into the eye or cornea $\gamma \delta$ as if they were coming from the nearby point $\epsilon$. What now happens within the eye itself on the retina through the approach (or virtual approach) of the radiating point, has been explained above, especially in Prop. 27 preceding. Again, let $\delta y$ be a convex glass, let point $\pi$ radiate, let the radiations be $\pi \epsilon$, etc. Because these strike upon a convex surface, they come together towards the perpendiculars $\pi \epsilon$, $\epsilon \pi$, and are refracted into $\epsilon \pi$, $\pi \epsilon$, and these, coming out into the concave surface $\pi \gamma$ of the rare medium, fee from the perpendiculars $\pi \gamma$, $\gamma \pi$, extended, are accordingly refracted into $\gamma \delta$, $\delta \gamma$, and thus encounter the cornea at $\gamma \delta$, as if they had come in from a point of the greatest distance along $\gamma \delta$, viz. very nearly parallel. And so again, what is corrected by this fictitious removal of the point $\pi$ in the coming together of the radiations at the retina, see above, especially in Prop. 26 preceding.

And this completes the demonstration of those things that were said above, in Sect. 2, of the crystalline.

56. The Roman stadia was an eighth of a mile, equivalent to the English furthing.
57. In Section 31, Problems 6 is at 950; 28, in 2; Problems 15 at 908; 34-908; 2. Problem 16 is at 950; 3, 4; and Problem 25 is at 950; 37-908a 8.
58. Reading "cornea" for "crania".
4. Consideration of those things that the optical writers and the anatomists have said concerning the means of vision

We shall not enumerate the details, but rather the most important points, nor shall we survey the opinions of everyone, lest the reader be confused. Which, and from what lessons, when they leave the spider’s web, and the crystalline; in it, from the reforming tunic and the optic nerve, the attribute the foremost parts of perceiving to this humor. For he says that perception or vision occurs when the rays from the visible objects pass through the tunic, but particularly the web of the crystalline, and when the image is imprinted upon the crystalline to such a degree that it begins to inhere; to this end, that humor is made rather thick. But he says that in order that distinct vision occur, the rays should be perpendicular at the anterior surface of the crystalline. For the perpendiculars are ordered as the points on the object are ordered. Unless vision happen by the perpendiculars alone, the vision is going to be confused, just as perpendicular radiations, too, are confused by oblique ones. And nonetheless, most powerfully convinced by experience, he also attributes something to the oblique radiations: it seems that there is more than the amount that enters the eye perpendicularly from the hemispheres. Now Witsius says that the vitreous humor is the immotant of the crystalline: while the aqueous humor is the aqueous matter of the nerve, perhaps following the physiologists, for René says that same thing from the same sources. That they are pellicular; the latter, so that the image might enter; the former, so that the image, now sensed by the crystalline, might pass through the vitreous, and through the substance of spirit to reach the connivence of the nerve, and there be recognized by the common faculty or sense. And so by attributes flatness to the crystalline in its posterior part, and denies that the rays of object should meet, lest the right rail be exchanged with the left in the replica. And above all, he thinks that the perfect image of the object seems passes all the way through to the connivence of the nerve.

This entire opinion (although it prides itself as Aristotle’s being the author, who says that the “seeing power of the eye” is “made of water,” in On Sense and Sensibles, ch. 2) is knocked down when the crystalline is cut off from the nerve and from the retina, and joined with the eye, as was shown above from P虐amus. What is a consequence is to be thought of in this introduced form of sensations and excitement. In the physiologists consider. Surely, if none of the senses is more marvellous and sober than vision, these things are unworthy of being declared about vision. Then, moreover, he carries the sense of light over to the tunctus, on the basis of touch. If it were by the sense of heat, and if vision were not accomplished in an instant, I would believe it. But now light and colors, which are by no means heat producing, and enter into the eye at a moment, are far too subtle to be perceived by a corporeal tunic, under the principle of touch. So what we have aquired in Chapter I on the immaterial/ motion of light: and so is not the amitt, and not even the nerve, but spirit.

Witsius’s account of vision and the eye is in Optics 44. Thesis—pp. 85–87.

or indeed something perhaps more divine, which receives and becomes fully conscious of light, which faculty I said above could not be investigated by Optics. For in that same place, this too is related, that images inher or are impressed upon humans.

Now, as for his simulacrum which Witelo foras using perpendiculars alone, he distinguishes with hortens substantiality between perpendiculars, and those very close to them. If light acts upon sense, and sense is affected in this action, it follows that it will itself also be affected more severely according to the strength of the action. And indeed, there is hardly any difference in illuminating between the perpendiculars and those very close to them, because of the luster being refracted very little. Therefore, the effect, i.e., the sensing of perpendiculars and of rays close to them, is nearly equal. And thus perception is continued. Witelo’s labors being in vain. But no need for more words. It has been demonstrated by most certain arguments and experiments that the picture of the object or the hemisphere is situated at the concave wall of the reform, with all confusion of rays removed. Next, it is certain that the rays of many points of the observable hemisphere come together at a single point of the crystalline. Finally, it is evident from sense that the apparition (identum) or image of a contrasting object appears in the black of the eye. Thus, according to Aristotle, on Sensibilia. Democritus of old said that vision occurs through this image or apparition.\(^{80}\) Witelo said that vision occurs through the confused illumination of the crystalline.\(^{81}\) I say that vision occurs through this acknowledged and irrefutably demonstrated picture. All three suppositions are certain: if it is uncertain to you by which of these vision is accomplished, follow any of them through, but pay attention to the arguments. Aristotle did indeed refute Democritus, noting that the apparition is not a “that,” some object, but a mirroring (τοιαύτης), or image, as in catoptrics, not a picture; and that it is not a reception of action by the eye in which it is thought to exist, but that by an eye placed opposite this eye, seeing itself in it.\(^{82}\) Similarly, I refute Witelo by the very confusion of the rays. For his statement that oblique rays are also seen insofar as the oblique radiations flow together with the perpendiculars, implies that oblique and direct radiation is received by the same point.\(^{83}\) Therefore, two objects will be considered to be in a single place. However, what confirms me is the most universal form of argument, used by Witelo himself. The effect of vision follows upon the action of illumination, in manner and proportion. But the retina is illuminated distinctly, point by point, by the individual points of the objects, and most strongly through the individual points. Therefore, it is at the retina, not

\(^{80}\) Aristotle, On Sense and Sensibilia II, 438v 5 ff.

\(^{81}\) Witelo designates the crystalline (which he usually calls “glacialis”) as the sensitive part of the eye in Optica III Prop. 4, in Theaurum II p. 87. His description of vision occurring by dissimulation of the perpendiculars is in Optica III Prop. 17, Theaurum II p. 92, where the confusion of rays is also discussed.


\(^{83}\) This is also in II 17, Theaurum II p. 92.
elsewhere, that the most distinct and most evident vision can take place. This is all the more so in that the defects of vision also follow from the perturbed arrangement [aratus] of the picture, just as has been demonstrated. And I do not know whether Democritus would discuss this picture under his name of "image" [idolium], rather than that mirrorklike "reflection" [eidosim]. It often beheld the ancients, that they were undeservedly refuted by Anaxagoras under a fabricated contrary of opinion.

As for the inversion of my picture, that might be raised against me in objection, which Witelo avoided with great care: he first attributed flatness to the crystalline, contrary to obvious experience, in order to maintain that opinion. And since this opinion affirms that that surface is bulging, by the testimony of Witelo, there occurs an inversion of the thickness. And I, for my part, tied myself in knots for the longest time, trying to show that low ends that are made of the right ones in entering the opening of the uvea, are again cut beyond the crystalline in the middle of the vitreous humor, and that another inversion occurs, and that the parts that were made to be left again become right before they reach the retina. Nor was there an end of this useless anxiety until I hit upon Prop. 11 and 12 among those preceding, by which this opinion was most evidently refuted. And even if I had upheld what was proposed, there was still to be a remaining complaint the hemisphere was going to be reversed. For those who stand facing us or the outside, judging these parts to be right, those left, have images directly opposite whose right parts will be taken to be left, as to be seen in mirrors. For the eye which to you is right becomes the left one for your image. I shall say nothing about how the picture's concavity was verging towards the head, while the concavity of the object was verging in the opposite direction.

And so, if you are disturbed by the inversion of this picture, and fear that inverted vision also follows from this inversion, I beg you to ponder thus. Just as vision is not an action because of illumination's being an action, but is an effect contrary to an action, so also, in order that the places correspond the recipients of the action must be directly opposite the acting things. Further, places are perfectly opposite when the same center forms the midpoint in all the lines of the oppositons, which was not going to occur if the picture had been erect. And so, in the inverted picture, even if from a universal perspective and with respect to some common line the right parts are turned into the left, nevertheless the right parts of the object are perfectly opposed to the right parts of the picture, and the upper parts of the object to the upper parts of the picture (each in relation to itself), as well as the concave to the concave. Nor is there any fear that the sense of vision might err about the region. For when it perceives an elevated object it clearly turns the eyes upwards, acknowledging them to be low in position and opposite the object, with respect to place. Rather, it would have been in error had the picture been erect. For then the interior wall of the eye would in some places be directly opposite the object, in others not, as from the sides for it would diverge from opposition. Therefore, no absurdity is committed by the inversion of the picture, which Witelo to seek a degree avoided. Jessenius too followed this opinion, considering that refraction at the cornea has the effect of preventing right from becoming left, which is clearly refuted in Prop. 12 of the
preceding. Finally, what is to be said of the image’s inhering in the crystalline (which is itself also Aristotle’s opinion, who chose, for vision, “water, because of its being more easily confined and made determinate” than air,44 because it receives and preserves better than air), is evident from the conclusion of ch. 1. However, concerning the penetration of the same image through the extremely narrow way or concavity of the cornea to the branching of the two nerves, enough has been said just above in sect. 3 of this chapter.

Platter is much better on the function of the crystalline, although even he did not quite hit the mark. He says that vision happens through the agency of the retinum tunic, but that the crystalline accomplishes for the retinum what convex glasses accomplish for those suffering from night blindness; it makes objects appear larger. Absolutely, Platter, the crystalline does do something like this, but it doesn’t do exactly this. First, because the dium boundary of the intersections of the crystalline45 is in the proximity of the retina, it was said in proposition 18 above what is going to happen regarding an entire eye applied there. Namely, an entire eye put by itself in the place of the retina would see no more than one point, and that in great confusion and throughout nearly the whole surface of the crystalline. If objects were going to appear with greatest distinctness to an entire eye located there, the same thing would not as a result also follow in part of the eye. The cause is this, that in the entire eye there is not only a center, in which the rays of all luminous objects come together (or are so gathered as if they had come together), but also a surface, upon which the picture is distinctly unfolded. This does not, however, take place at the retina, which is a part of the eye. For if it be said that those distant and larger objects are represented by the crystalline on the retinum, which objects project rays from all their points to the same point of the retina to grant this point for now, in itself false), by what procedure will that point of the retina gather the distinct parts of the visible object, when the point is in accordance with what was supposed above? Further, unless you bring in the object here, so that whatever of the likeness is represented through the whole surface of the retinum, is represented larger by the crystalline, you have now quite destroyed the analogy transferred from the entire eye and external mirror to the retina. For many visible objects illuminate the eye in breadth, in such a way that they nevertheless both come together, as it were, in the same center, and, having made an intersection (or something like it), are received by the same single point on the retina. Then, as a consequence, convex lenses can bring it about that a greater surface of the retina be taken up by any object you please. But between the crystalline and the retina there is no point of meeting of the rays coming from

44 Aristotle, De Sensu et Sensilibus, II, 438a 15. Kepler has a similar phrase: “more determinate;” where the standard Greek text has εὐκαταλείποντα, “more dense.” Evidence for this problem here: W. S. Litt, for his translation (Loeb Classical Library, 1937) p. 224, substitutes τῆς ἐξαπάτησαν, “more easily caught.”

45 The line of rays passing through the crystalline.
different points of the visual object, by 12 above, as a result of which the portion of the retina illuminated is made smaller, rather than larger because of the crystalline. See 23 above, and those things that were said in Section 2 on the means of vision. Finally, this is not the same as the use of convex lenses to illuminate objects by enlargement, as you have seen in Proposition 28 above: indeed, the vision would be false if, in making the picture, the object were to occupy a greater quantity on the retina than is correct. For as the picture is, so is the vision.

In this opinion, Platter appears to have brought in the anatomical example which I have heard from other physicians, namely, if the crystalline humor be shelled out from all the other humors and placed upon tiny letters, it displays them magnified. This is in fact extraneous to this enquiry. For vision occurs through the mediation of the picture on the retina. And this fallacy occurs not through the picture but through accounts of the image. Therefore, this magnification of the letters by the crystalline (or a certain analogue to it in the eye) does not give shape to vision.

Now compare the true mode of vision proposed by me with that of Platter: you will see that the illustrious gentleman was not farther from the truth than he was from the object, who is not deliberately treating mathematics.

Cornelius Gemma, a physician and philosopher of profound learning, in Book 2 p. 120 of the "Cosmocrator," compiles and jumbles his account of the means of vision from nearly all the respected authors. In this work you may see the ambiguity of words compete with the obscurity of the subject, both here and in practically the entire book, in that he prefers to rhetorize and waffle outright in a grave style concerning matters replete with majesty in the highest degree, rather than to argue, nor did he design to instruct any; but a serious and most attentive reader. First, he takes this from a Lexico and from the optical writers: vision is accomplished by a cone whose base is in the visible object, and whose vertex is in the center of the crystalline. Next, this from the philosophy of the ancients, with rules that are not optical, but words that are: another case gets in the way of that exterior cone, with vertex at the center of the crystalline and base set up within. The former is the cone of light, the latter, that of spirit or of the visual faculty; by the former, images flow in; by the latter, they are received, weak images by a large and broad cone, strong ones by a narrow one. But the riddles of this author, to the extent that they have something in common with other opinions, which I refuse, I refer to those. As for the rest, since they roll away by means of a slipperness of words, no matter how you might press, things that ought to

---

56. See p. 582, above.
57. "Cosmocrator" is an alternative short title for Gemma's "De naturae divinæ characteristicâ." Variae 1575. Cornelius Gemma (1534-1579) was the son of Rainier Gemma Friesius, mentioned in Ch. 2, above. He was Professor of Medicine at the University of Louvain. The passage cited by Kepler is certainly not on page 120, which is entirely concerned with the new star of 1572; Kepler may be thinking of pp. 157-8.
be done have been disregarded, primarily because we have surely found the true way.

Finally, comes Joannes Baptista Porta, whom I proposed that I ought to consider in this sequence. In the Magia naturalis Book 17 chapter 6 he presents the first consciousness of that thing of which I provided a formal demonstration in chapter two above: by what means do all things which are illuminated outside by the sun appear in darkness with their colors?190 Afterwards, having presented a few agreeable devices, in the course of concluding the passage, he now added these few words on the means of vision:

Hence it is clear to philosophers and optimists in what place vision occurs, and the question of vision in that was so thoroughly ventilated in antiquity is pilled apart, nor will it ever be able to be demonstrated by any other device. The likeness is sent in through the pupil, as [through] the opening of a window, and a portion of the crystalline sphere located in the middle of the eye maintains the function of a tablet, which time is going to be acceptable in the highest degree to those of intelligence.

Indeed, thou hast blessed us, excellent instate of nature: now that that quarter has been renewed, whether vision occurs by reception of emission. For, what many have wondered at everywhere in or in passing, they had either ignored when seen, or when it was studied more carefully, they referred to show it privately, through leanness and to the greatest injury to divine glory and harm to the human Republic: or, though it was considered, they were unable to give a treatment of it proportionate to its dignity (like basket weavers of Schwabia, hidden away in rather large barrels, with a little orifice in an opening). You alone have both noticed it and pursued it thoroughly, and have made it public in a worthy manner, so that, in proportion to the knowledge and love of the mysteries of nature which is in you, you might accommodate it to a most useful treatise. And so, as far as I am concerned, you have a grateful admirer and publicise of your name; as for the rest, I hope for the same things as you. I also do not think that the reception of images in the sense of vision can be more rightly confirmed in any other way, and the emission of rays refined for which see Macrobius’ Saturnalia, Book 7, which Witsch examined in Book 3 Prop. 5, and Aristotle, in the book On Sense ch. 2.199 Now do I think that there is going to be anyone who is going to raise even the slightest further doubt, praetided that the professors of physical subjects shake off fatal terror and deign to recognize even these few things. Indeed, the Philosophers will learn how to philosophize from his discovery of yours concerning light, colors, and the transparent, more rightly than from Aristotle. To Aristotle, in the book On Sense, Empedocles is absurd, because he said that ήδηκετίς are ἐξικελεύοντες.190 For he thinks it incongruous that vision

192 Colvin. are aestiva. Cf. ασημένια 23-439a 5.
should occur when the eye is contacted by an efflux made by colors. But let him into this camera obscura of yours: he will see the wall touched why can the eye not therefore be touched? The motion of the rodent which he introduces, caused by the sensible object, can also not be understood itself, and is far more incongruous than that emission. Not to mention that "diaphanous" is defined by him as if it were, as they say, a positive word for something necessary for vision, while it is a privative word for something obstructing vision; namely, opacity: thus, the reason why something is diaphanous is not because it is illuminated by light, but because it lacks roughness of surface, color, and high density, regardless of the light. Rather, light, illuminating the pellicid, most often impedes vision. But these are covered above in the notes to Chapter 1. Meanwhile, your words, Porta, on the means of vision should be considered somewhat more accurately. You say, "Hence it is clear in what place vision takes place," and afterwards, explaining, you say, "the likeness is sent in through the pupil," through the opening of the eye, "like the opening of a window, and a portion of the sphere of the crystalline matches the function of a table." Therefore, if I understand you well, if you were asked, "At what place does vision occur?" you would respond, "in the surface of the crystalline," as if on a tablet. For you will say that vision belongs to that class of picture which you have treated in this chapter 6 of yours, and I have demonstrated in ch. 2 above. It is clear that you are on the way for I have used the same words in Section 2 above. However, I would not have said that it has made its way through to the destined place. In fact, if you have a fixed target here, if you do not descend below the crystalline, you have erred at your opinion. You will no doubt reply, "It is stated more fully in our Optics," which, though I have searched for it diligently, I have not had a chance to see. Meanwhile, with the help of conjectures, I fear that even in the Optics you will not have attained the true object. For why is there no mention of refractions here, while you assigned chapter 10 to the affects of the crystalline lens, chapter 11 to mirrors, chapter 13 to the crystalline ball: why do you make no mention whatever here of the means of vision? And it is in itself easy, upon seeing that device of yours in your chapter 5, to immediately be seized by that position, that vision is carried out exactly through this device. I myself, although I duplicated the device, i.e., that one associated with my Prop. 23 above, which was not related by you which confirms my suspicion, since it is impossible to come to know the means of vision without it, was very much of this persuasion, that vision is nothing other

101 See the Appendix to Chapter 1 for Kepler’s commentary on Aristotle’s theory of vision.
102 Kepler’s conjecture is correct: Porta’s account of vision is described by Lindberg as being “in most respects traditional” (Theories of Vision p. 183).
103 Latin lens, literally, “lentil.”
104 Although Porta, and Leonardo da Vinci before him, had likened the eye to a camera obscura, neither had studied the camera as fully as Kepler did, and neither managed to follow the argument through to the discovery of the inverted image on the retina. Cf. Lindberg, Theories of Vision, Chapter 9; “Johannes Kepler and the Theory of the Retinal Image.”
than intersection, but that the aequus globe accomplishes precisely this with its refraction, that the intersection becomes parer which is not yet perfected right at the surface of the crystalline, owing to its nearness to the opening. For I believed that with the help of refractions, it happens that in so narrow a space of the eye there is represented an intersection so exquisite as can hardly exist in some large chamber without the use of refraction. For the rest, I am immediately referred at the eye by the aequus globe, which had only one point beyond it at which it would distinctly portray a portion of the hemisphere; before and after, not at all. It should, however, have portrayed more distinctly at distant points, if the intersection alone generated this picture. To say nothing of what was said in Section 2 above, that intersection by itself (without the assistance of gathering through refraction) would never carry through to the most distinct vision of objects smaller than the opening of the urea. And so, to conclude, most ingenious Porta, if you were to add to your opinion this one thing, that the picture on the crystalline is still rather confused, especially when the opening of the urea is dilated, and that vision happens, not by the joining together of light with the crystalline, but that it descends to the retina, and by that descent the radiations of diverse points are more separated, and also the radiations of the same point are joined together, and that the place of the gathering is to a point is in the retina itself, which gathering brings about the manifestation of the picture; and that by that intersection it is brought about that the image becomes inverted, and by this gathering, that it is most distinct and most manifest; if, I say, you were to add this to your opinion, you would have completely settled the account of the means of vision.

5. Those things that recoil upon Astronomy from the means of vision; or, on flawed vision

Although the perfection of vision depends upon both eyes, astronomers nevertheless do not always use vision to distinguish between objects, but most often use it to establish the angle between a pair of stars, in which procedure there is need of a single center at which the angle is evaluated. Therefore, they select one eye, imitating the artificers, who, when they are going to examine the straightness of a rule, use one eye. The same is done by those who aim "scorpions" and bombardas at a target. For all of these, the use of one eye is sufficient, though it be for a variety of reasons. Aristotle considered it worthy for a philosopher to make an inquiry into this subject, in Section 31 Problems 2 and 20. But neither does the application of a single eye have absolutely no
need of precautions against the possibility of its leading us into error, in a twofold way.\textsuperscript{117} For in the picture by which we have said that vision occurs, two regions of the eye are chiefly considered: the center of the eye, and the surface arranged around the center, receiving the picture. For it has been said that the picture is so arranged upon the retiform tunic, that straight lines projected from individual point upon it through approximately the same center of the retina\textsuperscript{118} are going to fall upon joints of the visible object corresponding to themselves, if they be produced.

And so, when the distances of stars are to be taken using astronomical instruments, the more careful astronomers, as has been said, do not trust the eye. For they know that although the eye be tangent to the very center of the instrument (which, however, is accomplished with difficulty), it is nevertheless not tangent, except with respect to the surface, in which the lines, in fact, drawn from the two stars through the upper sights, do not come together. Let $F$, $G$ be stars, $BAC$ the instrument, with center $A$, $DA$ the surface of the eye, $F$ the center of the eye. Since, then, it is not from $A$ but from the center of the eye $E$ that the straight lines falling upon $F$, $G$, are to be imagined to go forth, therefore, when the sights $B$, $C$ are so applied that $EBF$, $EGC$ are in a straight line, the angle $BAC$ will give a false measure of the distance, and will be greater than is correct, because it is within $BEC$ upon the same base. And so the arc $BC$ is greater than is correct, because the depth of the eye $EA$ does not permit the centers of the instrument and the eye, $A$, $E$, to be joined together.

This is much more to be guarded against when very small quantities are called under the measure of an angle, such as when we seek the diameter of the luminaries; and this must be laid in the place of a foundation by all who enter into a reformation of astronomy.

Accordingly, Archimedes, in his book on the number of the sands, took this precaution: First, by adopting a certain procedure, he investigated a quantity not less than the sight,\textsuperscript{119} set a cylinder equal to it at the center of the instrument $A$, and drew lines from the sights $B$, $C$, tangent to this cylinder, and coming together: let them be $BE$, $CE$.\textsuperscript{120} And so he set up angle $BEC$ slightly smaller.

\textsuperscript{117} The two sources of error are, first, the breadth of the pupil and the effective position of the center of vision, discussed immediately below, and second, the apparent enlargement of angles, objects, discussed afterwards on p. 332.

\textsuperscript{118} The center within the eye, not the middle of the retina itself.

\textsuperscript{119} That is, equal to or greater than the diameter of the pupil.

\textsuperscript{120} Archimedes, Sand revisions, in Archimedes opera omnia II pp. 223 ff. Kepler was using Commandino's edition: Archimedes opera omnia multa (Venice 1568). A summary (that is not a translation) of the procedure is included in The Works of Archimedes, pp. 223-4.
than that angle by which the distance FG was perceived. But the angle BAC was slightly greater.

Further, the particular quantity that is not less than the sight is to be determined thus. Let AE be the surface of the cornea, and from the visible point E, let the extreme radiations which are admitted through the opening of the uvea, be EA, EB, which. represent at A, B, meet at the point F of the refracting tissue, or close to its surface. Now let CD be a smaller quantity than that which passes through EA, EB, and closer to the eye AB than E. You see that even if CD is squarely presented to point E, nevertheless not all the radiations of E are turned aside, but they come in from the sides of about, and meet it F, the place of destination for all, by which rays, all gathered into one, vision of the point E takes place, but with an admixture of the vision of the object CD. For the right end D will image the left part N of the retina F; the left end C will image M, the right part of it. Consequently, the middle of the object CD will image F, which was previously also the image of E. Thus two objects will be seen in one place, but with this difference, that E will be thought to appear more distinctly, while CD will be like a shadow or a spider web. No wonder. For it was said above that radiations of the point C at the eye, when C is so nearly, are gathered far beyond F; therefore, they strike the retina F not yet having been gathered. And in just this way, CD will be less than the breadth of the vision. Archimedes, not much differently, sets the more distant cylinder GH equal to CD, and directs that it be white, for the sake of distinct vision. If the whole of the cylinder GH be seen, with no interference from the cylinder CD, he pronounces them to be much smaller than the vision itself, if, on the other hand, a certain part of GH be left out, he pronounces that to be a little smaller. If, on the contrary, all of GH be hidden (the two cylinders always remaining equal to each other), he states that they are greater than the vision. There is a little something lacking in this assertion. He did indeed define greater and less than the vision sufficiently, and required nothing else for his demonstration; but otherwise, the proportion cannot be established that the less CD is than the vision, the more of GH is seen. For if GH should approach, less of it will be seen then if it were to recede from CD, even though GH and CD remain equal. Not to mention that the breadth of the pupil determines the breadth of vision, which nevertheless is different at different times. This I have said only for the sake of information, so that the words of Archimedes and my opinion on vision may be compared with each other all the more rightly. As for the demonstration itself, that still remains for Archimedes.111 For lines

111. Here, Kepler refers to the following endnote:
drawn from the ends of the visible object, so that they are tangent to such a

cylinder located at A of the prior diagram, exactly equal to the vision, do not

always coincide at the center of the vitreous humor. For it was said above that if

straight lines should be drawn from the points of visible objects, they would pass

through this center and would be perpendicular to the retina. Accordingly, in

the latter diagram, let J be the center of the sphere of the retina, and from C, D

let CM, DN be drawn through J to the retina. M and N will be the approximate

places of the picture of C, D, by what was said above, although C, D would

make an image of themselves, clearly not by the lines CM, DN, but by ones

that are refracted and different in place. Therefore, in order that the edges of the

object seen, joined with the quantity CD by vision, might meet at the center J, it

is sitting that its boundaries clearly belong at MK, NL, as if the object seen were

KL, which, if it were smaller and more compact, Archimedes’ lines would meet

beyond J; if wider, before J. And so, as regards the diameter itself of the sun, in

the measuring of which Archimedes was busy in this passage, it is too compact to

be held in the span KL or CD, with the consequence that KC, LD of KL be the

sun meet beyond J, and the angle really is somewhat smaller than is correct.

However, that Rabbi Levi, whose words Commandino relates in comments on

this passage of Archimedes, was also not negligent in investigating the depth of

this point of meeting by means of the Radius Astronomicus: it comes out to


To p. 214 l. 10. Archimedes looked to this alone, that he search for an angle that

was certainly less than that angle under which the sun is seen. Moreover, as regards

the sun, he accomplished this with his demonstration. For the rays from opposite

extremities of the sun, grazing it, have a quantity equal to the vision, and closest to

it: they meet in the center of the vitreous humor, where the measure of the angle

would be correct, but beyond it, where they really make a smaller angle, which is what

Archimedes sought. Therefore, in line 13 an objection arises. For someone might say

that these radians of the extremities of the object sometimes strike upon the very

center of the vitreous humor. The answer is, that this does not happen always, but

only when the seen object is equal to the span of KL. And just as, when the seen

object exceeds this span of KL, the meeting is before the center of the vitreous humor

J in which case the object would appear greater than is correct, so, if the object

seen should be deficient from this quantity, as is the sun (the visible object, following

what Archimedes proposed), then the meeting will be entirely beyond the center of

the vitreous humor J. And therefore these three members are demonstrated in what

follows, containing the elaboration of both the objection and the solution.

112 Here Kepler refers to the following endnote:

To p. 214 l. 15. Read in the corretive opinion on p. 212 l. 14, namely, so that

straight lines may be drawn from the points of the visible to the points of the picture

corresponding to them.

But you might say, “Who will measure that angle, whether it be in the center of

the vitreous humor or outside it?” I answer that Archimedes taught it, as was indicated

above on p. 213 l. 3. For he used another cylinder greater than one that would be equal

to the vision, and placed it in such a way that (in the diagram of p. 212) it would belong

between FE, GE; that is, at RC, and compared the distance of the cylinders with their

quantity. See the passage itself.
he the very center of the optical globe, which he says he found himself. After he sought it out deeply with the greatest labor.113 In this connection, he marvelously confirms what we were saying above in Section 2. For the center of the sphere of the vitreous humor, which we marked out for the imaginary intersection if lines were to be drawn, and about which is arranged the picture representing objects: that, I say, is simultaneously the center of the retina and the uvea and the white or sclerotic tunic, and is therefore also the center of the whole ocular globe. For what that Rabbi added, that he had found this point at the center of the vision "that is at the center of the solidified moisture," he appears to have described from Alhazen, his tribesman, who "judges this to be fitting or fair," and Witelis, too, carried this over from the same source in book 3 Prop. 7.114 But anatomical evidence refutes this most clearly, asserting that the crystalline humor is in a position anterior to that of the center of the globe; and further, that its anterior surface has a curvature that is flattened, not protruding. In the rest, the Hebrew uses a good demonstration. For, supposing that \( K \) \( L \) and \( C \) \( D \) in the last diagram are any two quantities, the eye \( AB \) is made to approach so that when \( C \) \( K \) are set upon one line, \( D \) \( L \) are also in one line. For then, \( K \) \( C \) \( L \) \( D \) being extended, a meeting occurs at \( J \), a distance \( I \) \( A \) beyond the surface of the eye, which, compared to the globe of some eye, shows its semidiameter. For as the excess of \( K \) \( L \) over \( C \) \( D \) is to the distance between \( K \) \( L \) \( D \) \( C \) \( I \) \( J \) so is the whole \( K \) \( L \) \( D \) \( C \) \( I \) \( J \) to its distance from \( I \), while \( K \) \( L \) is nevertheless found to be a lesser distance from \( A \) \( B \).

But our Tycho Brahe, at the beginning less experienced as yet is the handing of instruments, also encountered obstacles of this kind from the joint of the instrument itself. For, in the next to the last diagram, since the straight lines of the rules meet at \( A \), and the transversal around \( A \) has some thickness and extends all the way to \( E \), it was impossible not just for the center, but even for the surface of the eye to be at \( A \); but it stands below at \( E \); as he advises in the Mechanical part folium D4 and in the Prosthæmatologia, fol. 341.115 And while he supposed that, by the use of a table whose structure is found on fol. 342, he had entirely removed the observational errors that had arisen thence, he nevertheless allowed afterwards, on fol. 343, that the cheekbone kept him from placing the pupil of the eye near enough to the circumference of the head of the instrument.116

113 Archimedes opera nova edidit a Frederico Commandino aequo in latimam converteris et commentariis illustratae. Venice 1558. Commentarius in Librum de Atrio Numero, fol. 60v-61r. The work quoted by Kepler both here and below, which he changed slightly from the original, see on fol. 61r. For Levis ben Gerson and his "rattus," see Bernard L. Goldstein, The Astronomy of Levi ben Gerson (New York: Springer Verlag 1985), Ch. 9, pp. 51-54, and the commentary on pp. 443-4.

114 See the preceding section abs. Alhazen II 12, in Thesaurus I p. 6, and Witele III 7, in Thesaurus II pp. 48-9.

115 Tycho Brahe. Astronomiae instarum mechanica (Kamburg 1588), in TBOO V p. 82, Prosthæmatia Ch. 4, in TBOO II pp. 335-5. Kepler evidently thought of the latter book as the "mechanical part of astronomy," just as the latter part of Kepler's own Optics bears the title "optical part of astronomy."

116 TBOO II p. 336.
Thus far he does indeed assign a cause for the vision's still being aberrant. But while I would grant the chief blame lies in this, that "it is not from the pupil," which Tycho Brahe supposed, "that the visual ray chiefly proceeds." (whether you understand that the opening of the uvea or the cornea covering it, as he himself appears to mean), but from the middle point of the globe, to speak technically now of the visual ray with the ancient school of Euclid. Examining this same thing better, Tycho wrote to the Landgrave of Hesse in 1585, in this way (Epirotitiae Astronomicae Vol. 1 p. 8): "For it is not possible for the center of vision to be efficiently joined with the center of the instrument, nor does the pupil of the eye remain motionless in viewing a pair of stars, whence it turns out to be unavoidable that the center of vision also changes somewhat."172 Up to this point he had considered the visual ray to come forth from the pupil as from a censer, but when he saw that the distance of the surface of the eye from the center of the instrument did not suffice, he ascribed part of the aberration to the translatory motion of the pupil, which is plainly nothing else but this, that the center of vision lies beyond the pupil, and that when two lines are drawn from the center of vision, some portion of the surface of the eye is subverted; and the vision of the two stars cannot be direct unless the center of the pupil be carried over from one line to the other while the center of the eye stays the same.

And so this quantity of the eye brings in some obstacles to accurate observations, which Hipparchus, intent on solidifying the dioptras,173 once introduced, so that, when a very narrow opening is applied to the eye, if the eye should look through it to the dioptra of either of the stars or the edge of the sun, these paired acts of vision would certainly cut each other in this opening, and the point of this cutting would not have to be sought in the breadth of the eye. And Polemy imitated this. Today, in place of the dioptra, they make a very subtle slit by setting up two sights nearly touching each other. But the authorities do not think this slit worthy of trust, especially at the shorter distances. Further, there is that obstacle noted by Tycho in dioptras viewing the stars, that if the openings be narrow, they obscure the stars; if, on the other hand, they be wide, they do not provide the required precision in the small divisions. For that reason, some replace the more distant dioptra with a very tiny globe, supported by small handles in the middle of a perforated tablet, and line it up with the stars by covering them rather than looking at them. Others fix a pair of strings crosswise in a wider opening in the tablet, so that the intersection of the hairs ought to take the place of the dioptra or the little globe. However that may be, the little globes or the crosshairs disappear in the darkness, and when a candle is brought near, the viewing is impaired. Because of these impediments, in both very close and more distant dioptras, there

172 TBO 51 p. 36. The letter is dated March 1, 1586, not 1580. Kepler changes Tycho’s words somewhat, combining two sentences into one, omitting a clause, and changing the punctuation.

seemed to Tycho Brahe, after long experience, to be no more effective way of sighting than by two observers for two stars, sighting along a pair of straight lines directed to each star, and by the single eye, inseparable on so many counts, being ordered to be entirely unused. This procedure, first used by Tycho, was enthusiastically adopted by the Landgrave’s staff, and to the great advantage of the observations: see Epistolae astronomicae Book 1 fol. 3 [119] Indeed, for that form of the Landgravian instrument, Tycho feared (on fol. 8) that it might be like his sextant as first outlined, but on fol. 22 its accuracy is defended by the Landgrave. And on fol. 28 Rothmann relates to Tycho the authentic form of the other instruments, which is equivalent in effect to the Tychonic. [120] It differs in this alone, that, in the Tythronic sextant each of the observers encloses the same cylinder in the center of the instrument between parallel slits in his own sight, while in the Landgravian, in the place of one cylinder, each encloses his own parallel slit somewhat removed from the center. And so Tycho, writing to the Landgrave on fol. 38, and to Rothmann on 61 and 62, greatly commends both the form and their industriousness. [121] So anyone who wants a more particular account of observing should consult these passages. See also Tycho’s Mechanica folum HS. [122] And the center of vision assures us of these things concerning astronomical observations.

We need to take note much more attentively of this property of vision, why is it that, to all people without exception, all things that are luminous appear greater in proportion to things placed nearby that are less luminous. For in the first or last phase of the moon, the luminous horn appears to be enclosed by a far larger circle than the rest of the body, illuminated by the light of the earth and very clearly visible. The same in the lunar eclipse on 1603 May 1525: certain observers were able to grasp visually the edge of the darkened part, although it remained by more than a third part. They therefore pronounced the circle of the darkened part be narrower. On 1600 August 7/17 is the evening I saw the moon in conjunction with the heart of Scorpio [22] an hour before its setting. It was going in above the

[117] TBOO VI p. 31. The letters states that the instruments had been improved under the direction of Paul Wittlich; however, a letter from Tycho to Thaddaeus Hauck of 14 March 1592 (TBOO VIII p. 323) relates that Wittlich had shown the Landgrave how Tycho had improved the sights. This letter is translated in Ginzburg and Westman, The Witch Connection, p. 58, and Edward Rosen, Three Hundred Mathematicians (New York: Abaris, 1986) pp. 269 and 271. Wittlich’s work on the sights is further described by Rothmann in his letter of 14 April 1586, TBOO VI pp. 55-6.

[118] The passages referred to are in TBOO VI pp. 36, 49, and 55-6. From this last passage, it is evident that Wittlich made an improvement which was not equivalent to the Tychonic arrangement, and the Landgrave’s staff subsequently modified the sights to their ‘authentic form.’


[120] Kepler is evidently referring to the “Supplementum de subdivisione et dispositio instrumentorum,” TBOO V pp. 153-5. It is on fol. HS and HS.

[121] Antares.
heart on the northern side, so that about a third part of the division projected above the heart, and while the division differed slightly from a straight line, the sun being in 25 Leo, nevertheless, the way across the luminous part appeared much wider at that place, than the distance of the sun from the line of the section. This was at the common boundary of Stygia and Hungarit, the polar altitude being 47° 12'.

Those who are weak of sight, and who are otherwise blind to distant things, imagine for themselves a rippling214 series of ten phases in place of one phase. The same ones, seeing people at a distance with bright collars, do not recognize the people's faces, which, without this condition, are evident enough. In full moons, it is occasionally the experience, as may be seen in Tycho's observations, that when five or six people are observing the same moon, the estimation of the diameter is inclined so very, ranging from 31 to 36 minutes, according to the vigor of each one's vision. This is, moreover, the chief controversy about the moon. On 1590 February 22 the moon was observed 22 times: twice at 31°, six times at 32°, seven times at 33°, six times at 34°, once at 36°. In the beginnings of lunar eclipses, the eclipse is noticed first of all by me, who am laboring under this defect, as well as the direction from which the darkness approaches, long before the beginning, while the others, who are of the most acute vision, are still at doubt, as happened in the month of May of this year 1603. For the rippling of the moon, mentioned above, stops for me when the moon is approaching the shadow, and is in great part removed from the sun's rays. In eclipses of the sun, the beginning is hidden for a long time, and suddenly some rather large part is seen to be missing, even to people with very sharp eyes. And the horns do not go off into a point, but are blunted or even cut off; and some people (as in December 1601) exclaim that they see precisely the shape of a horsehead. The magnitude of the eclipse is always diminished in the eyes, as the light spreads itself out everywhere and, as is seen, enters in at the ends of the moon's edge. In fact, also evident is this proof of this dilution: if you apply the moon's body to the edge of an opaque ruler which is close to the eye, the edge, while it is an unbroken straight line, will seem to be diminished on that side when the moon is placed in between, as the moonlight effaces the image of the edge.

All these things, and whatever others there are, draw their origin from the retina tunic, but in a different respect. First, whatever of this affects those with defective vision finds its occasion from propositions 26 and 27 above. For the more distant bodies, such as the celestial bodies, gather the radiations from a single point, into a single point, before they touch upon the retinum, and, cutting

214 ἀφορίζειν. Kepler uses the Greek to indicate that he means the line of division between the light and dark parts, for which the modern term is the "termination."

215 This was at the time when, together with the other Lutheran still in Graz, Kepler had been expelled. He spent several weeks in the area of what is now Burgundy before proceeding to Prague in September, at Tycho's invitation.

216 Reading "cristatum" instead of "cristatum." Later in this paragraph, Kepler refers back to the "rippling of the moon mentioned" in secta Lunaris cristatum, which I take to be a reference to this word.
each other at that point, they now strike spread out upon the retina. Thus, it
is not a single point of the retina, that is illuminated by a point of the object,
but a small part of its surface that is illuminated by a point of the object, and
thus it is encircled by many points; white things, however, and bright things
illuminate its surface strongly. They therefore bring it about that those things
which are depicted less bright in the same place, where they themselves showed
their own boundaries (they show them, however, either too far away, when the
intersection of the rays takes place before the retina, or too near, as just now the
opaque ruler did when cut by the radiant cone before the cone ends in a point; see
prop. 26, 27, 28 of this chapter), become entirely invisible, and give way to the
white things. And so nearly the same thing happens in the eye which, above in
chapter 2, with regard to the configuration of the ray, I demonstrated to happen
on a wall. However, that the image of the luminous body is not simply magnified,
but is in a way multiplied, and that some one larger thing is mixed up of many
distinct ones, this appears to be either because of the wrinkles in the uvea, which
is dilated at night, when we are looking at the moon, and comes together into
itself and into its wrinkles; or because of the gaps in the cellular processes. For
when the eyelids are shut and the brow wrinkled, many of such false images are
cleaned away, but not everywhere, because by shutting the eyelids the eye is not
equally covered everywhere, being left open transversely as long as the eyelids
are the least bit open.

On this account, therefore, the visual picture is corrupted, which is a nec-
essary consequence of defective vision. If all things were equally bright, vision
would be confused; now, however, because the luminous things predominate,
they are flouted in their quantity. And even if precursh the pictorial cone is
spread out at all observers, nevertheless, not everyone sees so subtle a visual
faculty as to perceive by means of all the radiations, but those alone who are af-
tected by all radiations imagine for themselves that the luminous things are larger.
Hence, those who have a conspicuous weakness of vision have double vision, not
of luminous things only, but also of dark ones, if they be narrow and exceedingly
distant.

There is a kindred question in Aristotle, Section 31 Probl. 28: why it is
that light radiating into the eyes takes away the view of nearby objects, which
returns when the light is diverted from the eyes. For, precisely as has been said
up to this point, some parts of the reform are illuminated very strongly, and
thus the perception is very strong, while the radiation of the remaining parts,
which correspond to nearby visibles, is in no proportion to the former, and as
a result there is almost no perception. For just as the reform's distinct picture
is to distinct vision, so is strong illumination of the picture to strong perception.

112 Here Kepler refers to the following endnote:
To p. 219. Confused vision and erroneous vision are different. Vision is confused
when it sees two objects in the same place, as on p. 213 1. 21 above. It becomes
erroneous when the entire place is attributed to just one visible object alone,
while the vision of another thing seen in the same place vanishes.
126 Problenn 960a. 24-28.
in which the spirits are strongly affected. This probably looks more to another defect of vision, which follows, Thus, another consideration, in which the retina enlarges the pictures of luminous things, seems to enter in beyond the laws of optics. It is indeed a commonplace among recent philosophers (physicists) that the rays of the sun are scattered by a bright color, gathered by a dark one. And in fact this does not appear to be false: if one places a white paper in the way of a globe of water or crystal at the point of burning or of intersection, it will appear to be widely illuminated, but with squinting eyes because of the brightness; if, on the other hand, the paper be black, a narrower surface will be illuminated. Who would not consequently conclude that the rays are scattered by the white, and come together by the black? Particularly if he sees that black things are for the most part ignites, not the white. But it does not follow from this that this is the nature of light and of white things. For it is impossible for any surface to have the power by its own color to make any ray strike upon it in a line that is different from that which its optical laws force it to strike with. For the cause of the apparently wider illumination of white things can be the extreme brightness, as will now be explained, and that the white areas lying round the point of burning are brightened by even a gentle ray, and are opposed to the strongly illuminated part because of the squinting of the eyes. But the "reason why dark things are most of all ignited" (besides what we have said in ch. I prop. 36), seems also to be because whatever things are colored with this color smack of dryness and of burnness, and their matter is consequently more inflammable. Therefore, as to white things at the burning point appearing wider, this same thing and most of the phenomena just brought forward seem to attest that what is widened is not the ray gathered upon the white object, but the impression of the white object, and of its picture on the retina, upon the visual spirit. For, by 50 of the first chapter, there is more radiation from a white surface than from a dark one. The brightness is therefore extreme, or, if it is the sun that is under consideration, then this is self-evident. Now, as we have ascertained above that the image inheres in the spirit, because by optical laws it could not happen in a humor or tunic, so here too the image of a white thing very strongly illuminated, or the image of the sun received into the spirit, appears to spread itself out because of the kindship of its nature, no otherwise than the way a red drop that falls upon the surface of water (liquid on liquid) spreads itself out, while dark images received into the spirit withdraw into themselves, as if the drop were to fall upon dust, and this happens by laws that are not, as previously, optical. "For if you consider well, the inheritance and persistence [shape] of the luminous image are in exactly the same class as this spreading out. For from the presence itself the spreading out appears to follow, as the vision flies such brightness, but nonetheless, having absorbed it, carries it around and falls upon it with others of its parts. Whenever of these causes is to be taken into account in whatever individual case, it is certain that this spreading out of luminous things exists either in the retina, by reason of the picture, or in the spirits, by reason of the impression: this was, in fact, the thing whose cause I wanted to investigate in this whole chapter. And so, from this chapter, Astronomers will ponder this, that occult perception or reckoning is not always to be trusted, however much they are taken into account in the quantity of the diameter of the full moon, or of the defect in
an eclipse; and consequently, that other more certain procedures must not only
be brought into consideration, but also one must not rashly disagree with them,
on the testimony of vision, when it happens that they disagree with vision. For
it has been demonstrated most clearly, from the very structure of vision, that it
frequently happens that an error befalls the sense of vision, in overestimating
the size of bright things.

It also appears appropriate to this place to state the cause why, for those
with midnight blindness, the heavenly bodies are more confused in the depth of night;
but when dawn begins to break they are seen more distinctly. For if much light
interferes with them, the light added on by the light of the dawn should have
interfered more.

Now the cause is this, that in the darkness of night the opening of the pupil
is widened by its natural motion, while in the light of dawn it is closed more nar-
rrowly. But the cone of the radiating point enters with more power through the
wider opening, and makes a strong impression, while the same cone contracted
by a narrower opening moves the eye more weakly. Hence, however, we have
always made use of this, that by an extreme sensation of one object, the weaker
sensations of the others are suppressed. Moreover, in chapter 2 this served in
the place of a foundation, that distinct lights upon a wall, coming as through a wider
opening, are hidden, the weaker by the stronger; but when the opening is nar-
rrowed, they are not affected in proportion; although all the lights are weakened,
more light is removed from the brightness of the stronger, and the weaker light
emerges.

279 Here Kepler refers to the following endnote.

To p. 221. The sense is this, that although the spaces of the heavens between the
large heavenly bodies have their own radiation, just as the smallest stars also have;
nevertheless, the points of the retina upon which they radiate are occupied by the
broadness of the stronger radiation of the large stars, with the result that those stars appear
large and tinged with rays. But in the dawn, these fringes of the large stars are erased,
because the radiation of the lightning heaven was out at its place on the retina; over
the radiation of a bright star that occurs at the sides, and that wanders in a foreign place
about the axis of the cone of vision. It can do this all the more easily as this radiation is
now weakened by the constriction of the pupil.

Further, this question must be understood to concern the first rising of the dawn,
when the light is doubtful, and not of the full dawn. For that now fully overcomes the
heavenly bodies, just as does that brightness of the air, which comes from the moon at
night, illuminating the air from on high with its full orb.
Chapter 6
On the varied light of the stars

It does not belong only to the natural philosopher to treat of the light of the stars; the astronomer too has something to say here, especially about the illumination of the moon. We shall join the two together for the sake of study.

1. On the light of the sun

The sun's power in the world is incredible and almost divine, for from the sun comes the life and preservation of all motion, and the embellishing of things heavenly and earthly, so much so that the more closely you study it the more miracles you find in this one power. And accordingly it befits the philosopher to probe all the treasures of nature for the purpose of proposing theories [dogmaton] befitting such a miracle. And since we are now discussing optics, let us winnow out what is most prove in this science, and apply it to this body. We learn from the divine Moses that at first everything consisted of water, and when the beginning there existed a rough and unorganized mass consisting of moist and dry or of water and earth, right away that very first day light was created, which on the fourth day was distributed among the different kinds of bodies. Casting off from this harbor of most certain opinion, with conjectures that are not absurd as ours, I struggle into this most vast ocean of contemplation, and state this: that the sun's body consists of the densest material of all in the entire world, so that within its very restricted orb is enclosed as much material as is spread out in the whole asthecal air through the almost infinite amplitude of the world's solid sphere. 1

And in fact, that the density of this body exists in the highest degree, is required by a heat-producing power, proportionately fierce and proportionately extended. For, if one may apply, sublunar instances to celestial things (and that this is permissible was long ago established by Moses, who showed that all is of the same matter, among things of the same quantity, a certain one burns more vigorously and longer according as it is denser, more moist than a flint, burning into more than a coal. In this regard, one of the ancient philosophers, Diogenes Laertius has it, did not so much speak absurdly, as receive an unfair hearing, in stating that the sun is a glowing stone. 2 That is, he did not fear, along with Aristotle, that it might fall to earth if it were a stone; whether he did so rightly, you will learn from the Englishman William Gilbert's magnetic philosophy, to which I fully subscribe here. 3 And when Paracelsus expressed the fear that if the moon were earthlike it would fall to earth, Plutarch answered knowledgeably, saying.
Just as the sun turns towards itself all the parts of which it consists, so too the earth receives the stone, similar to itself. The fact that it is a body is not ascribed initially to the earth, nor is it a torn away from the earth, but is something established for itself by its own special nature. What, then, prevents it subsisting separately, compacted and held in restraint by its own parts, . . .

and below.

It is probable, if the world is really animate, that it has in many parts earth, in many waters, fire, air, not by necessity, but by a disposing reason. For the eye is not driven into the head by its brightness, nor has the heart fallen down into the earth by its heaviness, but both are located this because it was more expedient thus.

And over most beautiful things follow.

Further, my position of a certain quantity of matter in the sun’s body, making it equal to the rest of the matter by which, according to the divine Moses, an extension or faciation is created between the waters and the waters, seems to be required by the elegance of proportion, that the same thing whose force ought to have permeated that universal space has received as much body as there is in that universal space. There is also to danger that such a body could be impossible that either the force should be so tightly condensed or the body so widely extended without the intermingling of a vacuum. For why should this proportion not hold between extremes, if between water and air, which are intermediates and very close to each other, there exists such a proportion that a drop of water comprises the same amount of matter as is in a fairly large chamber of air, as we have proved above in ch. 4 sect. 6 prop. 10? Besides, because it befits this image of the solar body to be utterly simple and to the greater degree one, it must necessarily be devoid of the two notions of the Opaque, which this word acquired in ch. 1 prop. 17 above: For it will have neither many surfaces beneath itself (in which case it would not be simple), nor color. For I defined color above in ch. 1 prop. 15 as “light in potentiality,” but to the sun belongs the pure actuality of light. And things that are in themselves colored reveal an impurity of matter, while it is right tha the sun is a body most pure. Consequently, notwithstanding its surpassing density, the body of the sun will nonetheless be pellucid. And so, by 11 of the first chapter, it must necessarily have been composed of some fluid substance, and, finally, have come forth from

4 Plutarch, The Envo, in the Mona., 924b ff. and 928A ff., in Mora (XXI pp. 60 and 91). Pllatonic is an introducer in the dialogue. Kepler has treated the price of the two passages with considerable liberty, changing words and altering a phrase.

5 Kepler has the following example in the paragraph, though there is no reference to it in the text.

To p. 223. I declare the sun be a body pellucid unto its own proper light, not to exterior lights, and it will therefore not be transparent. For neither will the higher density give passage to the sight of values of things in the sight, and will extraordinary light leave place in the eye or on other objects which are beyond the sun’s orb.
water that has been condensed and purified in the highest degree, which the divinely instructed Moses also implied.

Again, because by general admission the sun's function in the world is the same as the heart's in the animal (for I shall prove in the physical part of astronomy that the motion of the planets is dispensed by the sun, namely, that it dispenses life to this perceptible world, there must also exist in the sun's body a soul that is a handmaiden in such a task, or a vital faculty, if you prefer. Therefore, it is consistent that from the indwelling of this soul or faculty in the densest and purest body, and from its most powerful vivifying or formative faculties—this is, from the victory of the soul and the subjugation of extremely continuous matter—light should result; by what procedure, is uncertain, but it is certain nevertheless by the examples of many sublunar things. For tell, O Natural Philosopher: where might you see a rising flame without heat, which is either from an animate faculty or was once given birth from it? Tell further: what matter might you see inflamed, which is not born through some animate faculty in its proper body, such as oil, resin, and so on. Lest you hear some suspicion about subterranea things, those are the actions of an animate faculty in the globe of the earth, generation of metals and of rivers from sea water, warmer and protector of subterranea things from the cold of the upper world, perserverer of the harmonies of the motions of the heavens (though without dispassive thought), form-giver of the marvellous figures in fossils, so that you might be able to touch it, if you are unable to make it out. In fact, light is always conjoined to the animate faculty, so much so that the foremost among the physicians compare the source of life in the heart of an animal to a flame.

So, this animate faculty in the sun, the producer of light, although it influences the whole, being poured through the whole body (not as being about to consume foreign matter, but as being able to give form and protection to its own), it nonetheless will fix its seat chiefly at the center, and by the principles of chapter 1 will spread light from the center to every body. And since the lines that are drawn from the center are perpendicular to the surface, the light will therefore everywhere rebound from the concave surface, and will be gathered again at the center, and, passing through the center to the opposite surface, will reflect the same rebounding by 18 of the first chapter. Therefore, whether the entire body of the sun be everywhere equally inflamed by its soul from the beginning, or more at the center, there is always much of this immensurable configuration at the center. This is because here is no opposition of a lower medium occurring outside preventing something from rebounding inward, which suspicion I removed in ch. 4 sect. 5 through the example of the mirror.

6 The “physical art of astronomy” was first presented in chapters 33–34 of the Asteonome nova (1609), which is subtitled “physica coelestis.” See also Kepler’s Preface, p. 1.

7 The sun’s body is broader than ours, though our world, too, originally just meant “something dug up.” The Bolker Bad is near Göppingen in Württemberg. Its fossils are described by Kaspar Bohn, Floraes novae fontis helveticae Bolmer (Monthemard 1598; German translation: Stuttgart, 1682), Book IV.
Nevertheless, the greatest part breaks forth through the surface into the open
aether, on which note the following, point by point. First, because we attribute
the highest degree of density to this body, all refractions will be right on the
perpendicular, by the discussion in ch. 4 sect. 6 prop. 7. And because by Vitello
X.9,3 a form comes forth from a dense medium to a rarer one in the same lines
by which it enters into the dense from the rare, while the entry, as has just been said,
is made only by perpendiculars meeting in the center, therefore, no radiation will
occur from any point existing within the sun's body (even though it radiates in an
orb) except by perpendicular lines.

In turn, because to one point of a denser medium an infinity of straight lines
can be drawn from the rarer medium, following an infinity of inclinations, while
in the medium of highest density all are refacted into that one perpendicular,
which is drawn from the selected point to the center, therefore, by the same
method of reasoning, even though the center of the solar body is hidden within
the body, nevertheless, after coming out by a straight line to some particular point
on the surface, it will be spread from that point through the whole hemisphere,
and thus will be spread through the whole hemisphere an infinity of times by
the infinity of points of its surface.

But the points outside the center, if they themselves also maintain some light
from the form given them from the soul, will not be spread outside in any other
line (as was said previously) except that which is drawn from themselves through
the center. And thus the hemispheric radiation of any point whatever other than
the center into the world coincides with some one of the infinite hemispheric
radiations of the center. Nor will there be any privilege in the closeness of the
center, and the single center will be equivalent to the entire body. Indeed, as
the form given the center is more sublime than that given the rest of the body, so
is the center's radiation also stronger at a certain point of the surface than that of all
the intersected points of the body.

One might almost say that it is the center of the solar body alone that arouses
such surges, and should occur to the mind to think about what it is that you are
perceiving when you look at the sun's body, do not believe that you see only the
surface; a force of such magnitude does not inhere in the surfaces of bodies, but
lies hidden in the depths, as in the magnet; in fact, in ch. 1 prop. 15 above, we did
not even derive color from the bare surfaces. But you, when you are looking at
the sun's body, know that you are perceiving the center of the sun under refraction
everywhere on the whole perceptible surface, as it is that whose illumination is
strongest. If it were not so strong, you would indeed make out something of the
sun's body.

The center is therefore that from which light originates; the surface, that
which receives thence force from the center, and dispenses it to the whole world, and
that which, in itself, because of its narrowness, was going to escape the eyes; the
surface spreads it out and places it before the eyes; and finally, the intermediate
part is that which, for the center, mediates the passing out to the surface, and,
for the surface, mediates the force of spreading light through the hemispheres.

3 Theocritus B pp. 413-4.
Take note of these and compare them with divine things and with what I said in
the preface to the first chapter. You will see in the sun a palpable image of the
world, in the world that of God the Creator. But although from every part of the
surface everywhere one center shines forth, it is nevertheless not brought about
that the force of illumination is everywhere equal. For from the middle point
perpendicular light goes out, and strikes more strongly, [while] from the sides,
being refracted, it is weakened. For this is an inherent property of refraction from
the theory of optics. Further, a larger part of the sun also acquires a greater force
of illumination; the smaller part acquires a smaller force.

2. On the illumination of the moon

This was the most ancient part of astronomy, to look into the causes of
something obvious to everyone, namely, how it came to pass that in its monthly
circuit the moon waxes and wanes, and takes on various shapes.

Berosus the Chaldean, as Diogenes Laertius reports, taught that half of
the moon is luminous, or, as Cleomedes reports, is half fiery, being deprived of all
light from the remaining part; and that by the rotation of the globe it happens that
the luminous part hidden beyond the body gradually comes out more and more, 4

Thales of Miletus gives a better opinion of this, and was the first to say
what is in fact true, that this whole hemisphère is illuminated by the sun, and the
moon does not give light by itself in any part. Plutarch attributes this opinion to
Aristagoras. 5 Berosus was able to conclude what lies before the eye, or rather,
to conclude from that, that it is not always the same parts of the moon that shine.
For the face which we imagine for ourselves in the moon from the arrangement
of the spots—struggles out successively into the light, and, always motionless, is
turned downwards towards us, granting to the light a passage above itself. There
is at the western edge of the moon, a little above the eye, towards the zenith, a
very black spot, like a pebble, in medium and bright light, separated from the other
spots the way an island is from the continents, distant from the extreme limb by
barely a digit. 6 You will see this there, that is, at the western edge and somewhat
towards the zenith, whether the moon is at half, or gibbous, or entirely full. Thus
the circle of illumination first touches that spot, then goes across more and more,
and after the full moon leaves it altogether. Consequently, it is not the same parts
of the moon that are always in the light. 7 Berosus thought, but the same parts
of its body are forever turned towards the earth, the center of its gyrating motion;
however, in going around the earth, it presents one and another part to the sun, as
if it were to be turned by itself as 'on a spit, roasting itself every month towards

6 Berosus, who was a Babylonian priest, was not mentioned by Diogenes Laertius. Since
Berosus lived in the third century B.C. E., there can be no question of Thales having
informed upon his opinion. The passage from Cleomedes is from De moone circulari

5 7 The spot is evidently the Mare Humorum. The digit is a measure used by Ptolemy to
estimate the extent of eclipses; it is one sixtieth of the apparent lunar or solar diameter.
See the footnote to Ch. 2 Prop. 12.
the sun, in very nearly that manner in which Copernicus said that the earth is 
turned and roasts itself daily towards the sun, as towards a fire.

Even though this opinion of the illumination of the moon is certain and 
beyond any chance of doubt, nevertheless there is no lack of those who take it 
upon themselves to attack it using opacal arguments. The opinion of these people 
is fully argued in Plutarch, On the Face; and is repeated by Cleomedes, book II  
(although Scaliger would attribute this same instance to a certain Arab, the son of 
Amram).12 If, they say, the sun communicates its light to the moon, that will be 
diverted to us by reflection, so that we would not see the light of the moon, but, 
with eyes directed towards the moon, would take in the rays of the sun itself. This 
being granted, it will follow that the moon is a convex mirror. As a result, we will  
see the image of the sun in the moon. Besides, it will not be possible, whether in 
the full moon or in the halfed body of the half moon, that we could see the rays 
of the sun all around it. For the law of reflection is this, that it is generated solely 
by that point of the body where the angles which the rays of both incidence and 
reflection make with the sphere can be equal. And so, we neither perceive the 
sun's image in the moon, nor is the latter illuminated in that one particular place 
in which there can be an equality of angles, but it sometimes appears in 
its full body, sometimes halved, and sometimes harned at its extreme edges; they 
think it follows that the moon does not shine with rays coming from the sun.

This instance will be able to do the job for one who distinguishes absolutely 
all light into direct, reflected, and refracted; he does not acknowledge a fourth. 
It is as a result of this that Cleomedes, reduced as it were to narrow straits, 
concludes wrongly that the moon too has its own light which is aroused by the 
solar light, so far as it is touched by that light. Plutarch, on the other hand, seeks 
out another illegitimate way to escape. But let, by very evident reasons given above 
in ch. 1 prop. 22, have introduced a fourth form of light, arising from reflected 
and refracted light, to which have given the name of communicat light.

As a consequence, even though it is very truly established by the reasons al- 
ready presented that the moon does not make use of light simply reflected from 
the sun, there still remains this fourth form of communicat light, to be at-
tributed to the moon. For the account of that light which descends from the moon 
us, having arisen from the sun, is note other than the account of that which, 
descending from the same sun upon any wall in its way, radiates to the whole 
hemisphere, and, whatever darkened Camera may be set up opposite, it enters 
into it by the open side, and depicts itself with its color upon the opposite white 
wall, as has been demonstrated above in ch. 2 prop. 7, in view of this I bring 
back those words of Plutarch which he finally appeals to the rest of his useless 
excuse, saying, "the moon has many tough places, many irregularities of shape, 
so that the brightness that descends from the great body may shine brightly with 
light reflected by the not insubstantial heights, and be beneficially reflected, in-
ternally, and extend the reflected brilliancy between itself, as if it were borne

to us by many mirrors. By these words, he has described very nearly what I am accustomed to calling communicated light. Cleomedes, however, does better, in adding this necessary condition that the solar light is not just reflected by the moon, but is also slanted at the moon, as the brilliance of fire in iron and thus is made its own.

And from this it has now been deduced that the body of the moon is dense, as is the earth with which Plutarch asserts with many arguments in the book frequently mentioned, and is also heavily colored, as, with this removed, communicated light has no place (according to prop. 22 of ch. I), and with a rough surface, so that it will not be paludal. For all the instances sought from all sources bear witness that the moon's body is opaque.

The opinion of Posidonus, as reported in Macrobius, is also proved from this evidence: Renhold recalls this on fol. 184 of his commentaries upon the theories of Porfich. While he correctly attributed to the moon the same matter which the earth also comprises (which is fast appears to have been the most ancient opinion of the Pythagoreans; nor perhaps, did they mean by the "An-fichron" of theirs, cause of eclipses, which was given a futile refutation by Aristotile, anything other than the moon, 15 he nevertheless intended it to differ from the terreone globe in that the moon, like a mirror, sends forth again the light that is received from the sun, while the earth, overspread with the sun's rays, only becomes bright, and does not give out light. In fact, however, the account of the earth and the moon is the same. For the moon is not a mirror, as Plutarch and that Arat and Cleomedes correctly proved in refutation above, and it is false that the earth does not shine with communicated light, which is manifestly proved in ch. 2 prop. 7 by the instances of all walls, with Plutarch adding the instances of clothes. If Cleomedes suffers that which he himself celebrates: the moon's proper light, to be called "color", which I have defined above as "light buried in matter", he speaks, completely with me in the text, and reaches the same conclusion with me, finally attributing, with me and against Posidonus, proper colors to both the moon and the earth, which, aroused by the sun's light, radiates on half the orb, so that in that matter of resemblance the earth and the moon are equivalent. So that the astronomers are perfectly certain about this illumination of the moon, it comes from the sun, and that there the moon's body itself shows itself to be no different from any wall that is in the way of light.

Once this foundation is laid, astronomers now derive from it various ways this illumination occurs. They argue, first, that since the sun is of spherical form,

229

24 Georg Porfich, Theoricae novae platonicorum (Weinigenberg 1552 and other editions). Later editions incorporated revisions, the edition Kepler used, which he cites by page number, is that of Weinigenberg, 1553. There is also a Paris 1555 edition, in which the cited passage appears on fol. 103v.
25 The Anfichron, or sun's earth, was discussed by Arnocle in On the Heavens II 13, 520A 23 ff., and Metaphysics I, 5 986b 12.
the moon likewise of spherical form, but smaller, the moon is therefore illuminated over more than its half by the sun, and the boundary of the illumination is a circle; but one that is less than that which is the greatest described upon the spherical body of the moon, by Wirtelo II 2317. Now, from this careful preparation, the distances and diameters of the two celestial bodies being known, they show how to investigate the size of the circle of illumination.

3. On the circle of illumination of the moon and the earth

Knowledge of this will be useful to us later in the problems. Reinholt, in his most erudite and worthy commentary on the Theories of Pehrbech, which should be read carefully by those who study celestial subjects, gives this quantity for the circle of illumination on fol. 165. 15 Let ABC be a great circle of the moon's body, drawn through the pole of the circle of illumination. Let the pole or the middle of the illuminated part be A, let FG be another great circle from the pole A through D, the midpoint between AE, parallel to its circle of illumination BC cuting the globe of the moon below the center D. Now Reinholt gives the quantity of arc BAC as 181° 45', but according to Ploenter, the moon at apogee appears equal to the sun. 18 Whence it happens that the circle bounding the vision coincides with the circle bounding the illumination, if it should happen that the moon run precisely beneath the sun. Therefore, at the same time, Reinholt gives the remainder from CAB, namely, the arc CEB, which measures the part swept through the middle, 178° 15', so that the two together make up 360°. However, the other foundations upon which this computation of the most learned gentleman rest, are unclear. The quantity itself (I would say for the benefit of the art despite this man) is in error, which I demonstrate thus. Since Reinholt, from Ploenter, assumed a distance of the moon from the earth at which the body of the moon is viewed in the same visual angle as the sun, let the straight line CH accordingly touch the body of the moon, at the point C of the circle of illumination CB, and let the axis of illumination AD be extended until it meets CH at H. Now, because CH is tangent to the moon at the circle of illumination, it is also tangent to the illuminating sun, whose center is by supposition high up on the axis of

17 Phenomena II p. 7.
18 Georg Pehrbech, De motu stellarum, in the Paris 1553 edition, it is on fol. 164c.
19 Almayer V (4), trs. Tourret p. 255.
illumination DA: and this is by Wisdom II.27.28 And because the sun and the moon are viewed under the same angle, while the line AD through the centers is one, and the line CH to the edge of the two luminaries is one, and further, the angle CHD is also one and the same, CHD is therefore the same angle under which the semidiameter of either body is viewed. But, from Polenry, the semidiameter of either body is 15° 40', as Reinholt himself assumes, on p. 209 of the Book mentioned.29 Therefore, the angle CHD is 15 minutes, 40 seconds. And because FD is perpendicular to DM by construction, while CH is tangent to the circle at C, whence DC from the center D to the point of tangency C, is perpendicular to CH, therefore, FDH and DC will be equal and right. Next, in right triangle DCB, the remaining angles CBD, ADAH together are equal to the right angle DCB. Moreover, CDH, CD together are equal to the right angle FDH. Therefore, the common angle CHH being removed, the remainder FD is equal to the remainder CDH, and consequently the arc FC is 15° 40', while GB is equal to it, and the sum of the two is 31° 20'. And FAG is a semicircle, or 180°. Therefore, CAB is 180° 31' 20'', and the remainder CEB is 179° 28' 40''. This is the greatest quantity of all of the arc CAB, that is, when the moon is both at apogee and full. For consider that E is the vertex of the moon's shadow, and likewise also the place at which the sun and the moon are viewed under an equal angle. And since a longer shadow of the same globe becomes more acute, if, that is, the globe to be illuminated by a larger illuminating globe recedes to a greater distance. Therefore, since the moon has receded further from the sun in a straight line, the vertex of the shadow will also be more acute there than it is the angle of vision at apogee of 31° 20'. As a consequence, when the moon is set at perigee, of the same sense of the word 'perigee,' it has come closer to the earth, and has correspondingly receded from the sun, it will bound the shadow with a more acute angle. Again, when the moon is full, when it is nearly 60 semidiameters of the earth further from the sun than the earth is, while at new moon it had been the same number of semidiameters closer to the sun than the earth was, it will close the shadow with a much more acute angle, and therefore the arc FC will be much less than 15° 40'. And this is following the opinions of Polenry, which, in accordance with Reinholt's views, we have followed here. But it order that we may know the magnitudes of the traces between FC or GB, when they are at absolute minimum, let DHB accordingly be extended to I, and let H1, HD be made equal (ignoring for now the semidiameter of the earth), so that the moon may again be at apogee, and, in turn, full. In this place, let the straight line IL touch the moon and the sun, meeting with HC extended at L, and let JK and QJ be extended to their meeting at M, and let HL, from the opinions of Polenry, be 1210 semidiameters of the earth, while HC and HK are 64, so that CK is about 128, and CH being subtracted, CL is 114.25 For here there is no need for precision. Therefore, in triangle LCK, the sides LC, CK are

27 Theastraeus II.71.
28 Portus 1553, Int. 136
29 The moon's distance is calculated in Armoyou V.3. In: Tootorp pp. 247-51, the apparent diameters, in V.4, pp. 254-5, and the sun's distance in V.2, pp. 245-7.
given, as well as the angle LCK, for since CK and DH are parallel, DHC and HCK will be equal; consequently, LCK will be the supplement, 179° 44' 20", hence, the angle C.LK comes out to be 1° 32'. Noting in triangle HLM, angle HLM is 1° 34'. But the exterior angle LHR or dHBo is equal to the interior and opposite angles HLM, HML. Therefore, HML, half the angle of the sharpness of the full moon's shadow, is less than L.H.D, half the sharpness of the new moon's shadow, by the space of HLM, 1° 34'. It will therefore be 14° 6', twice which is 28° 12', as a result, CAB will be 180° 28° 12', less than before. But the arc C.B.R, measuring the circle of vision CB, when the moon is again at apogee and full, remains the same as before, namely, 179° 28° 40', if indeed it is true, as Ptolemy taught, that the apparent diameters of the new and full moon are equal.

![Diagram]

On the earth's circle of illumination

Since the full moon measures the earth's shadow by twice one and a half of its diameters, the earth's shadow will be more acute and longer than the shadow of the full moon; therefore, the circle of illumination will be closer to a great circle. Whetelo shows how to investigate this in X V 59, in approximately this way. He assumes, from the astronomical writers, that the distance of the sun from the earth is 1,110, where the semidiameter of the earth is one part, while the semidiameter of the sun is 1/4 of the same parts. In the above diagram, let N be the center of the earth HR. Let the straight line RQ be tangent to the bodies of the sun and the earth, and let perpendiculars NR, OQ be drawn to the points of tangency R, Q from the centers. Finally, let NP be drawn from N parallel to RQ cutting OQ at P. Now since OQ is 1/4 where NR is 1, OP will be 4/5 of these parts, and ON 1.210. Therefore, in the right triangle ONP, the sides OP, ON are given, as a consequence of which the angle ONP, 15° 48", will not be hidden. If QR be extended, it will meet ON in the same angle, forming the boundary of the point of the shadow. Therefore, the angle of the shadow is 25° 36'. And

---

23 Duplo se suspemplersee diamettr; I have rendered this literally because it is so odd. In the Almagest V 4, tr. Toomer p. 251, Ptolemy calculates that the shadow's diameter is about 31 times the size of the moon; however, according to Taylor Bache's Priorastronomy of A.B. 10 p. 135, in TBOO II p. 148, it is larger, averaging almost three times the moon's diameter. However, on p. 251 below, Kepler says that the shadow's semidiameter is 21 times the moon's semidiameter; perhaps that is what he means here.

24 Thesauri II pp. 451-2.
further, the earth’s circle of illumination will cut the great circle through the axis of illumination in an arc of 180° 25′ 36″.

4. On the phases of the moon

The manner in which it now happens that, as a result of the rays coming from the sun, the moon first appears humped or sickle-shaped, afterwards bisected, then gibbous or convex on each side [amblykystos], and finally flat, and thence in the opposite order gibbous, bisected, and humped, and finally hides itself again beneath the sun’s rays, Reinhold has sufficiently shown from Witelo, and the books on Spherics repeat it everywhere. The summary in Witelo is briefly this. In Book 4 Prop. 65 he proves that the moon’s disk necessarily appears flat (which same thing Aristotle also [proves] in ch. 7 Section 15 of the Problems); in prop. 66 and 70, [he proves] that the moon appears less than a hemisphere; and that the boundary of vision is a circle, [he proves] in prop. 67. [He proves] that the matter the moon gets to us, the greater it appears but the less is grazed than the real thing by the vision, which Witelo borrowed from Euclid’s Optics. Now, in prop. 74 he takes up the phases themselves, and first, concerning the full moon, he argues that it appears full when the vision is between the sun and the moon, and the circle of vision is entirely contained in the illuminated part, or is tangent to the circle of illumination. In Proposition 75 he demonstrates that when the circles of illumination and vision cut each other, while the axes make an obtuse angle, the moon appears gibbous. In Prop. 76, that when the axes cut each other at right angles, the moon appears bisected. In Prop. 77, then when they cut each other at acute angles, the moon is now placed almost between the sun and the vision, or approaches nearer to the sun [than the earth], and appears sickle-shaped. These things, wildly demonstrated by Witelo, do not require many words. I would only add a few short notes. To Prop. 65, therefore, Reinhold makes the following remark. When the sphere of the moon is narrow, the moon is very close to us, “there must appear in the body of the moon,” not just a plane, but “something swelling and protruding, somewhat brighter, from the middle of which, to the extreme circumference, certain spotted parts, like cracks or

25 Reinhold proves this in an exhaustively on ed. 108c-110d of the Paris 1553 edition. The books on the ‘Sphere’ were introduced at the university courses of the universities. They mainly presented the fundamentals of cosmology, together with trigonometric spherical geometry. The foremost of these treatises was the thirteenth century Spherae of Johannes de Sacrobosco, which was widely used well into the 17th century. The relevant propositions from Witelo are IV.24-25, in Theoricae II pp. 350-2.


27 These elations are summarized in the latter part of Prop. 77, Theoricae II pp. 151-2, which is in effect a cardinal to Prop. 74-77 pp. 150-11.

28 Peterbach, Theoricae et Definitiones Physicarum, 1553, ed. 165°, Paris, 1553 ed. 105°. Kepler has made a few significant changes in quoting Reinhold’s text below.
fissures, run out. That is to be credited to Reinhold’s sharpness of vision, which Plutarch appears to complain of in the book, On the Face, in these words. “Dark spots appearing on the moon, sort of like inhomogeneous dividing the shaded parts from the bright, are so distinguished that, separated from themselves and their surroundings, they are circumstancesized, and the penetration of the bright parts into the shaded ones makes the figure of a kind of weaving.”

5. On knowing the age of the moon from the quantity of the phases

Now, as regards the moon’s phases themselves, Reinhold again remarks that the increase of light approximately corresponds to the moon’s departure from the sun, which is borrowed from a certain obscure passage in Pliny, but it is meaning being hinted out by conjectures. Pliny seems to me to speak somewhat differently than he seemed to Reinhold, with the gist of his opinion nevertheless leading towards this same axiom: I shall therefore first explain Pliny’s passage. He says, “the moon, shining, adds, three quarters plus twenty-fourths of the hours” (accumulating them to its bright part) “from the second” day of its age; for before that it is hidden or dormant “to the full orb, and taking away” from the full orb “in waning,” and finally to total extinction. That is, because at opposition the moon comes to mid-sky many hours after the sun, and further, the whole body of the moon, like the planet, is itself also divided into twelve parts or digits; therefore, however many hours far moon is distant from the sun, it shines with that many twelfths or digits. But since from its first rising a is filled, not in twelve, but in fifteen days, it is therefore not a whole hour farther from the sun for each day of its age; nor does it add a whole digit to the luminous part, but three quarters plus twenty-fourth, that is, 45 minutes plus two and a half minutes, 47 and a half minutes in all, for one digit, because it departs 471 minutes of an hour from the sun daily; for, when 12 hours are distributed among 15 days, 48 minutes come to each. However, a more precise denomination than three quarters plus twenty-fourth did not occur to Pliny. And so Pliny also gives the farmers and the household heads, a rule for finding the age of the moon, that is, its departure from the sun, from the breadth of the shining horn, in its mean motion. If anyone should wish to follow this rule exactly, Reinhold warns that it

29 Plutarch, On the Face in the Moon, IV, 921nd., trs. Cherniss pp. 123, 124. Keplcr’s Latin depends considerably from Plutarch’s Greek. Cherniss’s translation should: “The dark spots in the moon do not appear as one but as having something like inhomogeneity between them, the brilliance dividing and delimiting the shadow. Hence, since each part is separated and has its own boundary, the layers of light upon shadow, acquiring the semblance of height and depth, have produced a very close likeness of eyes and lips.” No significant textual variants are noted.

30 The Latin is “vis,” which can be either a unit of money or penny or a unit of weight.

31 Keplcr regains this more clearly in his principal astronomical Cosmographicus Viti-Humor (1625) VI 5 3 p. 839, in DGW vii pp. 178-9 as follows: “However many days there are in the age of the moon, it will shine three fourths plus one twentieth of the times

235
is somewhat in error. We shall demonstrate both its closeness to the truth and its slight error by mean of the following diagram.

About center $A$ let a great circle $BCD$ of the moon's body be drawn, $BAC$ being its diameter and $EAT$ at right angles. And let $E$ be the center of the sun; therefore, the circle of illumination will be parallel to $BAC$, let it be $FG$. Now let a point $H$ be taken not on the line $EA$, and let this designate a point upon the earth's surface at which the observer stands, and let the line $AH$ of the axis of the moon's circle of vision $KL$ be drawn, and let $K$ be joined. Now, since in triangle $AEH$ the side $AE$ is about twenty times as long as $AH$, the angle $AHE$, determining the departure of the moon from the sun, will be much greater than angle $AHE$, while the two together are equal to the exterior and opposite angle $TAH$. Therefore, $TAH$ is certainly greater than $AHE$, the departure of the moon from the sun, but only slightly greater. But $TAH$ is the angle by which the axis of vision $AH$ and the axis of illumination $AE$ are inclined, upon which angle the breadth of the shining part at the horn is consequent. Therefore, the breadth or increase of the shining part at the horn does correspond approximately, but not completely, to the departure of the moon from the sun. Reinhold shows that the difference at the quadratures, where it is greatest, is three and a half degrees, more or less.\(^{32}\)

If it be desired to pursue all the subtle points, there is another thing wanting in this procedure, namely, that the circle of illumination $FG$ is not often equal to the circle of vision $KL$, but is generally smaller. Therefore, as a result of this cause as well, the ratio between the increase of the shining part and the departure of the moon from the sun is somewhat perturbed.

But even if the ratio were not at all perturbed, neither by the latter nor by the former cause, the sense of vision would nevertheless not be free from error if it should wish to reckon scrupulously the days of the moon's age, or the

that number of hours (of which any night has twelve) until the time it sets, and this holds from the new moon to the full moon, or to the moon's age of 15 days. But from that time, however many days are added to the moon's age above 15, three-quarters plus a twenty-fourth of the same number must again be subtracted from the number of hours 12 accumulated with the age of 15 days. Three fourths of an hour is 45 minutes, a twenty-fourth is $\frac{5}{2}$ minutes, the sum is 47\(\frac{1}{2}\). This taken fifteen times makes nearly 12 hours.

\(^{32}\) Theoriae naturalis (Paris 1553) fol. 110v.
departure from the sun, from the three quarters plus twenty-fourths of the breadth of the shining part at the horn. For even though the digits withdraw within the boundaries of the sense of vision equally, nevertheless, to the degree that they are closer to the circle of vision, they appear so much the narrower, and to the degree that they approach the center of the lunar disc more closely, appear so much the wider. The demonstration of this fact is very nearly the same as the one that shows why the chords subtending arcs of a circle do not come out proportionally to the arcs, but to the degree that the arcs are smaller, the chords subtending them are so much more nearly equal to them. and since at the beginning of the quadrant 2099 small parts of the diameter add one minute, while at the end, all of 13 degrees 31 minutes correspond to the same number of small parts. Thus, that is, the whole visible hemisphere of the moon appears flat, as we had said above from Vitruv IV 64, and further, any of its semicircles drawn through the side of the visible hemisphere appears to be a straight line. From this it happens that the parts of the circle correspond to parts of a straight line; and for equal parts of one straight line, upon equal arcs, those are wider that are present to the vision directly, and narrow that withdraw to the parts of the globe that slope away towards the boundary of vision.

Moreover, this is the cause for the very slow appearance of the first increments of the waxing moon, and of the last increments of the waning moon, while when the moon is divided in two, by the careful viewing of the division of the moon’s face, you may perform a judgement of the true quadrature within a few hours.

In 1062, on the evening of November 12, December, when I had not looked up the hour of the moon’s quadrature, the moon appeared to me still conic at six and one half hours after noon. And indeed, it had now come to the boundary of schismotomy. For the arc’s position was 29 30’ Sagittarius. The moon was clothed in a narrow half, such that the diameter of the moon might be placed between it and the edge of the moon, and I had proved through a certain small star in the constellation of Pisces, so that this star was embedded in the halo.

The error in this statement evidently arose from Kepler’s frequent use of different numbers of sexts for the diameter at opposite ends of the table. The correct arc should have been 1 25’, not 13 55’.

Theorem 11, pp. 246-7.

16. The 1852, 1881.

17. Here, in the marginal note, Kepler refers to the following enodice:

To the margin of p. 297. I do not pursue this suspicion of a new phenomenon really, or without precedents. For it seems to be not so rare that, like comets, stars are also seen wandering in this way. Indeed Fabricius sent Tycho Brahe certain observations made in Friesland, having acquainted the distance in Mercury from a certain bright star in Gongs, which could no longer he found either by Fabricius or by anyone else. (Translator’s note: This is the first recorded observation of a Ceti or Oeta Ceti, a true variable star with a period of 3.5 days. It was reintroduced by Johann Phaethon Hoham in 1600, the first instance of a supposedly new star that had disappeared and then reappeared. Fabricius’s observation is in TRIO XIII pp. 114-5.) Similarly, Jonas Biring, the Langgrav’s maker of mechanical devices, when he was compiling
6. The paradox that never ever was there a real new moon

Now, as regards the phase itself of the full moon in particular, Wiinio himself, and Reinhold after him, advises, in the explanation of Prop. 74, 29 that this phase has its own breadth of time. That is, because at full moon the sun illuminates 180° 28' 12" parts, while we see the moon at apogee under an angle of 36°, at perigee a few minutes greater, and the arc of the great circle through the pole of vision is that much less than a semicircle; when the former excess and the latter defect are added, the total is about one degree. And so beneath the most exact instant of opposition the two circles of vision and of illumination are everywhere distant by about 30 minutes. Therefore, the phase of full moon lasts until the moon completes one degree; that is, for about two hours; this is, from that time when the circle of vision is tangent to the circle of illumination at the moon's eastern part, to that point where the tangency occurs at the western part. This is the reasoning of the authors just cited.

However, since they are splitting hairs, 30 here, I should also be allowed to split hairs in contradicting them. Let them hear this new and marvellous voice, which I build upon those foundations of theirs: a perfect full moon has never been seen nor can any ever be seen. So far from one full moon's lasting for two hours! For because the semidiameter of the shadow is equal to two and a half semidiameters of the moon, when one semidiameter of the moon is added, so that the centers of the moon and the shadow allow the right amount of space outside the shadow, about 64 or 66 minutes will be used up. That is the amount that the moon has to be distant from opposition to the sun, whether in longitude or latitude, unless one wants it to have a taste of the shadow. But at so great a distance from opposition, the circles of illumination and of vision now have come to intersect, because they are tangent to each other at a distance between the moon

a celestial globe with the most notable fixed stars and was inauguring it with the heavens themselves, found a certain star in Antinous, overlooked by Ptolemy. But of the one which today is shining in the breast of Cyprus, he steadfastly asserts that it was not seen at that time when he was occupied along with the Lord's grace's staff with the description of this star. In fact, Tycho's observers likewise did not note it (while simple mention of the one that was seen in antiquity in the breast of Cyprus, which was solitary, was made six hundred times, though this new one was close to it in both position and magnitude; nor did Ptolemy and Gerber note it in their most accurate description of the Milky Way, while this new star is nevertheless located in a very obvious position at its edge.

30 This is Book IV Prop. 74, Nævius II p. 150.
31 inqjdlbodqieim.
and the point opposite the sun of 30 minutes. Therefore, the moon either cannot be in the full because of northern or southern latitude, or if it can be made full by traversing the points of opposition, it begins to be eclipsed before it becomes full, passing through the earth's shadow. Unless perhaps when it is off to the north, the southern parallax gives it support, or the contrary.

And so much for the full moon. Very nearly the same things might be said of the new moons. For if wherever place the moon passes by the sun is not considered, at new moon, the horn has a remnant, for the causes just stated. In fact, at an exactly central conjunction, if it happen to appear less than the sun of which below, the circle of illumination will descend within the circle of vision, and (if the sun should meanwhile happen to be hidden from our vision) the edge will seem illuminated circularly. But now, as I said by way of preface, I am splitting hairs with the authorities. For the fact of the matter is that this defect in the new moon is extremely slight. You will easily understand this from what I just said, that those parts that slope away near the circle of vision are imperceptible, even though they be fairly large. For let the distance of the luminaries be 179 degrees, and let there correspond that a whole degree upon the great circle of the moon's globe drawn through the poles of both the circle of illumination and the circle of vision, by which degree the two circles shall have overlapped each other. Now when 90 degrees is subtended by 100,000 parts, 89 degrees is subtended by 99,983. Therefore, the remaining degree will be subtended by the remaining 15 parts, which is barely the seventh thousandth part of the moon's semidiameter. From this it appears that there is no danger to the observations when the moon's altitude at the moment of opposition has been investigated by the highest and lowest edge of the moon, even if, because of northern latitude, the moon were not yet completely full from the south, or the opposite, I would not deny, however, that this cause makes the beginnings and ends of eclipse obscure, just as they usually are very doubtful, because at the part where the moon usually enters the shadow, it is usually still more demured of light because of the intersections of the circles of illumination and vision. Therefore, in the neighborhood of the shadow, where a very narrow part of the sun's body illuminates this edge of the moon, the light of the moon correspondingly also grows very faint.

7. On full and partial illumination of the illuminated bodies, and the earth's penumbra

For it is an additional consequence that on that side of the moon which is just about to have a taste of shadow, the circle of illumination is broken, and withdraws within the circle of vision farther, and for this reason is less distant from the spot of illumination, than the great circle. This I demonstrate thus. About centers A, B let great circles be described, CD of the sun, EFGH of the moon. And let the straight line DE be tangent to the right sides of the circles, CG to the left sides. And again, let the straight line DE be tangent to the right side of the sun and the left side of the moon, the straight line CF be tangent to the remaining sides, sitting DE at point J. Therefore, whatever is between the points EF is illuminated by the whole visible hemisphere of the sun. But
for the inquiry’s sake, let it be called the “full illumination.” \(^{41}\) Let \(EF\) and \(GH\), representing complete lesser circles, be joined. Therefore, whatever is between these two circles is indeed illuminated by some bit of the solar body, but no point of this edge is illuminated by the whole shewable body of the sun, insofar as something of the sun is always missing, and is hidden behind the body of the moon. Let this be called “partial illumination.” \(^{42}\)

Should you understand \(EFGH\) to be the earth, the same theory would apply, and when the lines \(DE \), \(CG\), \(CF\), \(DH\) are extended somewhat to \(K\), \(L\), \(M\), \(N\), respectively, let that which is between \(KL\), \(NM\) be called the penumbra, but let \(LN\) be called the umbra. Therefore, since the parts \(C\) of the sun illuminate the moon \(GE\) all the way to \(F\), while the parts \(D\) illuminate \(EF\) all the way to \(H\), it is evident to the eye that when point \(D\) is blocked, point \(H\) would not receive any light, and when the parts \(DA\) towards \(C\) are gradually blocked by the interposition of the earth, the light will be gradually extinguished from \(H\) all the way to \(F\). As a consequence, the circle of partial illumination \(GH\), which in the preceding was the circle of illumination, will be broken from the side \(W\) and will withdraw towards the circle of full illumination \(F\), doing so long before the eclipse begins.

But to investigate the quantities, let us again suppose what Wieldo and Reischold supposed, that the sun’s distance from the earth is \(1210\), the moon’s from the earth \(64\), and that of the full moon from the sun \(1274\), which is the line \(AB\). \(^{43}\) Further, let the semidiameter of either luminary appear to be \(15° 40’\). It is evident without calculation that the point \(I\) is going to be in the earth itself.

For if at true opposition \(EF\) and \(CD\) be equal, \(I\) representing the observer, then \(EID\) will be one straight line, and we will \(FIC\). But they should also be straight lines by construction. Therefore, \(J\) is the earth, and \(E\) is the circle of full illumination; \(I\) is also the circle of vision. If that, it is, were possible that the luminaries be truly opposite without darkness. Hence, as before, \(EG\) or \(IH\), an arc of about half a degree, is given. Accordingly also, when the circle

\(^{41}\) illuminated: Although the word which also appears in the title of this section is used in Plinšs philosophorum to mean “meeting of rays” (cf. LSS p. 1700 and Plinšs philosophorum IV.13.11), it appears that Kepler has in mind the root meaning of the components: rov : “complete” + σηρη : “light of the sun.”

\(^{42}\) illuminated (Also in the section title). This as it stands is not a classical Greek word; however, there is an adjective form, ἱλατηρικός, which means “deserted by light,” dark.

\(^{43}\) Theorema novum (Paris 1555) fol. 110r. The figures ultimately derive from Poleni’s Almagestum V 15, tr. Toomer pp. 255–7.
of vision FF is inclined so as to be tangent to GH at H, the portion FH extinguished before the beginning of the eclipse, as before, will hardly appear to be a forty-thousandth part of the moon’s semidiameter. I wished to add this so as not to strike fear into the less experienced by this pruning back to the quick. For at the beginning or end of an eclipse there is no danger, even if the circle of partial illumination be broken before its tone, and perhaps vanishing completely and gathering entirely within the circle of full illumination; this is only useful in providing causes of that matter concerning which astronomers are now already in agreement, namely, why it is that the beginnings and ends of eclipses of the luminaries are so doubtful and untrustworthy, and why the light of the moon at the edge of the shadow appears so pulsed and as if diluted by water, to such an extent that Wiele, in Book 4, prop. 77 asserts that the moon often seems to be partly eclipsed without having entered the shadow at all.\footnote{Theorms I pp. 151–2. This is not quite what Wiele wrote: what he actually said is, however, the light is somewhat darkened as the moon approaches the shadow.}

Let us, however, also explore for the earth how much the breadth of full illumination and how much that of partial illumination is. As above in Section 3, let R be the center of the earth HB, and on it let RS be the circle of illumination or of partial illumination. It has been demonstrated that when the center of the sun is on the line NH, so that one who is at R sees one edge of it, one at S the opposite edge, the arc RSH is 180° 25° 36'. The arc of full illumination THV is sought. Now since the sun is supposed by Ptolomy to subtend 31° 20' of a great circle, it is evident that one who proceeds from R to T through 31° 20' of a great circle, if he previously at R saw the uppermost edge of the sun graze the horizon, is now, at T, going to see the whole sun. For the sun’s parallax does not confuse this at all, which is the closer the more of the horizon at S for, having gone forward towards V through 31° 20' of a great circle because the arcs on the earth there correspond similar arcs in the heavens, he will see the whole sun, and will be standing at the edge of the region of full illumination. When T and V are joined, this will be the circle of full illumination, differing from the arc RHS by 62 minutes 30'. Consequently, THV will be 179° 23'.

Now, from this, let the following also be investigated: how great the umbra is in the moon’s passage. Let NH be extended and let some line touch the circle at T, cutting NH at X. And let XT be extended as far as necessary, and let TX be joined. TXV will therefore be as great as half of the supplement of the THV, which is 37°. By what was demonstrated in Section 3. Therefore, TXV is 18° 30'. Therefore, when TXV is 1, XX, which subtends the right angle JT.
Chapter 6

is 186. To XY let there be appended the straight line \( XY \), to denote the distance of the full moon from earth, and let it be according to Polenary, 64, as before. Therefore, the whole \( XY \) will be 230. Let the straight line \( YZ \) be erected cutting \( XY \) extended, and forming the boundary of the depths of the penumbra and the umbra at the same time. Therefore, in triangle \( XYZ \), the angle \( X \) is known, and \( Y \) is right, while the side \( XY \) is 250. Therefore, \( YZ \) will become \( \frac{135}{150} \) semidiameters of the earth. \( ZN \) being joined, a new right triangle \( ZYN \) is formed of given sides about the right angle. Therefore the angle \( ZYN \) is given, 1 12° 14" or \( 72 \frac{1}{2} \) minutes, the depth of the penumbra with the umbra. But that of the umbra is about 45°, the difference 27 more or less. For when the umbra shrinks, from the earth’s approach to the sun, the penumbra grows, and when the moon is going to traverse a thicker umbra, it meets with a narrower penumbra, and so, while the moon is not yet ready beginning to be eclipsed, nearly the whole body of the moon is already in the penumbra, and a slim portion of the circle of partial illumination remains, and finally also an extremely thin portion of the moon enjoys the full light of the sun. Whence it happens that as the eclipse begins the moon is very pale, and the brightness usual in a full moon is exceedingly darkened.

It is also worth while considering here whether the moon’s light could appear of equal brightness everywhere. Reinhold had indeed asserted above that it appears brighter in the middle. And reason seems to require that where the light of the sun is more scattered, it is also more attenuated. But it is more scattered near the circumference of the apparent disc, for the account of both vision and illumination is the same, but we said above in Section 5 that the angle under which the body of the moon is viewed be divided into twelve equal parts, a smaller part of the surface is seen by the middle parts of the angle, and a greater by the outer ones. So let the angle of illumination, or the light itself falling upon the moon, be divided into the same number of parts; therefore, the same amount of light will correspond to the large outer parts of the surface as to the intermediate narrower parts, with the consequence that the light will be dispersed more widely in the outer parts, and will fall more obliquely. Finally, since partial illumination surrounds the outer parts, while full illumination covers the middle, the middle will be correspondingly brighter.

But, however, am moved by these arguments not at all, or at least very little. For a regards full illumination, that covers the whole moon in such a way that the circle of partial illumination, even granting an exact opposition without darkness, vanishes in comparison with the rest of the diameter, while in oppositions that are not perfect it is a large extent hidden, and in the place where it contacts the full illumination it hardly differs from it. But even if the sun’s light is more scattered at the outer parts of the moon, it also in turn makes its way into our eyes more densely at oppositions, and indeed under the same angle by which it is distributed over that greater surface. This is confirmed by experience that is less fallible. For when the light of the full moon is let into a camera obscura dark chamber by that arrangement which I have described above in Chapter 2 it displays on a white floor an edge brighter than the middle, because of the spots that extend over the middle. But the moon’s being perceived as brighter in the middle to one looking at it, appears to be an effect of the eye’s sight, which when directed towards the
center is everywhere surrounded by bright parts, but if directed to the edge is now deprived of brightness on one side, while the blue color of the sky rushes in.

But when the moon is already waning, or not yet full, the proposed causes are quite valid, as experience also attests. For the light which forms its inner, someone or gibbous face is very weak and dilated. Because, being spread over a large surface, occupies a greater angle in vision than in illumination in this situation.

8. On the Lines of the Moon's Phases

Albategnius taught how to draw the image of the waxing moon from the known motions of longitude and latitude. That the method was barely dependable, you have seen above. If you have been taught with eagerness for a more reliable picture although you will be asking this of yourself, and which does not just the normal but also the gibbous moon, you will need to know the lines by which the visible shape of the moon is bounded. Of one of these, indeed, the one facing the sun there it is agreed that it is an arc of the circle of vision, but those which face opposite the sun are not arcs of a circle. Wittes said in book 4, prop. 25 that the gibbous part of the moon is irregular in kind, while in prop. 77 he asserts that the horns' shape is contained by two axes of virtually equal circles. Let it therefore be known that each is an arc of the cone section that is called "ellipse." As Aristotle notes in Section 15, problem 4, I prove it thus.

Apollonius defines the cone surface to be this: whenever some line, bound immovably to some point, goes around a circle (the points not falling in the extended plane of the circle) from one part of it, giving a surface, until it returns to that point of the circle where it began to move. Now, indeed, the boundary of the moon's illumination is a circle, as was said above, but our vision fits the ratio of the required point. For it does not fall upon the extended plane of the circle of illumination throughout the whole month, except about each quadrant of the moon with the sun, while the cutting of the moon is also considered to be a straight line; hardly curved at all. Therefore, whenever you look at the moon, you make a cone with the circle of illumination. For vision occurs in straight lines, coming together in the one center of the eye from all points of the visible object.

But it has just been said that the cone of the moon appears as a plane disc, upon which no lines which are drawn from the eye to the center of the moon.

47. The Arab astronomer Al-Battani. See Chapter 1, note 172.
48. Theorems 8, pp. 126, 30-31, 151. In prop. 77, Wittes says, "the shephl seems to be contained as if by two equal arcs, because of the imprecision of their inequality." (p. 151).
49. Aristotle, Problems XV 6, 915b 6-12. There is, of course, no mention of the ellipse in Aristotle's works, as the application of this term to the geometrical figure was first made by Apollonius, about a century after Aristotle's time. See Apollonius, Conics 1:13.
Therefore, an imaginary plane, or rather, the quasi-plane surface of the eye, cuts the cone just mentioned. Now Apollonius defines the ellipse to be this: when a cone is cut by a plane which is neither parallel to the base of the cone, nor placed subcontrariwise, so that the section meets with any straight line whatever that is drawn from the vertex of the cone to the surface; that is, such that the whole cone is truncated. All these conditions are satisfied in this cutting of ours. First, because the imaginary plane is perpendicular to the straight line from the eye to the center of the illuminative circle or base is perpendicular to the base. And then the cone does not appear horned or gibbous. Next, when the moon departs from the sun, the circle of illumination is inclined to the straight line through its center and the eye of the sun, and then the cone becomes scalene. And it is cut by a plane perpendicular to the axis; therefore, the section cannot be subcontrary. Third, the entire cone is also removed. For that always happens when the plane is perpendicular to the axis. Therefore, all conditions being satisfied, the appearance of that boundary of light in the moon, which is seen in a place farther from the sun, bounding the concave of the horned moon or the gibbous side of the gibbous, will be elliptical, which was to be demonstrated.

Consequently, if the whole circle of illumination could be seen, it would appear with the figure of a perfect ellipse. But because half or a little more of it is hidden behind the moon’s body, the amount that is seen is therefore only the arc of an ellipse. This not only must be known for the picture of the configuration, but is also useful for a particular problem below. It might be proved from Wieleb IV 56, but the demonstration does not accomplish what the proposition promised; that a circle beheld visually appears to be a cylindrical segment, because not everything that is close to a cylindrical is cylindrical. If Wieleb had demonstrated this, Serenus would already have supplied the rest, who showed the section of a cone and a cylinder are the same. Thus the defect of the horned moon differs from that of the eclipsed moon, in that the former is bounded by an ellipse, the latter by a perfect circle.

But in the same way that the section of the horned or gibbous moon is elliptical, that of the half moon necessarily had to appear straight. See Wieleb Bk. IV p. 76 and Aristotle in the passage cited. However, it seems to...
him that something slight was lacking as to straightness. In 1662, in the evening of 11/21 December, having observed carefully, I judged the upper horn to be acute, the lower somewhat obtuse. It is open to anyone to investigate this more carefully.

This must also be considered: whether the moon is exactly bisected. For first, even if the circle of vision were a great circle, nonetheless the circle of illumination, which represents the section, goes beyond the middle circle. In Section 3 above, CAR was 180° 35' 20" at new moon, 180° 28' 12" at full. Therefore, at half it will be 180° 29' 46". And the circle of illumination goes beyond the middle by 14° 53'. The sine, 433, is the two hundred fourth part of 100,000, the whole size of the moon. From one side (the bright) there would stand 241, from the disappeared and dark side 239.

Now, however, the circle of vision is also not equal to the great circle, and in this the ratio of the bright part to the dark becomes greater. Let FAGE be a great circle of the moon, the circle of illumination FG, and AE at right angles to it through the center, so that the ratio of breadths of the bright and dark parts be that which AC has to CE.

Now let BIDH be the lesser circle of vision from the same center, so that the bright apparent part may be 1CHD, the dark part 1CHB. Therefore, since from the ratio of AC to CE the equal, AB, DE are removed, the ratio of the remainder BC, CD will be greater.

These things can indeed be said against bisection as to the appearance. But since the breadth of AB with respect to BC is completely imperceptible, as was said above, the ratio will also be imperceptible different. But also, the ratio of 239 to 241 cannot be distinguished by senses from the ratio of equality. For the one would exceed the other by about 15 seconds. Consequently, it can safely be ignored. Moreover, this will be helpful below in freeing us from this suspicion.

It is superfluous to repeat here the farmers' rules of how to distinguish, from what has been demonstrated, the coming from the waxing moon, by the orientation of the horns, which are always turned away from the sun. On this subject, the passage from Book 18 ch. 35 of Pliny is usually added.54

Nor is it very important that astronomers be informed how to imagine the zodiac and particularly for themselves using the inclination of the horns, and how, when the horns are standing on the perpendicular, they may be made more certain of the ninetenth degree of the ecliptic.55 Finally, how the line through the horns, extended, leads the vision to the pole of the ecliptic. For these things are everywhere encountered everywhere.

53 Recalculation shows that this should be the 230th part.
55 If the crescent moon is at the 90th degree on the 270th degree of the ecliptic (0° Cancer and Capricornus, respectively), the horns lie on a great circle which passes through the north pole and is perpendicular to the equator.
9. On the moon's spots

Even if it is left to the natural philosophers to argue what the spots on the moon are. nevertheless, since these chief—since forth in the full moon, this consideration should also be added to the enumeration. And really, the natural philosopher, even if he were fully to treat of the question, that would hardly be of benefit to the astronomer (as perhaps he might learn the same thing from the position of the spots when the moon is full as from the inclination of the horns when the moon is horns, since the circle of illumination, passing over the surface of the moon, does not cut it squarely into right and left, nor into an upper and lower pair, but transversely from the right eye to the left corner of the mouth); however, the astronomer supplies the natural philosopher with many things for untangling the arguments of the question. Even though Plutarch considers this argument in an entire book, with the title "On the face of the Moon".26 In it, the first opinion is that of those who think the face of the moon is a product of vision. This is refined there by arguments that are many and tightly packed. I am adding occasional evidence.

On 1602 21/31 December at 6h in the morning, through a device described in Ch. 2 and an instrument made for this purpose, a description of which is furnished below,27 the moon made an image of itself brightly upon the paper lying below, inverted in situation, just as it was in the heavens, gibbous. The edge all around was very luminous and bright, except from the gibbous side, for there it was perceived as more washed out. The middle, however, was one continuous spot or darkness, darker in one place, lighter in another. You should not think that what I would consider to be in the moon's ray was in the paper, for both the gibbous face and the spot in its middle were carried over to all parts of the paper whatever that were placed beneath it; rather, indeed, it was from moving the paper that this spot was first discovered. The shape of the spot, so far indeed as was able to be formed somewhat confusedly through a sufficiently wide opening at a distance of twelve feet, was the representation of the Hebrew namech (יִשָּׁה) with its belly full; that angle (for at the other sides it was nearly round) being pointed approximately towards the middle of the gibbous side. Nor was it just this one time, but, a trial being made of it more frequently, the spot always thrust itself forward along with the light, so that it could not possibly be a reflection of light.

Plutarch in that book brings forward many opinions on that face of the moon. Reinhold proposes others also, of which he most approves this one of

---

26 Plutarch's first chapter seems to begin with the "first opinion" mentioned in the next sentence, though in all surviving texts the beginning is fragmentary. The relation of it begins in Ch. 2. Cf. Plutarch, The Face of the Moon, pp. 25-27.
27 The instrument is described at the beginning of Chapter 11 (see Chapter 9, as in the note in JGGW II p. 451).
Witelo, which attributes to the moon the transmission of the sun’s rays, differ-
ently in different parts.\(^5\) Those which appear to us to be spots are the denser parts, to which the sun can pour in only a little light. Hence, he even thinks it
happens that the moon is seen even when it removes the entire sun from us by the
interposition of its body, for then the sun’s rays pass through to some extent, and
flow into our sense of vision. True as it seems, Witelo was not concerned with
spots when he chose this opinion, but with this phenomenon, why it is that in a
total eclipse of the sun it is possible for the moon to be seen, on the side turned
away from the sun. The spots came upon him in the role of something extra. And
so if we will accommodate another demonstration to this phenomenon below, we
shall be freed of this opinion, which has a very great deal of difficulty, and shall
beseech the cause of the spots elsewhere without these authorities objecting. For
Plutarch elegantly concludes, from that surpassing brightness of the moon bor-
rowed from the sun, that its body should be of the highest density, that sends
the least light into its depth. This is easily evident through a comparison carried out
with other things that shine with reflected light, among us, of which, so far as
any one is more transparent, it reflects correspondingly less of the rays. Nor does
the moon thus withdraw from us by many diameters of the earth, so that we may
not be able to argue from terrestrial things to lunar ones, under the guardianship
of vision. If our air, with a depth of a few miles, placed against the setting sun,
so weakens its rays that opaque objects set out in the sun almost lack a shadow;
and, as the same will be said below, when this space is doubled, it extends the
shadow all the way to the moon’s body; then would we not have made the moon
rater than this very air of ours if we should assert that the sun’s rays can pass
through five hundred German miles of the moon’s body (for the moon’s diameter
is about that great), and, indeed, pass through it in such a way as to enter into
the eyes? \(^6\)

That little work of Plutarch’s is most elegant and lively, and worthy for a
philosopher to delight himself in at some time when more weighty studies have
been set aside. Furthermore, it is the cause of my at length concerning, not unwilling-
gly, with this very author in this opinion, of which, for me at least, my teacher
Maestlin too was already the author; and I would say that the moon’s body is
such as this our earth is, forming a single globe out of water and landmasses.
Plutarch in fact persists in saying this, and fortifies it both eloquently and clev-
erly with many reasons against various objections, so that some Peripatetic might
well wonder that so many solid things can be argued against the opinions of his
sect. What especially convince me are the following: First, it was said above that
the moon, when it displays a bisected face, shows an uneven cutting, and to some
extent a twisted one. This is evidence that some of its parts are low, others more
raised up, and these to such an extent that it can be perceived from sixty semidi-
ameters. Next, in certain lunar eclipses there appears a great unevenness, and one
not coming from the earth’s shadow. For it is known that the peaks of the moun-
tains are very rarely raised by the distance of one mile, of which there are 1600

\(^5\) Theoriae novae (Paris: 1553) fol. 10r-4v: Witelo mentions the moon’s transmission of
rays in the introduction, Theoriae II p. 3.
in the earth's diameter. And so if there is some roughness in the earth, this must be imperceptible in the moon, which is 60 semidiameters away. For let the height of a mountain which the circle of illumination transits be a mile, that is, about the eight hundredth part of the semidiameter, and let this quintessence of its shadow also remain at the moon. And since it is sixty times eight hundred, that is, almost fifty thousand miles, out to that shadow of the mountain, the mountain's shadow will subtend barely 4 seconds. Consequently, an altitude of fifteen miles would in the end add one minute to the shadow, of which there are up to 90 in the diameter of the shadow.54 Consequently, if something irregular occurs in partial eclipses of the moon, this has to draw its origin from the body of the moon itself.

In 1599, on the night between 9 and 10 February, new style, when the sky was very beautiful, I observed an eclipse of the moon. And although I lacked instruments, I at least did not neglect to take note of those things that are made out by the naked eyes. In the morning, after the third hour of the town clock,\(^{60}\) the moon turned towards Cor Leonis\(^{61}\) in such a way that it was reckoned that a perpendicular from Cor Leonis would fall in the middle side of the face.

The face of the moon was inclined in that direction nearly like a portrait of a person whose right ear is hidden. Compare that with what I said above in Section 2 about the face of the moon. For this takes place in every full moon, which serves as indisputable evidence that the same face of the moon always faces the earth.

Further, there appeared to be a sort of gap above the right eye, which for us was opposite our left, as if at the edge of the circle something were missing in roundness.

When quarter to four was striking, it was reckoned to be the beginning of the eclipse. There was, however, doubt whether it was a diminution of the circle or a fissure first come into shadow, just as valleys are first cloaked in shadow. And this was a little below the point that stood opposite Cor Leonis.

When it was striking four, a good part was already gone. And so that the clock might not have made me err too much, (as they are usually set carelessly), I made a conjecture of the altitude of Jupiter, by a comparison of my distance from the window with the elevation of the edge of the window above my eye. And so the altitude of Jupiter was considered to be about 6 degrees. In the quarter-hour past four it had already gone behind the mountain, but could not yet have been on the horizon. It was not yet half in the shadow.

I have added this, so that the little observation would not lack the circumstances of its time, and so that a reader who is attentive to horological error (if there was any such) would not be thrown into difficulties by the exact statement of the time. Other things which I have noted in this eclipse will be said below in Ch. 7 Sect. 3.

54 That is, the earth's shadow at the place where the moon passes through it. Tycho Brahe, \textit{TBOO II}, p. 134, gives apparent semidiameters for the shadow at different distances of the moon, ranging from 45' to 47'.
55 This was in Graz.
60 Regularus.
In 1598 on 11/21 February, in the morning, when a little before five on the town clock the moon stood erect to the perpendicular, eclipsed by half its diameter, and when, from that moment inclining gradually more forward, it was traversing the northern par of the terrene shadow, and when, finally, a little before the sixth hour, still appearing to decrease, it removed itself beneath the clouds with the very meagerness of light; during this whole time (but especially when dawn was breaking and the light of the remaining part was dilated), it was seen torn to pieces or cut up, as it were, by certain luminous channels working their way into the shadow; and that which ought to have been an arc, dividing the shadowed from the luminous part, portrayed the edge of a broken timber with its unevenness. I believe that dawn was indeed a contributing factor. For when the moon was affected by the same northern part of the shadow in December of 1601 and in May of this year 1603, it experienced nothing of the sort, because the eclipses fell in deep night.

Add to these that if you look carefully at the moon even exactly at full, it seems to depart perceptibly from roundness.

All these things provide me with evidence for my statement that the moon was correctly described by Plutarch as the kind of body that the earth is, uneven and mountainous, and that the mountains are even greater in proportion to its globe than are the terrestrial ones in their proportion. And to jest along with Plutarch, because among us it is usual for people and animals to follow the spirit of their land or province, there will consequently be living creatures in the moon, with much greater bodily size and ruggedness of temperaments, than ours, for the simple reason that they endure a day fifteen of our days in length, and indescribable tides, because of the sun's being so long overhead—i.e., indeed, there are any there. For that place is not absurdly credited by the supervision of the people with being destined for the purification of souls.

But to the point. When Plutarch said that the moon is an earth, he then stated that the moon's spots are seas, which almost absorb the sun's rays transmitted into the deep, and do not reflect them as strongly as the earth'soaked parts usually do. His words are:

Just as our earth has certain large gulfs, so we judge that the moon too is opened up with great depths and rifts, containing water or foggy air, into which the sun does not penetrate with its light, but abandons them, making a diffuse reflection. Thus Plutarch, with whom we do not agree in this part, it is more fitting that the bright parts that are in the moon be considered seas; those that are spotted, lands, continents, and islands. For this which is to be demonstrated by the optical writers is most thoroughly confirmed by experience: that watery surfaces most of all become radiant with light, if they are placed next to the land. I believe because of the uniformity of the surface as a whole, but the roughness and rippled state of the tiny parts, or because they participate less in dark color than the earth. The former of these makes: it suited for reflecting the sun's light in almost all directions, the latter assists the light that is communicated. For to the extent that it is less black, it is more white. But white is suited to taking light in and vibrating it back out again, by 30 of Chapter 1, and so is the moderately transparent, by 22
of the same chapter. When I returned to Styria in 1601 for the sake of business, I climbed a mountain of considerable altitude, called Schöckl,25 of the Styriac bergs' dominion, intending to measure the orb of the earth from two mountains by means of a suspended plumb bob.26 And I in fact demonstrated that the mountain itself rises above the peaks or the other mountain, which has the Wilden stronghold placed upon it,27 by the height of five of the towers of the Strassburgers.28 And truly, looking down on the other hills of the lower region through ten German miles and more, it provided no other opinion to the one observing, than as if I were looking down at some meadow in which hay is distributed in sheaves. Therefore, from this mountain the earth presents itself to the observer with incredible brightness, so much so that when a cloud suddenly arose and cut off the face of the sky from me, a paper spread out was in many places more brightly illuminated from below than from above; the cloud covering the mountain, but its region illuminated by the sun. And the land as a whole showed this brightness, partly verging toward black in the forests, partly exulting in greens in the fields and meadows, in some places also, with plentiful plowed ground, being red. But that which furtoward the middle region, the river Mur, at that time overflowing into pools and meads, easily overcame the dimming brightness of the earth with its exceeding splendor. These things happened because the precipices mountainous had raised me up somewhat on the perpendicular, where more direct rays from the land lying below were able to fall upon me. What indeed was not about to happen if I had been able to perceive the whole orb of land in lenses that are nearly direct? And these things really made me believe my eyes that there is usually more brightness from water than from land, not by simple reflection, for the sun's position, the same as mine, at the left side of the downstream current, completely ruled that out, but also by communicated light.29 And so I conclude that in the moon the bright parts are a watery matter, but those that are dry are land masses and sands, while the whole moon, as will be said below, is surrounded by a certain airy essence, which transmits the rays of all sides.30

10. On the mutual illumination of the moon and the earth

It is widely enough admitted that the principal light of the moon is from the sun, the modes and varieties of which light have been the subject of discussion so far. But there are some who nevertheless ascribe to the moon some

25 The Schockel (1,466m) lies about 15 km north of Graz. For more on this outing, see Kepler's letter to David Fabricius, 2 December 1602, JKGW XIV no. 239 pp. 329-1.
26 This method is described fully in Epistola astronomiae Coelestis ab Avicenno, in JKGW VII p. 38.
27 The Wildoner Berg (1,550m), with the ruins of the castle Oberwilkening (1,452m), is on the south side of Graz.
28 The single spire of the Strassburg Cathedral, built in the 15th century, 134m tall, was one of the tallest structures in Europe at the time.
29 After reading Galileo's Sidereus Nuncius (Venice 1609), Kepler reconsidered this passage in his published response, De lighte communis Nuncius (Prague 1610), and reversed his opinion. Cf. JKGW IV pp. 297-8.
slight proper spark of light, by these arguments first, because in total eclipses of the moon there nonetheless remains to it some redness that is bright enough. Next, because in total eclipses of the sun the moon is again seen brightly in full face. Third, because in new moons, and two days and more thereafter, and even up to the quadratures, the moon is seen, not just with a whole circle, but completely with the whole round face. Really, none of these causes is sound. Of the light in an eclipse of the moon, it will be said below in Chapter 7 that this is not proper to the moon. But of the light of the new moon and of the moon covering the sun, an account is to be given immediately below. Moreover, that nonetheless the moon was sometimes seen when it had stretched the sun away from mortals, it is to be believed, on the authority of Witus and Reinhold, who were impelled from this phenomenon to provide causes, which they would not have done unless they were certain of the fact.69 Let them pay attention, who live in the Tyrolean Alps and who live in Italy, and the southern part of France, together with all of Spain, let them (I say) pay attention to this matter and the face or color of the moon, in the coping year 1605, on 212 October of which year, as indicated by the calculation of Tycho Brahe, the moon, nearly at perigea, will be set centrally against the sun in the regions mentioned. For concerning Tycho’s opinion that the moon’s diameter in eclipses of the sun is smaller than that which could cover the whole sun, something has been said in the appendix to Volume 1 of his Progræmatarum, and something will be said below in its own chapter.68 But Comenius, from the opinion of Posidonius, appears to deny this exact thing or the moon’s light in an eclipse of the sun.67 For he asks, since (in his opinion) it is settled that the moon’s body is translucent, why the sun’s rays do not therefore pass through that globe in a total eclipse of the sun, as they ordinarily pass through a cloud, and make it visible and make brightness pour forth from it; why therefore the moon is dark, escapes the sense of vision, and day is turned to night? And once the question is posed, he gathers little reasons from everywhere, by which he strives to establish that the sun’s rays are not obliged to pass completely through the moon or to procure brightness for it. Thus he says exactly the opposite of what Witus says; perhaps he is reasoning from the effect itself, the conversion of day to night, and not from the actual appearance of the moon covering the sun, as Witus did. For although for me the truth of this matter is not established by any experiment, nonetheless the light makes this happen, that the crescent moon, with

67 Theorium usque (Paris 1553) fol. 34r.
68 In the Progræmatarum itself, the moon’s apparent diameter is given to 37 at a distance of 60 earth radii, while the sun’s is 33 (pp. 473–4. TBOO II, pp. 422–3). However, as p. 817 (TBOO III p. 329 r), there is an anonymous appendix stating that there had been an error regarding the diameters: a lunar diameter of only 30 i is consistent with observations. In a letter to Marinus of 1 February, 1620 (AGW XVI, no. 551, p. 277v), Kepler admitted authorship of the appendix—thus he managed not only to write Tycho’s posthumous opinion, but also to cite it here in his own support! Cf. Kepler’s measurements of the sun and moon in Ch. XI Proo. 3–5 and the appendix v 3, below.
its greater part turned away from the sun, nevertheless shows in its entire body, so that I have no doubt whatever that it would strike the eyes much more obviously by the same light when it had switched the sun, which ordinarily blinds our eyes and suppresses visible things, from our eyes.

Witelo, then, and Reinhold, argue ambiguously about the residual light of the moon, as was said above.\textsuperscript{30} Witelo said that the body of the moon is passed through by the sun’s rays, and that after the passage is made they are certainly very weak but nonetheless visible and enter into the eyes; but at no other time than when the moon is nearest the sun, for in departing, it turns these traversing rays away from us, just as it does the shadow. When Reinhold saw that the days nearest new moon, in which the moon is perceived very brightly in its entire body, are forsaken by this cause, since the diameter through the luminaries has now diverged from our eyes, he said that the moon has its own spark of light in addition, which is perceived only in the days nearest the new moon, and that in an eclipse of the sun this spark of light is mingled with the rays of the sun passing through the moon’s globe.\textsuperscript{31}

In fact we have just now shown clearly enough, in Section 9, that the moon’s body cannot be pallid. Moreover, it will be clearly shown below that that spark of light is not proper to the moon; this will be most completely undermined where for authentic causes will be opened up. When these have been looked through, no one will think it necessary to beg other causes from elsewhere.

There are those who think that that whole circle visible in the nascent moon is nothing but the circle of illumination, which is seen as a whole because the moon’s apparent diameter is so much less than the diameter of the sun. This reason is completely false. For this light is perceived not only on the edge, but in the whole body, not for one day, but for two or three. Indeed even somewhat at the quadratures, where a semicircle of illumination is completely hidden behind the moon.

In Book 2 of the \textit{Ponestummanum}. Tycho Brahe ascribes the cause of this light to Venus, which may be able to illuminate the moon so brightly.\textsuperscript{32} But the waning moon is always blessed with this light, [while] Venus is not always set by its side. Besides, Venus is many times higher than the moon. Consequently, even if it might sometimes bestow its light to illuminate the edge more strongly, nevertheless the ray of Venus does not reach to the middle disc of the moon, which is equally blessed with this light, being hindered by the interposition of the moon’s body. Meanwhile, I do not deny that it sometimes happens that we view the moon from the earth by illumination created by bright heavenly bodies opposite it, concerning which see ch. 7 below.\textsuperscript{33}

\textsuperscript{30} See p. 260 above.

\textsuperscript{31} Witelo IV Prop. 77. \textit{Theophrastus} Ipr. 151; Peutnach edition, Wittenberg 1655, fol. 164v-165r; Paris 1553 fol. 164r.

\textsuperscript{32} There is no Book II of the \textit{Ponestummanum}, and neither Ch. 2 of Part I nor Part II appear to address this matter.

\textsuperscript{33} See p. 288 below.
On the other hand, my first teacher, Maastricht, to my knowledge discovered the true cause, and taught it to me and to all attending his lectures 12 years ago, and publicly explained it in 1596 in his theses 21, 22, and 23 of his Dissertation on Eclipses. That teaching is to be presented in no words other than the author's own. Here is what he said:

Of the light which is seen in the horned moon near the horns, diffused through the whole body, it is agreed by those who see it that it is not overshadowed by the brightness of the daylight, which either remains in the evening after the setting of the sun, before twilight (that is, while it is still bright day), or in the morning, with the dawn, precedes the rising of the sun. On the other hand, it is agreed that this same light is weakened on the rest of the days further removed from the moon, so much so that about quadrature, what the moon is a long way above the horizon in the dead of night, nothing, or very little, of it is seen to remain [and that only by people with the sharpest vision]. Therefore, from this light's being separable, it is concluded that it is not as certain people wish [among whom is Reinhold], born along with the moon and proper to it, but, just like that great monthly light, is similarly foreign and borrowed. For unless this were so, it would assuredly be seen much more brightly in the dead of night than in the illuminated air of daylight. Moreover, the derivation of this light upon the moon is shown by its position with respect to the earth. For in the new moon, the moon, placed between the sun and the earth, sees that face of the earth which the sun illuminates, placed directly beneath it. But we acknowledge that the strength and brilliance of the sun's rays reflected individually on the single parts of the earth is such that in sunny places it dulls the sharpness of the eyes; and furthermore, that it fills with light the inner recesses of buildings, wherever it is allowed to enter even through a little crevice. Who will deny this same thing of the whole of the light itself, gathered and reflected from the whole earth together with the water? We therefore say that the earth, by its gleaming light, sent to it from the sun, casts its rays on the equator and on the moon itself, as it were, as the sun itself, and in turn, since the sun itself is the center of the sun and the stars, the sun itself is illuminated; and the light so received from the sun is redirected toward the earth, as we have said before, so in turn each lights up the night of the other.

This opinion receives support from that weakening of this light. For, when the moon afterwards moves away from the new, it begins gradually to leave this illuminated central part of the earth and to perceive its remaining part more and more obliterated. At the same time, the strength of the reflected rays is both diminished and dulled. Hence it happens that whatever of this light

21 Dissertation de eclipthis solis et lunae. Tubingen 1596. Bracketed words are Kepler's interpolations.
is reflected to the moon that is halfed or swelling beyond that, can be seen on earth either not at all, or with greatest difficulty, because of the excessive attenuation.

So he spoke. Let them then stop seeking other causes, since they see the true one. If anyone cannot believe that the force of light that is communicated to the earth by the sun is that great, he should bring into consideration my experience, which I set forth in Section 9. where, from a distance, the sun appears to have a diameter of some four miles, and to be so high that a person standing at such a distance can see the moon about a third of the way to the edge of the earth. The latter was my own experience, and is confirmed by that of other observers.

In Book 1, ch. 8 of the Cosmocritica, Cornelius Gemma adds that Plutarch, from the little book on the glory of the Athenians, in which he says that the latter wrote, at the time when the victorious Greeks were at Parnassus, that he meant to say Salamin, the island in the Aegean Sea, that the moon, being still slender, was filled with a sudden addition of light.

Although this is not borne out in so many words in the book mentioned (for Plutarch's words are, "the full-moon goddess shone forth upon the conquering men"), nonetheless if perhaps Gemma happened to read something of the sort in another author, this has to be accepted concerning this light of the moon. It is borrowed from the earth, to which light a greater force from some external and adventitious causus would be reconciled.

Further, by this light, however tenuous, the moon wins among astronomers favor by its nature that is not slight, when, being visible with its whole body, it nevertheless does not extinguish the light of the fixed stars which it touches most closely. Indeed, I have more than once observed it standing very close to

75 Aristarchus of Samos (Third Century B.C.E.) proposed a heliocentric arrangement of the planets. His own work on this subject is lost, but was reported by Archimedes in his book The Sand-Reckoner in the Works of Archimedes, T. L. Heath, ed., pp. 221-
76 Gemma's "Alcmannatobstiva" is an alternative title for De mutauer divinis, chimera territa-
the Pleiades, so that it was not a whole diatmber away, while all the little stars of the Pleiades were twinkling distinctly. Not to mention that the diameter, then to be measured as correctly as possible, would not fit with a single method. And so in the discussion of the moon's diameter, below, arguments are to be taken from this place as well as from the preceding chapter on the eye. For most clearly, as was also said in the preceding chapter, the horn illuminated by the sun is judged to sketch out a circumference that is greater than is enclosed by the small circle of the remaining disc, whose light is borrowed from the earth. And so the circle of light from the sun appears to contain the circle of light from the earth, on one side, at least.

Hitherto, therefore, the moon not only has shown no light that it could bear, acquired of itself, but also has been convicted of destinct and opacity. But the suspicions concerning light in its total eclipses, I will dissolve below, where I shall treat of the shadow. For I am going to show that this too comes chiefly from the sun.

11. On the first phase or rising of the moon

The reason why peoples who used the lunar year paid such careful attention to the first moon, is shown in part by Reinhold in the commentaries on Peurbach, but most recently and most carefully by Joseph Scaliger in De emendatione temporum, whose polymath erudition everyone rightly and deservedly marvels at, but very few will imitate. Therefore, astronomy must have a theory of the first appearance, which Reinhold, from Peurbach presents adequately through the causes that advance or retard the seeing of the moon. Among these causes he places the obliquity of the sphere; long or short risings and settings of the sign in which the luminaries are moving; the moon's latitude, north or south, great or small; the visibility or invisibility in its period; to which he adds a more extended or briefer twilight. And nonetheless, when Peurbach said that the moon can be seen old and new on the same day, which agrees with the ancients, who used to call the day of the new moon "old and new" because of it, and with Pliny's statement that it was then seen by certain sharp-eyed person on the same day, both in the morning before the sun, and in the evening after the sun. Reinhold vociferously mightily to accommodate the causes to this pronouncement. Finally he appeals to experience: I—that I might not say nothing here—am of the opinion that this thing cannot be grasped by rules. For Reinhold truly affirmed that summer nights introduce a great impediment by their brightness. The whole thumb rests in the air, which, if it should be pure and clear enough, as in winter, while the observer stands on the ridge of some high mountain, higher than

78 Theorica novae (Paris 1553), fol. 95v-100r.
79 Joseph Justus Scaliger, Opus de emendatione temporum (Paris, 1583), Prolegomena.
80 Theorica novae (Paris 1553), fol. 98v.
81 Sempere venere.
83 Theorica novae (Paris 1553), fol. 100v.
84 Here Kepler refers to the following enthone;
the great part of this thicker air, then nothing would prohibit the moon from being seen at the very moment of conjunction next to the sun in mid heaven, at least if it be passing through one of the limits. For although it cannot be more than 5 degrees from the sun, with the consequence that it shows hardly the seventeenth part of its body to us, and that itself, being received at the edge of the moon, appears much narrower than it is, an objection which Reinhold carefully raises against himself, nonetheless, it can easily happen that the whole disc of the moon, enjoying in turn the full light of the earth, in accord with the condition of the place and of the one viewing, can dispel the air's brightness and pour itself down upon the seas, since it often overcomes the surpassing brightness of the air situated around the horizon. For it is known, those things that the optical writers would declare about deep wells; that stars set close to the underside can be seen from them, since the vision has been freed from the brightness of the surrounding air. Much more so, if one were to stand on high, beyond the bound of such thick air, which everywhere intercepts the sun's rays. And not only on account of this cause, but also because of this, that the moon's light can be perceived on the brightest day while the sun is present, I judge that the altitude of the pole and the varieties of risings are pretty well in agreement, and that it can easily happen that when the moon is swift and located at the limits, it may appear both old and new on the same day. And this I do not know, whether it happens more readily right at mid sky than about the horizon, where the more obliquely and the farther the solar rays traverse the vapor, the greater the brightness they make and the zone they blind the eyes.

In 1598, 21/31 July, in the morning, in Graz, I saw the moon about 16 Cancer very brightly, so that there was hope that it would be seen on the next day, but clouds took over the following day. The sun was about 7 Leo.

In 1603, 25 Aug. or 4 Sept., the sun being at 11 Virgo, 5° moon, at 24 Leo, and nonetheless cast down to the southern limit, was nonetheless seen very brightly.

In 1603, 4/14 March, in the evening, I saw the moon so brightly that it seemed absolutely necessary that it could also have been seen on the previous

To p. 255: Take this condition in such a way that the surface of the mountain above the observer should intercept the view of the air radiating beneath. For all these things are referred back to the eye; carefully note this on p. 256. For aside from squandering of the direction of the eye, the sun certainly illuminates the land and the air more brightly from this sky, and brings about a greater brightness of day, than when it is lying upon the horizon, as was shown previously. And since the question here is from where the moon be more sightly perceived, standing near the sun: is it from the high mountain with the huminities smitten at mid sky, or is it from the plains. while they are setting, for then the brightest of the air around the setting sun is in all circumstances wider than around the illuminating sun, and therefore takes up the eye more widely. And although the brilliancy of the air is generally brighter in mid sky, we have nonetheless placed the observer higher than that material which shines so because of the proximity of the sun.

35 That is, if the moon be at its greatest northern or southern latitude.

36 Variae actum (Passa 1535) fol. 100v.
day, had the sky not been clouded from the west. At hour 6 the sun was at 23° 49' Pisces, the moon at 14° 43' Aries. And between the sun and the observed position of the moon was 20° 10' on a great circle.\footnote{Here Kepler refers to the following enolone:}

To p. 258. In 1583 14 March, Tycho Brahe saw the moon at 15° Aries, while the sun had set at 3° Aries. The moon was at the northern limit. [C. TBOO X p. 241 - trans.]

\footnote{In the observation book for 1587, 24 February (TBOO XI p. 199), Brahe writes, “On 24 Feb, in the morning, before the rising of the sun, there was seen a great star in the likeness of Venus, when the sun was just then about to rise, which was perhaps Venus itself, although it was still west of the sun. But because of the excessive northern latitude, it was seen before the sun.” The observations for 21 February (on the same page) do not record the latitude; perhaps Kepler calculated it from the given declination and elongation from Procyon and Saturn. Note that Sichel, ABOO II p. 349, reports, Tycho’s note incorrectly, instead of “minimum” (excessive), he has “minimum” (minimum).}

In 1587 Tycho Brahe saw Venus on 21 Feb. old style, when it has run out to nearly ten degrees of latitude to the north in Piscis, at 6° in the evening, and 24 Feb. in the morning before sunrise, when in respect to longitude it was still after the sun.\footnote{Theorems II p. 124. Wierlo wrote: “The comprehension of the place of a sun object consists of the comprehension of the light and color of the object and the remoteness of the object, and of the part of the universe in which that seen object exists; and also consists of the comprehension of the quantity of the remoteness, when all these are comprehended at once through the path of cognition.”}

And to the moon and to Venus, the nearest bodies to earth, one may not unaptly accommodate Whewell’s Book Four Prop. 14, so understood that for more distant things the greater depth of air set in the way would represent those things as blue, something that painters scrupulously imitate.\footnote{Rauner Gemma Frisius and Cornelius Gemma, respectively. Rauner Gemma (1508-1555), born in Dokkum in Friesia, from which he derived his surname, was Professor of Medicine at Louvain. He edited Peter Apian’s Cosmographia (Amsterdam 1529) and many other editions, and designed a kind of cross-staff, which he described in De radio astronomica et geometrica (Antwerp and Louvain, 1545). His son, Cornelius Gemma (1534-1579), was, like his father, Professor of Medicine at Louvain.} Thus, for the moon and Venus at the periphery of the epicycle, the least amount of the aethereal substance is put in front of that which they send across by the rays; consequently, they are perceived much more evidently than the higher bodies, by rays sent down through the immense space of the aether. For even aether has its matter. But to the point. Since, therefore, Venus is so small, and is perceived so near the sun, what is not to be believed of the sun’s sufficiently long horn? It is also not so rare for the sun’s light to be dabbled occasionally through certain subtle causes. Gemma, both father and son,\footnote{Rauner Gemma Frisius and Cornelius Gemma, respectively. Rauner Gemma (1508-1555), born in Dokkum in Friesia, from which he derived his surname, was Professor of Medicine at Louvain. He edited Peter Apian’s Cosmographia (Amsterdam 1529) and many other editions, and designed a kind of cross-staff, which he described in De radio astronomica et geometrica (Antwerp and Louvain, 1545). His son, Cornelius Gemma (1534-1579), was, like his father, Professor of Medicine at Louvain.} relate that before the battle of Charles V with the Duke of Saxony, the sun for three days was seen as if flooded with blood, so that even most of the stars were seen in midday. Therefore, the cause was that the sun’s light was dabbled, but that of the stars not at all. The cause accordingly had to be high up by which the sun’s light was rendered dull not just at one point of the earth, but for the whole visible horizon, indeed for the whole region of land.
from which the air might be able to radiate to some place, even to a portion of the earth that is so much greater in proportion to the greater number of places it is observed. If you were to say that the air was so widely thickened, this would also have clouded over the stars. What remains, therefore, is that the cause of this dulling was nearer the sun than is the highest air, at least at the confines of the lunar course. Perhaps it was contrary matter, more widely scattered and thinner.

But to the point: with the stars sometimes shining forth by day, the moon too will be able to shine forth when very near the sun. In general, the cause for the heavenly bodies being hidden by day is not chiefly the sun’s presence, but only that the air, poured everywhere around our eyes, all shines brightly, illuminated very strongly from above by the direct light of the sun, from below by the communicated light of earth, all the stronger in proportion to the sun’s striking the earth more directly from on high. I once thrilled with this matter in the following verses. My conceit was that I was climbing the ridges of Atlas, projecting above the surface of the airy region:91

New beneath my feet it begins, black clouds beneath the heavens;
While a new light was on the lands, night was risen on the poor,
I speak wonders: a vaporous humor, which is wrapped over the earth,
Took the form of a heavenly body by reflected light,
Such as where the winter aether, with dark clouds,
Has painted the roofs and fields with glowing snow.
Almost as above, on Mount Schekel of Styria.
The heavenly fires were to me then as at midnight.
Hard it was to see by the torch of Phoebus, though present.

And now it ought to be noted that it very often comes to pass that the stars are seen by day, when the sun’s rays, along with the sun itself, are defined, and that excessive brightness of midday is extinguished. And by reasons and experiences approximating these it is made plausible that the moon can be seen on the same day before and after the sun.

12. On the light of the other heavenly bodies

Albategnius thinks, in ch. 30, and Wiele, in Book 4 Prop. 77, that for all of the stars there is its own measure of light from the sun,92 and that it even follows from this that they may be considered to be increased and decreased in light. However, the cause of their not appearing halved, like the moon, is their

91 Kepler’s source of information about Mount Atlas is evidently Pliny, Natural History V. 16–15, Vol. 112–13, where the peak is said to be “raised above the clouds and in the neighborhood of the lunar circle.”
92 Wielo IV 77, in Theocritus II. p. 152: “And the phases appear to us more evidently in the moon because of its proximity to our senses of vision; however, in all the other stars receiving their light and the actuality of their light from the sun or from other stars, the same tapers must be produced, from the preceding three theorems.” (The discrepancy in number is present in Kepler’s text.)
measureless distance, over which a shape cannot be perceived. Others persist in saying that the light of the stars is indeed from the sun, but passes through translucent bodies, and thus at last is made to vibrate. The opinion of these is weaker than the rest. For really, as we said before of the moon, if the stars shine by refraction of the solar ray, they will pass through only a diametral line from the sun, like the tails of comets, and will not project their most luminous rays to us, especially where, by the mass of their own body, they hinder their being able to be seen in its place of the rays departing from the sun, that place being turned upwards. So, next the sun, they would be invisible, the contrary of which is attested by experience and by Ptolemy's rules to the emergence of bodies departing from the sun. Venus, however, if not any of the rest of the planets, is enough to refute this. For since this star goes around the sun, as we above k, now below 1, on which point no one has anything to doubt, after Copernicus and Tycho Brahe have divested the general rudeness in these hypotheses today into two heads, the one that the opinion of one of these must necessarily be true; and both with one voice declare that the path of Venus is wrapped around the sun, it is thus fitting that the light of Venus, as that of the moon, is extinguished when it comes between the sun and us, but is seen at its broadest when Venus flies to its upper conjunction. But the situation is different from this. For it is invisible for a long time at the superior conjunction, and since it is very high, it casts a weak light, and casts a shadow from bodies poorly. Therefore, the view it descends, the brighter aspects, and emits a light that is a rival of the lunar light; and is finally of such brightness that sometimes at the actual moment of conjunction it can be seen with the form of a great star, as I have found noted in the Arabic observations, and is not made to vanish by the extreme brightness of the nearly are spread out all around, itself occupying a single point, by which will not move the moon and turn it towards us, itself being equivalent to the strength of light. It is therefore necessary that it have this surpassing light as its own.

Thus in my theses on the foundations of astrology, which I published in 1602, I in thesis 21 first made it plausible by four arguments that the planets have their own light. One was this very one which I have just presented. For Venus would really change its face and waste away, like the moon, if it is only shown with light communicated from the sun. Second, I showed by examples of substantial things that all light doesn't have to be gathered at the sun. For here many animate things (not to speak of inanimate) have something of a spark of light placed in them. Third, from a certain combination of geometrical differences and by the function of the celestial light, I showed that the stars need a twofold light, both their own and an extrinsic light from the sun. On this line of argument, I leave judgement to the reader, for it does not belong to this place. Finally, I said that it was fitting that shining and twinkling are evidence for proper light, nebulosity and sluggishness for extrinsic light, for it is necessary that all light

16 Dr. fisciamentii aereolaeque centraulis (Prague, [1601]). In: Rosi, Philadelphia 1979. This isolated work contained a proposition for 1602, and so presumably was published at the end of 1601 at the beginning of 1602. Capit Bibliothecorum Keplerianorum Second Edition, Munich 1968 dates it in 1601.
that is communicated and colored in matter be weakened, which Cleomedes also teaches, stating that light communicated from elsewhere does not radiate far. And by this rule, to the moon is ascribed borrowed light, to most stars proper light, least of all to Saturn. Next, in thesis 29, I stated the same thing that I demonstrated above in chapter 1, that when the light of the planets is colored, it is necessary both that they be translucent and that the essential causes of color otherwise in the matter of the globes, so that in this way the power that a particular planet shows in effect has some disposition in the body analogous to it. See Prop. 15 of Chapter 1. Afterwards, I compared the colors of planets with certain bodies, and showed that if a black surface were strongly illuminated, the color of Mars is returned; if bright red, that of Jupiter; if reddish or white, that of Saturn; if yellow, of Venus; if blue, of Mercury. It seems now that the same thing must be believed of the proper light as if iron were put in the fire, or a coal, it becomes red, and if from the redness much light slanders through, a shining similar to Jupiter is generated. If it be most bright and translucent, a yellow body, and there be much light from it, the bright colored extravasance of Venus becomes radiant with light; if it be of sapphire or crystal, lightly subtle, that of Mercury; if on the other hand it be of a coarser matter, the rays of Saturn will shine forth.

It is, however, entirely credible that these heavenly bodies are ignited by an inherent force, no less than the sun, and it is in accord with what I said of the impurity of their matter, or the visible form itself. If you look into it a little more carefully, without question as light is completely unmolested. From Saturn's given color there appears a certain small quantity of purple, from the most welcoming brilliance of Jupiter a little redness, like smoke through the fire; this is brightest in Venus. Thus the stars of Cantis and Arcturus, most greatly Cantis, put on all the colors of the rainbow in succession. And, lest my vision deceive me, I always had associates present, who, whenever they thought they saw some greenness shining out, indicated it with a very brief sign. We were unanimous; whatever was seen by the one silent person, the other signed at the same moment. This emission of flashes of light also concerns Venus. They are mistaken who think all these things come from optical illusions, from the instability of air. For why is the matter of emission not the same for all stars that are equally distant? Why does Arcturus show color, itself being chiefly reddish, but Cantis more so, whose color is more crystalline: why does it stir up sharper twinklings than Arcturus, why longer ones? And why does the heart of Scorpius? Why twinkle so fast, the eye of Taurus? So slow, you would say you saw a coal stagging alive under the ashes. Are not Capella and Lyra equally bright stars? And nonetheless you will note no colors in Lyra, while in Capella, while it does indeed shine forth colors, nonetheless they are almost purple. Finally, by ocular experience, twinkling is added to the planets either by some internal alternation of the body, perpetual and continuous, which you might say is like a paroxysm, or by an external revolution of the body.

Cleomedes, De motu coelesti II 4 p. 182, ms. Goulue (French) p. 158.
53 Saturn.
54 Arcturus.
55 Capella.
56 Aldebaran.
belonging to the parts and to the surfaces, proceeding by the unfolding of some parts after others, which Tycho in the *Programmata* favored.263

In 1602, 19/29 December, in the evening, Venus, now declining greatly, radiated most brightly through an open window into a dark chamber.264 The air was extremely cold. Venus was twinkling most astoundingly. When I looked back at the white wall towards which the ray of Venus flowed in conformity with the breadth of the window, it swayed, as if by smoke in the way of a flame, doing so very quickly, with urgent motions. For it was not ascending, so that I might believe it to be from ascending vapors, but, shining forth now at the middle, now at the top by a sudden alteration, it withdrew now downwards, now again to the side. And, I swear, that fluctuation, so unaccustomed, of smoky darkness in the yellow ray, shone up on an inconceivable hidden horror. The speed as well as the moments of fluctuation were coordinated with the twinkling, perceived by the eyes from the star itself.

On the following day the air changed, and there appeared a morning rainbow; winds ceased. And so, whatever this is, I have referred it to the alteration in the air. But a few days afterwards, namely on 20/5 January, 1603, in the evening, the sky was again calm, there were about eight degrees left to the three day old moon to reach Venus, and both were sending their rays through the same window. At that time the rays of the two were seen in an evident enough proportion, such that the moon did indeed cast rays much more brightly than Venus, simply because the horn of the moon is also much larger than it, but nonetheless the difference was evident between the edge where the rays of the two ran together and the edges receiving the rays of Venus alone. I conjectured that the ratio of brightness of the surfaces illuminated by the moon, by Venus, and by both, was 4 : 1 : 5. The ray of Venus was wavering a great deal, but that of moon not at all. Therefore, both this and the previous wavering were from a real twinkling of Venus, not, indeed, from the air then passing in the wind, as I had believed. For it would also have applied to the moon.

Further, I have inserted his mention of the light of the stars, although it is more physical than astronomical, all the more willingly, because I think it not impossible to measure the azimuth and the altitude of Venus, Jupiter, and other stars by the lights they cast upon walls. For in 1601 in the month of December, I saw the distinct lights of Jupiter and of Spica Virgins through the same window upon the same white wall. He who’s studious of celestial observations should ponder this matter, and have at hand, if necessary, plane mirrors as well, down into which he may look. For the eyes are always more inclined, somehow or other, towards looking downwards than towards looking upwards.

13. On the light of comets

What it was that ignited the lights of the comets, present day theoreticians have adequately shown, it being recognized that the heads of comets always are

263 Tycho mentions *rotation* as a possible cause of twinkling (among other causes) on p. 263 of the *Programmata* (TB/0018 p. 377).

264 Obscurum cameram.
spread on the side opposite the sun, unless, being opposite the sun or extraordinari
dy phenomena of immense altitude, they occurred their own tail or beards by
the interposition of their heads. Nevertheless, even now the wits of philosophers
are kept busy with difficulties far from ordinary. For if comets are illuminated by
the sun, which the direction or the tail attests generally, and which reason pro
ounces probable, why, then, do the tails not look precisely to the place opposite
sun? Why do they almost always bend away, why are they curved into an arc? Fi
nally, just what is that thing that receives the sun's rays, and shines by its striking,
showing the shape of a tail? For if you will say that it is xerular, looking to the
essence of the comet, you will have fashioned the most monstrous monstrosity.
If it be the aetherial air, I ask why it is not thus illuminated by the sun every day,
so as to shine even without a comet? Even Tycho Brahe, in the Fruegmnnmmae
vol. 2 ch. 7, so neatly agrees to this opinion, as to say that the comet's tail is il
uminated by Venus, against which very opinion, however, he inveighed shortly
thereafter, hesitating in ch. 9, and urged his readers to probe the mysteries of op
tics. Would that I may be able to give all that is required in turn here, through
his subservient admonition.

But because I am not able to do all that I want to, let me therefore be allowed
to accomplish what I can. First, from observations of that memorable comet of
1577, 101 it is evident enough that the question of the tail's inclination and of
its curvature is the same. For the direction in which the tail declined from the
diameter of the sun is the same in which the bending appeared. For Tycho thus
explicitly stated, in more than one place, that the curvature looked towards the
zenith, and the concave towards the horizon, and since the comet was in the
north, the line of the tail being extended to its intersection with the ecliptic, the
angle made by the line of the tail extended to its intersection with the ecliptic was
from 3 to 9 degrees less than if the form had been drawn through the head and
the sun, with the result that the tail was further south than the diametral line of
the sun. Therefore, the inclination or deflection of the tail from the sun's diameter is
nothing other than part of the curvature. And thus, that which had begun to be
produced directly away from the sun, by being curved in at every point, finally
gave the images of a declining line. Say, O daughters of Parus, 102 what may be

101 There is no such opinion in Ch. 7 of the Perseusioe, which is in TBOO III pp. 415-
435. Kepler meant to refer to Chapter 7 of De mundi aetheris recentoribus pheno
menis (Urnburg, 1586), which was described on the title page as 'Book II', although no
Book I ever existed under that title (Tycho evidently intended this as a sequel to his
1573 book De nova stella (TBOO I pp. 1-142), cf. TBOO IV p. 9). This chapter in
TBOO IV pp. 133-154 consists largely of a series of observations of the 1577 comet,
showing that its tail faced away from Venus rather than the sun. The subject is taken
up again in Ch. 9, and there is a diagram illustrating the direction of the tail on p. 203
(TBOO IV p. 173). Tycho's invocation of Optics is on pp. 204-5 (TBOO IV p. 178).

102 For this comet, see C. Doris Hellsman, The Comet of 1577: Its Place in the History of
Astronomy (New York, 1944).
the cause of this curving, and the question of the deflection will be settled. It is not parallax. For, as will be said below in ch. 9, this cannot represent straight lines as curved. It is not refraction, unless we conceive some sort of monstrosity, that the aetherial matter by certain degrees of nearness to this celestial body is more and more thick, and unless it is so in only one direction, that towards which the tail verges. Further, if we will readily to assert this, it will also be easy for us to reply about the tail's illumination itself. For the matter to be illuminated will be ready at hand. However, it cannot be illuminated by the sun itself, because the sun's light is simple and pure, without colors. But it will be able to be illuminated by the sun's light passing through the comet's body, for the reason that the sun's rays are gathered together and redoubled, and also colored, through the comet's body, which for this reason should be pure; translucent and rather dense. But if we shall not be allowed to attribute to the tail of comets its own matter, we shall be forced to state, as we already had above, that the aetherial air itself is not quite entirely lacking in matter, but is made of something suitable for being so strongly tinted by the colored ray of the sun, passing through the comet's body from so near a place, that it is able to run in to the eyes from a distance with this color or brilliance. The rest will be carried out from the diagram of Chapter 5 Prop. 19: the spreading out of the tail at the end, and on the other hand the extremely compact origin of the beginning, and that which some people relate of certain comets, that right from the head the tail comes together as n in a point, and from these, as if from a point, a new tail is attached, broader as it proceeds.

For all these things necessarily follow from the laws of a solid pelliculogic. Next, this too will follow for the consideration of the natural philosophers: the body of a comet consists of a certain moist substance denser than air, for that was in the description of the pelliculogic in Chapter 1. But not that that globe of moisture is hard like glass, for it is gradually dissolved. Thus it is necessary that the moist substance be not solid but flowing and soft, which the comet's yellow light that Tycho Brahe attributed to it also appears to confirm. This original opinion of mine is also confirmed, that comets' 266 tails are bubbles, that is, hollow or of a different nature on the inside. For unless they be solid and consist of the same material everywhere, such a refraction does not take place, that gathers the rays of the sun passing through and, after they intersect, disperses them again on the opposite side.

The opinion about the watery material of comets is confirmed by the new

266 See e.g. De mundi aetheri recentioribus phaenomenis Ch. 1. in TBOO IV, p. 12.
star of 1572, which the authors, and Cornelius Gemma in particular, describe in such a way that it appears to fill or nearly all the colors of the rainbow in the succession of time. For he says that it began from red next, when it was shining, it was greatest: afterwards silverly; and finally, it disappeared pallid. And this is the very order of colors in the rainbow: red, yellow, silvery or green, blue, and purple, or, what is the same thing in a star, pallid. And the colors of the rainbow are evidence for moisture, as appears from the first chapter. Therefore, it is also likely that that star consisted of moisture.

Notes

If only this be maintained, whatever cause is alleged for it, that the matter or the aetherial air is mass strongly illuminated by rays of the sun passing through the comet’s body, then by the base and pure rays of the sun, the remaining things will have been explained. For since follows radiation, so that if a portion of the aetherial air beyond the comet, opposite the sun, radiates it will as a result also be seen, and the more the former happens, the more the latter also. But the tail of comets radiate most strongly, such as the one which Cardano describes from Halcy, three times as great as Venus, whose light was so great as if a fourth part of the moon were to shine; likewise another, which the claim was seen right in the middle of day. And, lest you might think that this light is from the comet’s head alone, look you at that Mithridatic comet, described by Justus in Book 37, 106 in size, it had taken up a fourth part of the sky and so shone that the whole sky seemed to be on fire; this must be understood to refer to the tail. The tail was certainly seen to rise and set without the head’s being present, because rising and setting took an interval of four hours. Another one first emerged into the eyes of mortals by the brightness of its tail, after sunset, before the head had risen heliacally; it at last emerged in the following days.

Now, once the illumination of the aetherial air is granted, there will also be granted certain degrees of propriety for the location of the comet’s body. From these, you might not unfitly also conjecture the hallowed shape, of which sort that comet of 1577 seems to have been, 107 not that the sun’s rays, contrary to the nature of light, are bent into an arc, but that from one and another part of the comet’s body, according to their uneven distribution, longer rays came out here, shorter ones there, all of which, arranged in relation to each other, show the shape of one curved tail.

The same thing can also be the origin of the declension. For because we have said that by the common optical account’s happening that the sun’s rays, sent
through the comet’s dense body, are first driven together as to a single point, and then, upon intersecting, are again spread out; thus the lateral extremities of the tail will decline from the diameter of the sun in both sides. Grant now that one side of the head, whether because of its shape or density, does not transmit the sun’s rays; that part of the tail will therefore be cut back, while the other part, declining to the other side, will by itself display the form of the whole tail, which will consequently be declining. And furthermore, when the comet’s body’s situation in location with respect to the sun changes, this inclination will also be able to change, as in 1556, the tail first declined to the right, next was in line with the sun, and finally went over to the left. The same also can be believed about the changeable form, and the reason will be clear why, by P. S. ’s account, the form of a goat was once changed to a spear at the approach of the Macedonian troops. For as an example of these times it is known that the shape of the head tends to change: a terrible beam of light, like flaming clouds, was seen to proffer to two sides and move away in different directions. 

But what if we mingle the Arabian or opinion of the tail with the more recent one, so that some luminous matter really does exude from the head, and indeed in that direction in which it is sent forth, to the sun’s rays, as it were? Then if the tail were to touch the earth, no wonder that the air be infected by a poisonous influence.

I shall not conceal this direction from you, reader, that you might know how to form the image of a comet. Let a ray at the sun enter into the chamber described in Ch. 2 prop. 2, and place a water-filled globe in the way of half of it, so that the sun’s ray falls partly on the glass, partly on the surrounding air. You will see a comet on the wall.

108. Ptolemy, Syntaxis Mathematica II. xxvi. 900, pp. 232–3. Some sources read nimbus, gigantic, for nimbus. Pass, cf. OLD p. 796 s. nimbus. In Plo’s account, however, it was not a nimbus, but a "clean effigies," or figure-shaped, that changed into a spear.

109. The writer of this example is not known; it appears not to have come from Plo.
Chapter 7
On the Shadow of the Earth

We next proceed closer to the astronomical subject of eclipses. Although the considerations of the illumination and of the darkening of the moon are so tightly interconnected, as we have now said above in ch. 6 about the penumbra, many things about illumination still had to be put off to this place, and no completely conclusive opinion was reached as to whether the moon is entirely lacking its own light.

1. On the form of the shadow

The shadow of the earth is a cone, or in the shape of a cone, both on account of the illuminating sun’s being round and on account of the earth’s being round (mountains notwithstanding) and being smaller than the sun. This is sufficiently proved by Witelo and others: see Witelo, Book 2 Prop. 26, 27, 28, where 33 following is clearly intended for eclipses. For it shows that an eclipse of the moon occurs at a central opposition of the luminaries from the earth. The ones preceding this, 31 and 32, appear to be intended by Witelo to state: the reason why, when the shadow has been wrapped around it, the moon is made darker at perigee than it is at apogee, which question we shall here treat explicitly.¹

2. The paradox that the moon is not obscured by the earth’s shadow

And so, these things being as they are, although ever since the birth of astronomy it has been generally admitted that eclipses of the moon result from its entering in to the earth’s shadow, the computation also built upon these fundamentals did not correspond to the outcome with any great degree of exactitude. The greatest problem confronting this undertaking was presented by those things that were considered in chapter 4, on the refraction of the rays of the sun occurring in air. For if the rays of the sun tangent to the earth at sunset come to us refracted, they will therefore pass on refracted, and will shorten the shadow below the place of the moon’s transit. We have to unite this knot, so that astronomers may be freed from doubt, and so that there should exist no one who dates

¹ Theoricae II pp. 71-3. The theorems concern the shape and position of the shadow when one spherical body illuminates another.
quibble about eclipses from refractions, or about refractions from eclipses of the moon.

About center α let the greater circle βγ be described representing the sun, about center β the lesser circle εγ of the earth, and let the common tangents δα, εξ be drawn, extending until they meet at ζ, with the axis αβ. It is obvious that εξζ is going to be the earth’s shadow, if δα, εξζ were to come through unaffected. But now the globe of the earth εγ is surrounded by the sphere of αβγδ, whose medium is denser than the medium of the aether, with the result that δα, εξζ that are going to enter the air at the points a, μ cannot carry through to ε, ζ, but are refracted into ηη, σσ, and strike upon the earth at η, ζ, and are then hindered from proceeding any further. And since the sun illuminates as much of the entire sphere of the air as is presented to it, it sends forth other rays beyond βγ, γη, some of which, without ceasing, will be tangent to the orb of the earth, and will carry on past the earth to the opposite part of the air. Let δα, εξζ be the rays, which are refracted at ηη, μμ, tangent to the earth at ε, ζ. Now, from what Tycho Brahe has established by experience, the horizontal refractions λτ, μζ undergo a refraction of 34 minutes. As a result, the angle λτμ is 179° 26′; and the same amount farther, since by Weigel 10, 9 the same things happen in going out as in coming in. λτμ will likewise be 179° 26′, as well as μμη. As a result, the penumbra shadow of the earth will be equal to νοπνζ, in fact still shorter, as will be clear below, but at the sides of p there is cast some refracted sunlight. It will be weaker by half than the light of the setting sun for us, which can indeed by no means be called darkness, even though it has come in to us through the depth of the air ηη, μμζ. For it makes it day for us nonetheless, and casts shadows from bodies. Therefore, even though the spaces ηη, μμ are twice λτ, μζ, a total extinction of light will not be able to occur.

Further, from these suppositions the altitude δβ of the earth’s shadow, made by the rays ηη, μμζ, is easily obtained, although others present an ever shorter altitude, as will be said below. In ch. 6, sect. 3 above, the angle δβζ was determined as 0° 25′ 36″, from the Ptolemaic suppositions, but in triangle δβε, λμβ and λμε together make 94°. And because λβζ is entirely imperceptible, because the extremely short portion of the air at λβζ is imperceptible in proportion to λμε, which is equal to 1,200 sestertii of the earth, therefore λβζ alone is 34°. Let λα be

2 This is twice the angle GX P in the diagram in ch. 6, sect. 3, and is given at the end of the section, on p. 246.
extended to u. Thus in γευ, γρύ will be 34°, and εςοδ is 12° 48′, half of εςο. As a result, the exterior and opposite angle εςο is equal to the sum of the two, and is 46° 48′.

Continuing, in triangle ζυπ, ζυπ is given as 34°, and ζυπ as 46° 48′, and ζυπ is the exterior and opposite angle and thus equals the two together, and is 1 20° 48′. From ρ let a tangent to the earth’s circle ες be drawn, and let it be ρε. And let the point of tangency ες be joined with the center δ. Thus in the right triangle ρεδ the angles are given, because ρεδ is very small, and is imperceptibly less than ρεπ, 1 20° 48′. Nevertheless, let it be 1 20′. Now, as the sine of ρεπ is to the semidiameter of the earth εδ, so is ρεδ to ρε, the altitude of the shadow. Hence, therefore, the actual point of the shadow comes out to be no higher from the center δ than 43 semidiameters, while elsewhere, from these suppositions, ες was estimated at 368 semidiameters. But when the moon comes closest to the earth, it reaches a limit of 54 semidiameters, making its transit 11 semidiameters higher than the earth’s shadow, bounded by λος, μτ, comes to an end. And thus this demonstration is most true, except to the extent that a departure from the truth is made in the angle of maximum horizontal refraction, which is not of the same magnitude everywhere on earth.

What then is to be said in reply to the astronomers? Is not the entire theory of eclipses, and with it the measurements of the bodies of the sun, the moon, and the earth, as well as the general relative measure1 of the celestial spheres, brought to ruin, its foundations undermined? What must be said, of course, is what is the fact—hear us with unbiased ears, worthy antiquity, for in your honor we cannot forsake the consideration of refractions, most firmly established by Tycho Brahe. This, I say, must be replied: that which comes into the computations of the astronomers, removing the sun’s light from the moon, is the shadow, not of the earth (except in the middle, of which see below), but of the air surrounding the earth. Let the lines βε, εζ be tangent to the sphere of the air at υ and ξ, meeting with μδ at τ. I say, then, that υεζ, the shadow of the air, is that which deprives the moon of its light. And so, as the opaque body is, so is the shadow: the air does contain something of the opaque, especially about the edge, just as every watery globe exposed to the sun casts a shadow. But even so, just as the air transmits the rays of the sun to us, but now reddened, so its double (though in some places single, in some places half or even less) transmits the rays of the sun all the way to the moon; and at the beginning of the eclipse, the moon is not in the shadow of the earth, since it is still in the rays of the sun transmitted through the air. And this is the very thing which we were trying to confirm above in § 4. With examples, that

---

1 The alert reader will note that this is the third distinct job that the point ε for perhaps it should be ‘the point ε′ is required to do. First, it was defined as the point of tangency of the line ρε with the circle ες, then it became the point of tangency of λος with that circle, and now finally it is the point of tangency of ρε. This redefinition allows Kepler to simplify what otherwise would have been an impossibly complicated diagram.

2 Symmeneia. This is the often misunderstood word used by Copernicus to characterize what his system had that Ptolemy’s didn’t, namely, a common relative measure of the parts of the universe. The English word ‘symmetry’ did not acquire its common meaning until after 1800.
the moon was eclipsed while the sun and the moon appeared above the horizon. For since vision occurs through the reception of rays, it will not be the case that our vision, established in the plane of the horizon, will go across in one place, while the rays of the luminaries in another; but the rays of the sun, in going to the moon when it is in the shadow of the air, will take the same path through the air that our eyes indicate.

Here someone may object that the computation of astronomers will be disrupted. For if it is not the earth $E$, but the wider sphere of the air $V$, that casts the shadow, surely, since we are standing on earth and not at the surface of the air, we measure the parallaxes and altitudes of the moon in semidiameters of the earth $O$, not of the air $V$. Therefore, having measured the shadow at the place of the moon’s transit in semidiameters of the earth, not of air, even though it arises from the air, and supposing it to have the same thickness on earth as it has in our measure, arising from the earth, we have accordingly stretched it out, whence it happens that the sun comes out too high for us in our computation. Let $u$ be the place where the moon makes its transit; the thickness of the shadow from the earth would be $ug$, but of the shadow from the air it is $uy$. When we measure this by $ug$, it is exactly as if, in our false imagination, we were to draw the lines $uy$, $ux$. If these are to subtend the sun $by$, it will be necessary to raise the sun up far higher above it. To an astronomer who says something of this kind in objection, I have this one thing to say in reply: that it is indeed truly argued, but is beyond the limits of perspicuity. For from ch. 4 section 6 prop. 11, it appears that the altitude of the air is barely half a mile; even if the horizontal refraction is just 34. But the semidiameter has 860 of such half-miles. As a consequence, when we compute one thousand seven hundred semidiameters of the earth, we are short one of these because of the altitude of the air. But we can be short one semidiameter out of 602, since we have applied all the precision of observation. For this reason, this very slight uncertainty must not be compared with others under which, for necessary causes, astronomy labors elsewhere.

3. On the redness of the eclipsed moon

Let anyone be undecided about what was said, that the moon is still illuminated by the sun’s rays, admitted and refracted through the air; even when the eclipse is total, he should carefully attend to total eclipses of the moon: he will find this illumination flowing into the eyes. This is not a case of someone’s having mentioned this fact recently, for the sake of refractions, so as to fit a suitable epilogue to the tale. The thoughts of the ancients survive, and it is for just this reason that the opinion originated that that redness is the moon’s own light. In the Commentaries on book 4 of Ptolemy, 8 Theron philosophizes thus:

"As regards the eclipse of the moon, the earth brings this inconvenience upon the moon. For since the moon is an eternal body, participating in the divine

---

5 This is half a German mile, or about $2\frac{1}{2}$ statute miles (4 kilometers).

6 Theron of Alexandria, Commentaria in C. Ptolemaei magistri cosmographi libros, 4. 1, p. 956.
nature, and immune to the modifications that as a rule follow upon coming to be and passing away; it accordingly will not subject itself to alterations or diminutions. But since it is endowed with its own rather weak light, which cannot easily reach the powers of vision, it appears more brilliant by another light, borrowed from the sun, so as to illuminate the earth and the air. Thus when the earth intercepts the solar radiations, the moon is then deprived of the solar light and, darkened by the cone of shadow, it again comes to be weaker and darker. It does not itself undergo any alteration from this illumination, nor from the privation of the solar light, but since it is free from corruption, it always preserves the same color and appearance which it has as its own, and the same magnitude. Further, that the moon, though indeed always illuminated, is in certain full moons not illuminated with its usual brightness, but is darkened, they have called an "affect" and a "labor", because they believed that it undergoes that alteration in its own activities.

And below:

The moon indeed does also have a light of itself, but weaker, as we have said.

Cicero, in book 2 of the *Cicerotheoria*, follows Theon: he blends the light of the moon from the sun's light and its own. Reinhold, most recently on fol. 64 of the Commentaries on Pseustach, says:

There is in the moon a certain special or rather dim light, and its quality is openly shown by the total eclipses of that body, in which the entire orb is perceived with a foul and horrible color, which, however, is redder in some places, when the moon is higher and lifted farther outside the ecliptic, darker in other places, that is, to the extent that it is lower and for that reason more deeply immersed in the shadows of the earth. 7

By these final words, he transparently refutes what both he and his predecessors had said, that that light is the moon's own. For if it were the moon's own, it would appear more brightly out of the darkness. For thus also, the light of the moon that is borrowed from the sun is brightest at night, while by day it hardly moves the eyes. Now, by the testimony of Reinhold, the opposite happens. For upon entering deep shadow, it appears darker, higher and nearer to appear; it is redder, as well as when it verges to the side of the shadow to the north. Whence it is understood that at the edges of the shadow the moon finds a confused light, which I say is this very light that is transmitted refracted through the body of the air.

For this reason, Plutarch, more ancient than all of these, speaks rightly in the book *On the Face of the Moon*, when he introduces Pharnaces, arguing that, like the other stars, the moon has its own light. "It does not go straight into hiding in

---

7 Pseustach, *Theoriae nova*: ed. Reinhold: Wittenberg 1542 fol. 70r; Paris 1553 and 1556 p. 102. The relevant passage is under the heading: "De Illuminatione Lunae."
eclipse, but shines with a certain color suggesting a coal, a terrible color, which is its own." He himself objects in reply as follows:9

We see the eclipsed moon take on different colors in different places. Mathematician distinguish these on the basis of time; thus, if the moon is eclipsed in the evening, it appears horribly black, right up to the third hour and half an hour beyond. If at midnight, then it gives off that scarlet and fiery color. From half past the seventh hour, redness is shown, near down it now takes on a blue and hush countenance.

This rule in fact is false, and does not do what it promises to. The Arabs have prescribed a different one for us, on the color of eclipses, which Cardano, in the supplement to the Almanac, and in the conjunctions on the Retractation from Alfonso and Linerius, accommodates to the nearness to the node and to the apogee, which are more or less in harmony with Rinhold's tradition, although the varying constitution of the air seems not to permit a rule. But Ptolearch carries on, and accommodates this objection of his. "One might say instead, (and excellently, in my opinion), that this coal-like light is foreign to the moon, and is instead a mixture of the eclipsed light and that shining through the shadow, while its own light is black and earthly." But since it is a question of the causes that spread this light within the shadow, he brings in the other stars standing about the sun. The cause of this particular redness is very weak, although below his argument will also be kept for the rest of the colors. Our6 Witeh, when he was able to fully complete, deny the moon in its own light in book 4 prop. 72, appears to have stated book 2 proposition 31 at this redness of the moon in eclipses.12 He says there, "The shadow is less shadowy at more distant parts." Sound words, if the cause be sound. For the rest, Witeh ascribes the cause to the vision, which compares the shadow and the more luminous or weaker rays, placed next to each other. For the rest, the moon in this way would undergo nothing else in its body, traversing the shadow farther out at its darkest part, and so this phantasm of vision contributes nothing to this redness that it has. The cause, however, is entirely in the refractions, so that redness is nothing else but the illumination of the moon

9 Ptolearch, On the Face in the Moon, XLII 331D and 934C-D, pp. 132-3 and 136-7.
10 I have not been able to identify this work.
11 Cardano, In C. Polonaeae Quattuorpartitae CEutactiunae Libros Comentmoriae, Basel 1555, II text 52, p. 318-9, citing the "tables of Alphonsoius and Linerius." Cardano's table relates the color of the eclipsed moon to the sun's distance from the node ranging from black (close to the node) to light yellow (farthest). Cf. Ptolearch, Tetratabla, II 9. "Alfonsoius" refers to the astronomical tables prepared in the thirteenth century under the patronage of Alfonso X of Castile and Leon, which circulated in a number of versions and were the most commonly used astronomical tables even in the sixteenth century. "Linerius" was John of Lignacius, a fourteenth century Persian mathematician largely responsible for the diffusion of the Alfonsoic Tables, which he supplemented with tables of his own. Cf. DSF VII 132-8.
13 Witeh in II pp. 151 and 72, respectively.
14 Kepler has begun both this sentence and the preceding one with "caecum."
by the sun’s rays, transmitted through the density of the air, and refracted inwards towards the axis of the shadow, as becomes clear from the following examples.

Frotharius in A.D. 926: “The moon was eclipsed on the Calendars of April, and was changed into a pallor by the small part of the light that remained, as if it were a copy, and thus, as dawn was breaking, it was all changed to a bloody color.”14 Therefore, it grazed the shadow from the side, since it carried this redness before it.

Conelius Gemma relates in Cosmocrētice Book 2 fol. 64.15 In 1569, on March 3, “at the third hour of the morning, Phoebe suffered a terrifying eclipse, marked with frightful colors. First,” he says, “disky, it next shone out bloody, soon after scarlet, and green, and slate colored, and finally unsightly in unbelievable variety.” Here he practically said that in the moon were seen the colors of the rainbow. Go now, and place a spherical glass filled with water in the way of a ray of the sun entering through a small crack into a dark chamber: you will see all the colors of the rainbow on the opposite wall. Further, these colors are always the offspring of moisture, which the ray of the sun traverses. But what moisture is there that the sun’s ray could traverse, other than the humid sphere of the air? Indeed, the moon was towards the perigee, but was traversing the southern part of the shadow. And so in both instances it was in the part of the shadow that to use Wielo’s words, “is less shadow-forming”, that is, diluted by many rays of the sun that have undergone refraction in the thick air of the north. For when Saturn had a configuration with Jupiter, and Jupiter with Mars, and the sun with Mercury, they aroused a vaporous air from the warming recesses of the earth, as completely unravelling experience shows us. Nor did the seasonal weather happen to give moderate assistance, like that in another year (1597), which so greatly thickened the air in those places that the phenomenal refraction of Zembla occurred.16 But what Gemma adds, “the third hour of the night,” he seems to write from Stadus’s Ephemerides rather than from a sure observation.

For the middle of this eclipse could not have appeared far from an hour after midnight, by the evidence of Tycho’s computation from reliable experience.

I remember when I was still a boy in Württemberg being called outdoors by my parent to the contemplation of an eclipse, at 10 p.m. at night; the moon appeared entirely reddened all over. The circumstances attest that this could not have been anything but the one that was seen on 1580 January 31, whose northern

---

14 There is some uncertainty as to which Frotharius is meant. According to Frisch, JKGW II p. 420, Frotharius was an Italian monk who lived around the end of the tenth century, and wrote a Chronicon et Caesaris Osiaciani ad annum 990. However, Hammer, JKGW II p. 452, cites Frotharius, Annoles Rhenenses, Mon. Germ. Hist. Ser. III 376. This Frotharius is presumably the Frothard who was born in 893 or 894, probably at Epernay, France, and died in 966, probably at Reims. He wrote Annoles, a chronic covering the years 919-966. His works are in the Patrologia Latina Vol. 135; there is also a separate edition of Annoles by P. Lauer. Paris 1905.

15 Ars cosmo-cetricus is an alternative short title for Gemma’s De naturae divinis charac-
teristicis (Antwerp, 1575).

16 Cf. IV 9, p. 151.
latitude Maestlin, in the *Astronomia,* said it was a little less than the sum of the semidiameters, for which reason it was entirely hidden in the shadow for a very brief time. Compare it with Fredseus’s.

On 1588 March 3, in the morning, in the Tycho observation of the total eclipse of the moon, I find, at 2h 58m the annotation, “The moon was now seen brighter towards the east,” and because of this these words were added: “There-fore, it had passed the middle,” that the middle followed by a few minutes, as is clear from the beginning, end and other phases. Here, in one and the same eclipse, the moon at one part gradually puts aside, in the other puts on, that redness which the ancients considered the moon’s own light. And since it had not yet gone to the middle of the shadow, it shows, through the redness of the eastern bump, that the rays of the sun refracted in the terrestrial air are wider near the western than near the eastern part of the shadow.

I saw a similar eclipse of the moon at Graz between 30 and 31 January, or 9 and 10 February, 1599, concerning which see also above, ch. 6 sect. 9. For when the moon was just immersed in the shadow, with a very small part of the light from the sun remaining, it was nevertheless perceived almost totally by its redness, so obvious was the redness. Next, opposite the light from the sun (a little above), it appeared to be eclipsed again; this is, the redness itself gradually gave way, or rather, the moon on its eastern edge entered into the darker part of the shadow. The last light from the sun ceased at the high point of the redness, so that you would not know how to distinguish it, except that over a large area at the horn it had a red color, and it was equal, but not brighter, in the middle. A little before it moved itself beyond the mountains, even though it was in the bright twilight and the moon was deeply immersed in the shadow, the redness could still be seen, and further, it could even be distinguished that the western part was still the brighter. It was, nevertheless, about the middle of the shadow when it set, for no light appeared through the air, and no trace of it shortly afterwards, the air near the mountains being quite humid and thick.

For something similar, see also below in ch. 11 probl. 31, of the eclipse of the moon of 1603 8/18 November. But no clearer example can be supplied than what I saw in 1598 6/16 August near Graz. The moon rose in darkness, the sky being overcast near the horizon, so that the rising moon could not be seen. When it now had several degrees of altitude, the clouds opened up and it shone forth, with half of its body very bright with redness, so that it could not be considered eclipsed in that part, half being barely visible, even though the whole was in the shadow. I, being unused to this redness, was amazed, for I was unable to reassure myself in any opinion.

15 *ZOD.* 11 p. 358. Kepler’s words are slightly different from Tycho’s as transcribed by Dreyer, which are as follows: “N. B. It had now passed the middle, because the moon was brighter towards the east.”
19 *De nuptiis.*
20 See p. 414, below.
of whether it was entering the shadow or leaving it. It also powerfully held me in doubt, why the shining part was not clearly distinguished from the dark part, for the redness did not have a boundary, but was continuously diminished and finally faded out from the greater part of the moon into darkness. While I was thus contemplating, the clouds again interposed themselves. A little later, when three quarters past seven were ringing in the town, the departure of the clouds dissipated all doubt. For the bright horn had risen below, and I saw the redness vanish and become like the darkness, so great was the height of the narrow horn illuminated by the sun. The redness did not lie uniformly around this horn, for the redness was spread out most widely on the upper left side, and it was narrow at the lower right, between the bright horn and the part that was completely dark.

At that moment, I captured the phase of the moon like this. At the ninth hour on the town clock, it appeared completely restored, more than 5 quarter-hours being taken up in its exit, which agrees excellently with the Brahmic observation taken at Wittenberg. What I am seeking here can be said to be different from the idea that this redness arose from the rays of the sun refracted in the earth's air, the measure of which refraction was different in different places on earth at that moment.

For if the heavens, standing around the sun were to color the moon so, they would color its whole disc, equally set out before them, uniformly.

The entirely dark part, here and in the preceding eclipses, would have been in the full shadow of the earth, the narrow part in the shadow of the air, which was transmitting the refracted rays of the sun, and finally, the dark part, free from both shadows, would have been in the grasp of the pure sun, or of some part of it.

4. On the pallor of the eclipsed moon.

It is now appropriate to take note, though rather inadequately, of what concerning the moon's body is hidden in the earth's shadow. Thus, in 1588 March 3 at Maulbronn in Württemberg, when the moon was in the middle of the shadow, I was barely able to perceive it with the eyes because of the ashen color, and I was amazed when I remembered the one I had seen in 1580. In Denmark, when they measured its distance from the fixed stars, they had such difficulty picking out its

21 Sidern. This usually refers to heavenly bodies, but the context here and below suggests that Kepler meant the glowing "etherial air" surrounding the sun.

22 Where Kepler was a student at the Stiftsmühle from 1586 to 1589.
erges that the observed distances sometimes appeared to show it to be retrograde, near the middle of the eclipse.

So is it this pallor or ashen color that belongs to the moon's own light? We still do not have any need to say even this. In fact, we may rather reply here, with Plutarch in the *Face of the Moon*,23 what we were unable to reply above on the redness of the moon, because of its brightness, in this way.

Now since here on earth, on ponds and also rivers, receiving the rays of the sun from purple and crimson awnings, the shady places nearby imitate that color, and because of reflections of whatever origin, are illuminated with multiple splendors, what wondrous thing is it if an ample flow of shadow falling celestially as if upon the sea of light, which is not calm or at rest, but is stirred up by numberless stars, and receiving various mixtures and alterations into itself, carries back from the moon hither different colors at different times?

I have recounted these words because they seem to smack of an experience like that from 1566 which Conclus Gemma had set forth above, on the colors of the rainbow in the moon. Therefore, as concerns the opinion of Plutarch, if various colors of this kind appear in the moon, I would say, as above, that they arise from the humid sphere of the air, refracting the sun's rays, but not at all from the fact that the light of the heavenly bodies in the heavens themselves is so obviously marked out by this variety of colors, and that it imbues the moon with similar colors. If, however, it is a question only of the very faint pallor of the moon, or the ashen color, whence it comes to the moon, and whether it be of the moon's own light, I shall reply with Plutarch that this spark of light is acquired for the moon from the heavens standing about the sun, or whatever else is luminous in the heavens. This I shall do, though Tycho was impartial, who believed, above,24 (though less thoughtfully) that Venus alone is sufficient as to acquire light for the moon so strongly, in order that, when it had come out from the sun, it may thes-by struggle its way out through the exceptional brightness of the twilight.

No right is so black, even if the densest clouds hold the sky, that if you were outdoors you could not distinguish between snow and coal, between sky and mountains. Accordingly, in the densest part of the earthly shadow (which brings about the night) it never happens that absolutely all the rays of the stars are excluded, or that nothing of these terrestrial objects is illuminated. Why then would it not be even more true, that in that pure aetherial air, in which the moon has its passage, existing sixty semidiameters of the earth above this misty air of ours, and with the shadow now drastically weakened, the heavens might be able to do the same to the moon, lying in the earth's shadow, that they can do here in the mountains, in clouds, in snow, in coal? Not to mention that some of the rays of the sun refracted in air reach even to the middle of the shadow. As a result, with the support of each lucid cause of this spark of light, there is no need to ascribe to the moon its own light.

24 See p. 275, above.
5. Problem: To measure the refractions in the regions at the greatest distances by observation of eclipses of the moon.

In section 2 above, from a horizontal refraction of $34^\circ$, we marked the limits of the shadow of the earth from the outermost refracted rays of the sun, tangent to the earth, it being determined as 43 semidiameters of the earth long, and it was said that when the moon is lowest it crosses 11 semidiameters higher. In this way, this shadow of the earth never would touch the moon. But in section 3, when an account was given of the examples that were presented, we fully granted to the authentic shadow the darkening of part of the lunar body. In order that no one think something contradictory was proposed there, I ask that the following be considered; that refractions are not the same in all places, as was sufficiently stated in ch. 4 sect. 8 above. For in Kasel, in an inland region, they are found to be less. Next, it must be carefully considered which particular region of the aerial substance the rays refracted by the humid substance of the air can pass way cross. About center $n$ let the greatest circle of the earth's circle $b$ be drawn, and about this the wider circle of the air $a$, and let $be$ be the circle of the illumination of the air, and let the rays $\delta b$, or $\lambda b$, on the edges of the sun come together at $\pi$, forming the boundary of the shadow of the air, which causes the eclipse of the moon. Now let the rays $\delta b$ tangent to the sphere of the air at $\delta$ be refracted into $\delta e$, $\eta b$; and, existing there, let them again be refracted into $\zeta b$, $\eta b$, $\zeta b$ will be the nearest boundary of the shadow. For as regards the parts of the sun between the rays $\delta e$, $\eta b$, they do not reach $\delta$, $\eta$, but, tangent to the sphere of the air above, towards $\kappa$, $\lambda$, they make the same angles of refraction, and, since they entered at a higher place than $\delta$, $\eta$, the also exit at a higher place than $\zeta$, $\eta$, and thus become farther out than $\zeta b$, $\eta b$. Again, let $\beta$, $\gamma$ be places on opposite semicircles of a meridian of the earth, at which places, when the center of the sun is placed upon the line $\alpha a$, the entire sun is seen from both, refracted, grazing the horizon with its lowest edge, so that $\mu \lambda, \nu \lambda$ are refracted into $\kappa b, \lambda b$, and thereafter into $\beta b, \gamma b$, cutting $\zeta b, \eta b$ at $\pi$, $\pi$. By what was demonstrated in ch. 5 sect. 3 prop. 9, it is necessary that if $\zeta b, \mu b$ had been perfectly parallel, and likewise or, $\nu b$, still $\lambda b$ and $\eta b$ would meet at $\theta$ nearer the earth than $\mu b$, $\nu b$. This is now much more so when these lines are inclined, because $\zeta b$, or by supposition come.
from the sun’s upper edge, μνκ. λν, from the lower. Thus χ will be more distant than αν, κν, and the other boundaries of the refracted radiations. Now whatever radiations besides μκνκ should fall from the upper parts of the solar body upon κ, λ, will by that intersection encounter the earth, since μκνκ, are the last of the ones that do not encounter the earth but, because of the refraction that occurs, are the last of the ones that are tangent to it at μΚ Κ. With these things supposed thus, let ζαή αφι, δε, δε, be extended until they cut each other at χ, and farther, until they cut δη αγ, θτ, and εκ at ζ. θτ, jointly. There will thus be a conical region with a double vertex ριομιοι in the middle of the shadow, in which the refracted rays falling down circularly are gathered, around an altitude from the earth of 43 semidiameters. Again, in the earth’s shadow there will be a certain oblique parabola, ending in a point, which is represented in two places by the spaces GPA. ηο. ηο, in which the refracted light of the sun is spread circularly, but more concentrated at the circle P. Q., where again the hollow conical region begins, represented by Pho. ηοηο in which again the refracted light of the sun is spread, but now more weakly, because it is over a broader region. But whether either of these serves to illuminate chaosmata, which nearly always face north, the natural philosophers may judge. Finally, there remains the conical region of the shadow, hollowed out on both sides, represented by ζαή αφη and ζαή αφη, through which the refracted light of the sun is much more broadly spread out, and which is much higher than 43 semidiameters of the earth, and is hollowed out in the middle by the pure shadow P. Q. This, then, is the place that the moon traverses.

In order that we may measure this region, note first that, in ch. 4 sect. 6 prep. 9 above, the refraction of a ray from the aether tangent to the air (on the supposition that the ray that is tangent to the earth after refraction receives a refraction of 34° was 1° 1° 30′. Therefore, angle ζαή is 178° 58′ 30′. And hence the arc ζαή is 2° 1′, by Euclid 3. 2. And since from ζαή above, and ch. 6 sect. 3, p. 246, and εκ δη is 12′ 48′, and εκ δη 8′, δη will be 39° 47′ 12′. When 8νις, 2° 3′, is subtracted, 30°
will be 87° 44′ 12″, which is ζΔ. And because ζΔ is tangent to the circle, ζΔ will also be a tangent, by Thetio X 9.27 Therefore, αζΔ is right. But before, ζΔ is was 87° 44′ 12″. Therefore, ζΔ is is 2° 15′ 48″. Hence, where the semidiameter of the air αζΔ is 1, αζΔ becomes no longer than 25′ semidiameters. And because ζΔ is is 2° 15′ 48″, αζΔ will also be that much. But the exterior angle αζΔ is equal to the two interior and opposite angles αΔ and Δ, and αζΔ is 12° 48″. Therefore, αζΔ will be 2° 28′ 36″. But since the circumference 86° is was 179° 34′ 24″, as was said in ch. 6 sect. 4.28 and ζΔ is is 2° 3′, αζΔ will be 177° 31′ 24″, half of which is 88° 45′ 42″, which is ζζΔ or αζΔ, because ζΔ and Δ are tangents. As a result, αζΔ is is 1° 14′ 18″. Hence, where the semidiameter of the air αζΔ is 1, αζΔ becomes 46′ semidiameters of the earth. Here, then, the region illuminated by the sun’s refracted rays begins.

Again, because the upper arc βγ, by those things that have been supposed (and by ch. 6 sect. 8), had to be 179° 23″, if the rays of the sun had come through unretracted, while the two observers at β and γ see the sun 34° higher than it should be, that arc is therefore increased by 1° 8′, and becomes 180° 31′, with the result that the lower arc γδ is 179° 29′. Half of this, 89° 44′ 30″, is βγ, and consequently βγ extended, together with γδ, encloses an angle of 15° 30′, and the exterior angle βγδ is equal to the sum of that angle and the supplement of [βγδ or 34°. Thus βγδ is 49° 30′. And since βγ extended cuts δβ between β, δ near β, it matters little whether we take βδ or something slightly greater, yet something much less than δβ, in the place of a measure. Now, since βγδ is 89° 44′ 30″, and αζΔ (let it now be αζβ) is 49° 30′, αζβ will be 89° 26′. Therefore, where a certain line that is nearer to αζΔ than to αζβ comes out to be 70. Thus αζβ would be distant from the earth by 70 semidiameters of the earth, that is, much farther than p, δ. However, the rays from all the particles of the sun are not refracted all the way to 70 semidiameters, but only those from the edge, while the point where the rays of all the particles of the sun begin to be refracted was proved above to be raised up 43 semidiameters. And since p, δ are 46 semidiameters distant, while t, u are much more than 70 semidiameters, and the region pt, tu is illuminated by the refracted rays of the sun, and the moon makes its transit between the stated boundaries, between the altitudes of 54 and 66 semidiameters, as astronomy bears witness, it is therefore obvious that while it is in the shadow of the air, it is nonetheless somewhat illuminated by the sun.

Again, when the simple shadow τγδ is bounded at the apex χ at a distance of 70 semidiameters, the moon therefore would not make its transit through the simple shadow, because it transitions lower, and nonetheless when τγδ (if now it be understood as the region lacking any particle of the sun, even shining refractively) is bounded by the apex χ at a distance of 43 semidiameters,29 the moon, which transits higher, will accordingly pass through the point τγδ taken in its second

27 Theophrast. II pp. 413-4. This proposition shows that the direction of a ray is reversible.
28 This is not from section 4, but the very end of section 3.
29 JKGW II, like the 1644 edition, has a period here: following Frisch I am replacing it with a comma, which the sense requires.
sense), so that when it enters into the beginning of the air's shadow, it neverthe-
less enjoys the refracted view of the entire sun, but where it encounters the middle
of the air's shadow, it no longer will enjoy the full view of the sun, but only a cer-
tain small part. Hence what was noted from Gemma is entirely consistent, that the
colors of the rainbow are sometimes seen in the moon. For in a watery globe it is
perfectly certain that where a particle of the sun shining under refraction is hid-
en by the actual occultation of the refracted rays, colors arise one after the other,
according to whether much or little of the solar body will have been shining. You
thus see a remarkable consistency of nearly everything that has been presented.
If anyone should wish to refute it by false assertions, he will have to be artful
about it.

There remains a single observation put forward in just this context; and
though it is now almost entirely removed, it belittles a lover of truth not to hide
anything. In the above examples, the moon was divided, as it were, into three
parts, the bright, the reddened, and the hidden, and the bright was divided
from the reddened by an evident line; while the reddened was not
divided from the obscured by an evident line or limit, but the
reddened faded out successively into the obscured. It therefore
appeared completely consistent that what was perfectly reddened
was illuminated by the whole
body of the sun, refracted, while
what was perfectly obscured (to
the extent that it happens to
be obscure) plainly lacked all
exposure to the sun, and the
uncertain part in between was
exposed to a part of the sun. As a result, the simple shadow
and its apex ought not finally to arise from the seventeenth
semidiameter of the earth, but far
below, so that in the place of the moon's transit, some small part may be in the
simple shadow. The question arises, how this is achieved? The answer is easy,
if the horizontal refraction is assumed to be greater, for then D? D? will meet at
a lower point. Thus in August 1598, at AG is the ecliptic and ACG the earth's
shadow, CD the southern part, then the end F of the refracted rays will come
from a lesser refraction, F from a greater. Thus in the north, the refraction was
greater than in the west (because A is the eastern part of the shadow).!

Here Kepler refers to the following endnote:

To 292: The reasoning is as follows. DE are the southern parts with respect to the
eclipses. AG; consequently, as you see in the diagram on p. 279, they will be illuminated
by the refracted rays of the opposite region on earth, i.e., the north. And because a small
surface of the moon, namely, DE, is illuminated there, this proves that the refraction
of the north is large, by which it happens that if 279 the region TG becomes closer
to the earth, and thus fills up the moon's transit broadly. In turn, EF are eastern parts

---
But so that it may be known, or rather, gathered from the observation of an eclipse of the moon, how much greater a refraction is required, let \( K \) be the center of the moon, \( 54 \) semidiameter distant from earth, because the moon was at apogee \( H \) the center of the shadow; and let us suppose that it is at its nearest to the observation, and the beginning of the obscured part would have been quite far from the center of the shadow, by 20 minutes at \( T \), \( 36 \) at \( E \). In the following figure, let \( M \) be the vertex of the cone of full illumination [crepuscular zone], from chapter 6 section 7. \( N \) be center of the earth, \( F \) or \( E \) be the boundary of the refracted ray, so that \( O N = 36 \)°. \( O F \) is \( 36 \)°. Therefore (by ch. 6 sect. 71) \( 12N \) will be about \( 38 \)° \( 30 \)′, whether \( M \) be tangent to the earth or to the air, because the ratio of the air to the earth is imperceptible. And because \( 12N \) is \( 1 \), \( NO = 54 \), \( 10N = 1 \), if \( OF \) be tangent to the air, not to the earth, there will be added to \( 12N \) less than a thousandth part, with the result that the angle will come out imperceptibly greater. And because \( ONM \) is one straight line, the sum of \( 10N \) and \( IMN \), which is \( 1 \) \( 22 \)° \( 30 \)′, subtracted from both right angles, leaves \( OIM \), \( 178 \)° \( 37 \)′ \( 30 \)″. Subtract \( O1F \) or \( OFE \), for it [i.e., each respectively] is approximately as great as \( ONF \), \( ONE \), therefore remains, for the former, \( 178 \)° \( 17 \)′ \( 30 \)″; for the latter, \( 178 \)° \( 1 \)′ \( 30 \)″. Let \( TR \) be applied, tangent to the earth, cutting the air at \( R \), so that \( R1T \) is isosceles: the refraction, \( 1RT \) or \( 1TR \), will become \( 51 \)° \( 15 \)′ for the former, \( 50 \)° \( 15 \)′ for the latter. But from Tycho Brahe we had accepted \( 34 \), the amount it is in Denmark. However, in ch. 4 sect. 8-9 we showed that at different places and times the refractions are either greater or less, so that there is nothing absurd in this, that in a single eclipse in certain places the greatest refraction becomes \( 51 \)° \( 15 \)′, when the greatest and completely total refraction can be \( 61 \)°, as was said above.

It is, however, not necessary even to state that this, however slightly, it differs, from what would be entirely satisfactory, that what is cut off by \( EF \), the boundary of the dark part, was in the ample shadow, nor would anyone wish

of the moon, since \( A \) is the eastern point of the eclipse. As a result (in the diagram on p. 279), these parts \( EF \) will be illuminated by the refracted rays of the opposite region on earth, i.e., the west. And because \( CE \) is to a great part of the moon, this proves that the refraction is small, by which it is brought about that the region \( 12K \) becomes farther from earth, and it barely touches the moon with a narrow point.

31 Here Kepler refers to an ending switch, because of its length, is included in the appendix at the end of this chapter.
32 See the footnote on p. 253.
33 Here Kepler refers to the following ending.

To 283. In fact, there is another valid exception, namely, that everything is set up and measured from the extremity of some perception, particularly p. 282 l. 34
so be content with the statement that that part lacked the greatest part of the sun shining under refraction

In order that it may also be shown which regions of the earth the refractions now investigated arise, let us proceed thus. The observations of Tycho adjusted to Uraniborg give evidence that the moon began to emerge a little before 7° 50' 1, however, when I observed it now first shining forth, saw it increased by a digit14 or a little less; therefore that would have been at 7° 53' 15 at Uraniborg. Its degrees I about 117. 16 The sun was on the meridian this many degrees in the west in the region of Dacoton in America. 17 The sun was at about 23° 20' Leo, with a declination of 13° 47' north. Therefore, with one foot of the compass on the intersection of the said meridian and the parallel, 18 and the other extended to measure a little more than 90 degrees, the circle of the illumination of the earth will be drawn, from whence the shadow arose. This circle cut the earth's equator in the west, between New Guinea and the Solomon Islands in the desolate ocean; to the east opposite the African Guinea beneath the meridian of the west coast of Spain. Towards the south, however, it touched upon the Magellanic landmass, which looks towards Peru towards the north, the pole itself being held beneath the sun's gage, the circle swung across the Batavias' winter place, beyond Mesopotamia. 19 And since at that time at a polar altitude of 47°, the rising point was about 18 Piscus, from Copernicus's table of the angles of the ecliptic with the horizon on fol. 42, the intersection of the horizon and the ecliptic, GCHC is the next to the last diagram, was about 26°. 20 Therefore, in the right (spherical) triangle ZXY, the angle X is given with the base ZX = 25°, from 23° Aquarius to 18° Pisces. 21 Hence and 284° 20', and here, from the reliance upon the sense of vision, we easily make quantitative errors.

14 One twenty-fourth of its diameter, or about 1°. 1 See the note on p. 71, above.
15 This is how far the moon has apparently moved in the 7 hours 52½ minutes since noon.
16 According to the maps equidistant at Kepler's time, most notably that of Abraham Ortelius (Theatrum orbis terrarum, Antwerp (1570) and maps) other eds., the difference in longitude between Denmark and the Yukon Peninsula was about 120°; in fact, however, it is only about 100°.
17 This is at 13° 47' north latitude.
18 This is a reference to the Parent expedition to the arctic, for which see Ch. 4 sect. 9, above.
19 On the Revolution II 10, tr. Rosen, p. 76.
20 For the sake of economy, Kepler constructed this diagram with the preceding, placing the vertex X at the top in order to fit the triangle in. It has been rotated 90 degrees counterclockwise here to erect it correctly.
At this place in the text, Kepler refers to the following elucidation:
So 284° 20' be the rising degree, Z the moon, if the azimuth on the horizon, and XZ the intersection of the ecliptic with the vertical.
In the diagram, XY is the horizon and XZ the ecliptic. As Kepler says, the moon is at Z. If Y is the point on the horizon directly below the moon; hence, angle ZXY is right. Angle X was just found to be about 20°, and the angle on the ecliptic is 25°.
is obtained $XZ\over Y$, about $71^\circ 45'$. However, the luminous horn $CD$, which $k$ as close as possible to the observation, was also declined by about $20'$ from the vertical through the center of the shadow or of the moon. Therefore, the circle which passes through the centers of the moon and of the shadow cut the ecliptic at an angle of about $51^\circ 45'$. And the paths $F$ of the shadow were closest to the ecliptic, while $E$ was closest to the vertical. The former were cast forth by the Solomon Islands and New Guinea, while the refracted rays that were spread in between underwent refraction in the innermost deserts of Africa. The latter, on the other hand, arising from the Magellanic landmasses, were spread in between by rays of the sun refracted in Finland and the places around it. Thus the refraction was greater, at this moment, in our north than in Africa.

As much as I was able to accomplish in this, not following the beaten path of any antecedents, I have presented. There is nothing to prevent this theory, having arisen from these contemptible seeds, from growing to some degree of usefulness. This may you, whoever you are, a lover of the truth and ardent for the knowledge of things, not trample upon these feeble first sprouts with mockery; instead, may you try to support them and draw them out with applause.

Kepler's note on the 'cone of full illumination' (p. 293)

[The note begins on p. 447 of the 1604 edition.]

To page 283. The reason for my using the cone of full illumination.42 and its vertex $M$, which is a point between the earth and the sun, here, is to be sought on p. 279. For when the sun is so placed that $n\alpha$ and $n\alpha$ extended are tangent to the edges of the sun, $n\theta$ and $n\theta$ are drawn in such a way that when the true refractions happen at $n\theta$ and at $n\delta$, they would both be tangent to the earth at $n\beta$. And $n\alpha$ would contain the opposite edges of the sun with the lines $n\alpha, n\alpha$. Now if $n\alpha$ on the right is tangent to the left part of the sun, and $n\delta$ on the left is tangent to the right, they therefore previously meet in at $n\alpha$ on p. 239). On this account, even if $n\lambda, n\delta$ extended are not actually

---

42 overturned.
tangent to the earth by (for λγ, κδ are different lines), they are nevertheless nearly equidistant from the tangents because of the enormous distance of the sun. They therefore make about the same angle with each other that the lines marking on the circle of full illumination make. Further, from the same diagram on p. 279, it is apparent that those lines τς, υθ fall upon the parts of the moon which are indicated by the letter Ḵ, Κ on p. 282 [that is, the diagram that goes with the August 1598 lunar eclipse], for this is presupposed here. Therefore, in the same way as on p. 279, where from the known angle which Ḵτ makes with nor extended upwards (that is, 18° 30′), on p. 241 l. 22, and from the known quantity of refractions which the tangent to the earth λτ undergoes at λ and τ, 34 at both places, both the angle 170°, 49′ 30′, and ζτ, 70 semidiameters, and the divergence of γτ from γτ at any given altitude, are easily known; so here, in turn, the angle 18° 30′ remaining the same while supposing a divergence of γτ from γτ (on p. 283, of RF or RE in the opposite direction, and that at a certain altitude from the earth, the quantity of the refraction at τ, λ (on p. 283, in τ, Ḵτ), is easily known. And this very thing takes place on p. 283, except that, for the sake of convenience, things that are very close to each other are taken as equal. You should not be disturbed that the angles Ḵ at the eye are not really on earth, or in the air, but are outside, for they do not differ perceptibly from those that ought to have been set up at R, or at some other nearby point.
Chapter 8

On the Shadow of the Moon
and Daytime Darkness

For the sake of greater ease of understanding, this enquiry was pertinent to those things which we are going to say below on the quantity of the moon’s diameter. But the elegance of method does not allow separation of the shadow of the moon from the shadow of the earth.

1. The occasion of this enquiry

When Tycho Brah, the Hipparchus of our age, noticed that eclipses of the sun, whether the ray be allowed in through a notch or received by the eye, always show the moon’s diameter to be much less than it appeared as oppositions, he first conceived the suspicion, which he at length defended in the place of a just opinion, and openly professed in volume I of the *Programmata*: that the moon in conjunctions does not maintain the same visible diameter that it had at oppositions, but by the force of the solar light its edges are made thinner, a certain optical argument lending support. As a result, as if this were a universal thing, he displayed a special table of semidiameters for new moons. And when in 1600 he wrote to Clavius, he said he wondered at Clavius’s having seen a total eclipse in Portugal in 1560, and night-like darkness during the day. For his own observations do not allow the whole sun to be covered by the moon, no matter how low it is. Besides, that this is not to be established by this small table of semidiameters, the eclipse of the sun has taught, which, in December of 1601, followed directly upon the death of Tycho, of which an announcement is made in an appendix to the *Programmata*. As regards the opinion itself, this must find in any event be

2. Near the end of Chapter I of the *Programmata*, Tycho wrote: “It has been observed that in the eclipse of conjunction of the luminaries, the moon does not retain the same visible diameter that it has in other places, but its edges are contracted by the power of the solar light, optics providing this with a certain explanation.” He then provides a table of characters with a special column for conjunctions (*TBOK* II pp. 147-8).
3. Christoph Clavius (1538-1612), Professor of Mathematics at the Collegio Romano, the preeminent Jesuit college, and author of a widely read commentary on the Sphere of Johannes de Sacrobosco (15th Century). Many editions of this work were published both during and after Clavius’s lifetime, and he continued to revise it. In the edition published in Rome in 1581 (p. 425), he described the eclipse as having occurred in 1559; however, in the Rome 1607 edition (p. 564), he gives the year as 1560. No day of time is given in either edition. Cf. Lalande, *Between Copernicus and Galileo*, pp. 44-5. Brah’s letter to Clavius seems not to have survived.
granted, that the full moon appears larger than the truth, but differently to differ-
ent people. For this was the chief aim of Chapter 5 to show the causes, from the
actual structure of the sense of sight, why the edges of luminous objects are en-
larged, particularly in darkness. Thus, here, the edges of the moon are not made
thin by the force of the solar light, but are enlarged by the force of the lunar light,
communicated from the sun. And thus the astronomer should carefully take note
of this from that passage, that unless he be endowed with the sharpest and most
powerful sense of sight, he is not equal to measuring the moon’s diameter at the
full with the eyes without error, so much so that this belongs to hardly anyone.

Next, this too must be granted, that in the eclipsed sun, the boundaries, both
about the sun and on the side of the moon that has entered beneath, are spread
out in the vision, which was demonstrated above at the end of Chapter 5, again
from the structure of the eye and the principles of sight. In addition, one has to
distinguish carefully here between those things that happen to the sense of sight
and those that happen when the consideration of the sense of sight is removed.
For those things that happen is the sense of sight vary by individual cases, but
those things that really happen are uniform within a single horizon. And since the
question here is whether day or night is going to occur at the time of an eclipse
of the sun, and particularly, how great a part of the light of day is going to be
lost, one looks in vain to Aretics in the sense of sight, which, though it be very
deficient in one particular person, does not thereby extinguish the light of day for
the rest of the people. And so, in the second place, astronomers will now take note
of this: that one must not trust the sense of sight either for the number of digits,3
nor for the narrowness of the interior and concave circle bounding the sun, which
is the circle of the lunar body. For here in fact, in our sense of sight, the edges
of the moon, as Tycho says, are made thinner by the force of the solar light, or
rather, the edges of the luminous solar particle are enlarged.

It cannot therefore be argued from this accident of the sense of sight to
what happens outside of consideration of the sense of sight, nor can tables be
established for the sake of the sense of sight, which represent neither the object
itself nor the defects of all senses of sight. For the astronomer should not present
anything other than those things that in actual fact occur. The sense of vision,
however, we leave to the physicians to remedy.

But that, in addition, the ray of the eclipsed sun received through a notch
(since Tycho used this also) and through a narrow opening spreads out the lumi-
nous boundaries, and is evidence for a smaller moon, this is so far from showing
the moon’s true quantity, that instead, in Ch. 2 above, the occasions became on
this basis apparent, on which some deceived persons might be led into this opin-
ion, as if the moon really were that much smaller. And below, by this device (this
distinguishing of the enlargement), we shall teach how to enter in to a most cer-
tain procedure for measuring the quantities of eclipses. In this place it will also
be made plain by examples, that if the device be correctly applied, the diameter
of the moon appears decidedly greater than the amount that Tycho’s table shows.

Thus, these things being presupposed, it is to be demonstrated by us in the

3 See the note on p. 71.
Beginning, by many examples, that the diameter of the moon is fully large enough that it has at some times covered the whole sun, and has enclosed certain regions in deep darkness. Next, I shall ascribe optical causes, by which it can happen that sometimes, when the moon intercepts the whole sun, nonetheless the sun appears to protrude all around.

2. Examples from historical accounts, that the moon’s shadow brought night into the day

That the moon’s shadow is conical, both because the sun is round, and because the moon is round and also is smaller than the sun, is evident from the same principles by which we have pronounced the earth’s shadow to be conical, in Ch. 7 sect. 1 above.

As to the case in which the diameter of the moon is perceptibly not greater than the diameter of the sun, but equal: the point of the moon’s shadow ends right at our sense of sight, and enfolds only the most narrow little region of the air. And so in this case full darkness can by no means take place. For the sun, being turned away from only a single point of the earth by the intercession of the moon, illumines most brightly all the surrounding parts of the earth with some part of itself. These parts emit the light communicated from the sun everywhere upwards into the air; even sofar as it is in the moon’s shadow, and thus supply it with brightness nonetheless.

It is therefore necessary that as often as the darkness of night makes an onslaught during the day, at the time of an eclipse of the sun, the moon’s shadow be seen with a greater angle than the sun: such that the sun may lie well hidden behind the moon, and a sufficiently great part of the horizon may be covered by the shadow that the observer’s air cannot be illuminated by the neighboring air. In a word, it is necessary also to darken that matter which ordinarily lights the heap of sunlight for us.

As for the case where the altitude of that matter reaches 12 miles (I would note in passing), in order that this really be deprived of the sun’s light by the moon, the moon’s shadow in our neighborhood would have to have a thickness of 300 miles.

For let $AD$ be 12 miles, of which $DC$ or $BC$ is 860 miles. Therefore, as $BC$, 860, is to the whole sine, so is $AC$, 872, to the secant of the angle $ACB$, 9 51'. As the whole is to the tangent of the sine, so is $BC$, 860, to $AB$, 144 miles, from one side, and the same amount from the other. But in order that the moon’s shadow, standing at the zenith, cover 300 miles all around, the moon ought to be seen as twenty minutes greater than the sun, as much greater, that is, as the moon’s parallax, when standing at the zenith, varies at minimum for those travelling through 56 degrees or both sides on a great circle of the earth. But even if you give an altitude above the horizon of no more than 13 degrees, from here to 23 degrees the Moon’s parallax still

---

*This case was brought up at the beginning of Chapter 4. The rule used here is the German rule, or less Roman rules.
varies by 3 minutes on this side, and the same amount on the other as is to be seen in Tycho’s parallactic table. Therefore, it must surpass the sun’s diameter by six minutes in order to darken the matter producing the phenomena of twilight for 300 miles lying beneath the same meridian. And yet with this quantity of diameter it will not be able to cover that much in both directions, towards the east and the west, but the light of twilight will remain at the sides. Consequently, authors who have written about twilight should see about reconciling their teaching about the abode of the matter of twilight with the total eclipses of the sun and the complete darkness by day, without a prodigious magnitude of the lunar diameter.

For, that we might pursue more closely what was proposed, the occasional existence of deep darkness of this kind is impudently proved by authority and historical accounts. For this reason, Ptolemy, in order to prove by the strongest possible argument that the moon is not seethrough, and does not transmit the sun’s rays, refers to common experience: “If it,” he says, “so far from being bright that not only it is itself dark (invisible) in conjunction, but also on many occasions hides the sun.” And he brings forth Empedocles, who also expressed this opinion more evidently in verses:

And robs light from a space of earth so wide,  
As wide as the orb of the yellow moon encompasses.

Understand it as a little less space, as geometrical reasoning relates. Ptolemy follows up these words of Empedocles thus: “Exactly,” he says, “as if the light of the sun had fallen on night and darkness, not on another celestial body.” A little later, using the same argument, he made bold to compare ordinary night with this darkness at the time of an eclipse of the sun, and thence to attribute the same substance to both the earth and the moon, for the reason that each blocks the sun in exactly the same way, and brings darkness upon things. Here he brings in Theon, the mathematician at his age, defending exactly the theme that I am now treating, and by the same arguments derived from the authority of the ancients. “This Theon of ours, if you do not concede” that such darkness occurs, “will bring in Pomponius and Cydlius, and Archilochus, and Sessilchors and Pindar besides, who lament, in eclipses, that the most brilliant sun has been abducted from them, and that they are as if in the middle of the night, and that the sun’s rays are carried on a dark course: and above all,” (the most ancient of all, or before all, “Homer” (A Christian will append Isaiah Ch. 13): “who said that the faces of the people were invaded by night and darkness, and that the sun and moon disappeared from the heavens.” Indeed, this, in Plutarch, is readily admired. Of Sessilchorus and Pindar, Pliny writes in Book 2 Ch. 12 in this way: “...the human mind, in eclipses of stars, fearing critics or some sort of death from the

7 TBOO II pp. 132-4.
8 Ptolemy, On the Face in the Moon, XVI, 920C-41, pp. 102-3. Brackets mark Kepler’s interpolation. The quotation from Empedocles is from Fragment 310B 42.
9 Or the Face in the Moon XIX, 931F-E pp. 116-119. References to the other authors cited are given in the notes to this text.
heaven—it is general knowledge that the sublime voices of the seers Sismondea and Pindar were in this fear, because of an eclipse of the sun.

Clementines, being of exactly the same persuasion as Plutarch, proceed to give the causes of the same most bewildering phenomenon, that is, why the moon does not transmute the sun's rays, which was [sic] gathered from such deep darkness, and again, "why, when the moon is smaller, it may hide the sun, standing in the way of all its parts, and subtending its entire diameter."

And when Sismonde (I think it was) reviewed the opinion of certain of the preceding authors, saying "that in perfect conjunctions, when the centers of the luminaries are perceived on the same straight line, the sun's orb embraces the moon with a circle, exceeding iron all sides," Clementines adds bold to temperate a comment, as it to say "this is not perceived in observations. For," he says, "that limb would have been seen by us to be protruding, since it was most brilliant." Thus he clearly assures that total eclipses were seen by him as well, and makes this神州 general, Martianus Capella does the same: "Frequently," he says, "an eclipse of the sun, occurring in the circle through Mercur, overshadowed the whole of the same on every side." And Alciatius takes it as generally acknowledged that the whole sun is avowed. And Plebe also proposes a cause for why the moon might be seen in a total eclipse of the sun. And, in sum, these have been an astrono- mer who doubted that this happened; however many there are who relate that the moon's diameter is perceptibly greater than the sun's diameter, all likewise have taught this. No wonder; the histories are full of examples, some of which, from Maestin and Mercator, and the ancient historians, I will disclose. For this recounting is useful not only here, but also for another work I am contemplating, on the magnitude of the year; clearly necessary, if God grant life and ability.

Dionysius of Halicarnassus, in Book 2, says, "They say that at the concep- tion of Romulus the whole sun was eclipsed, and complete darkness, just like..."

11 Clémentines Du Monde Cercles, 4, p. 196; Ctes. Gouda (French) p. 160.
12 Martianus Capella, Marriage of Philology and Mercury VII 85ff., res. Stahl and Johnson p. 334. Kepler considers this passage from Martianus Capella at greater length in the papers relating to his unpublished Hypotheses, in JGGW, XX 1 p. 246. His words, there moc, is clear that "this in this context is actually the Greek proposition meaning "through," as the Goddess or the island of Naxos, despite the initial capital. The "circle through Mercur" is one of the several times, or celestial latitude, listed in the Hypotheses (cf. II 13). Toomer's translation pp. 123-3 and the note on sp. 19 (13).
13 Alciatius (c. 1550), see Chapter I, p. 155. In Nolino's modern edition (Vo. I, p. 54), however, Alciatius explicitly states that the moon at eclipse does not cover the sun.
14 Cf. Maestin, Michael, Historiae de eclipsibus, Tübingen 1596; Mercator, Gerhard, Cronologia Colognese, 1598. Many of the following accounts, as Kepler's words shows, come from Horaeis Brünning, 1545-1600. Chronologia, loc. cit., contain temporay et minora scis: calculo astronomico e posteri demonstrata: cui hic quasi picture ac Mutinii elipsim Sols et Lunae (Gerth, 1595). I have not seen a copy of this book, and have been unable to document all of Kepler's citations. 
15 No such work by Kepler ever appeared.
night, took the earth; and that the same thing happened at his death. This
might seem a myth. However, the Prutenic computation gives it confirmation,
although it is not fully certain; in this uncertainty, it nevertheless points to ex-
actly the same year and exactly the same interval of years as Romulus’s age of
55 years, in which two great eclipses of the sun happened. For although the prior
eclipse is given as 5 1/2 digits, the latter as 9 1/2, it can easily happen that an emended
computation (for it needs emendation) would give both as total.

Herodotus, in Book 2 on the war between the Lydians and the Medes: “In
the sixth year it happened that during a conflict, when the battle was closely
joined, night was suddenly brought about out of day. Thales of Miletus foretold
that this alteration of the day would take place, adding the time, this very year, in
which that alteration was also brought about.” However, since Herodotus does
not make mention of the moon, he was indeed inexpert in astronomy. So that
no one might think this to be some other wonder, Pliny gives us confirmation.

In Book 2 ch. 2, he ascribes this teaching to Thales of Miletus, first among the
Greeks “because of a predicted eclipse of the sun, which took place in the reign of
Astyages, in the fourth year of the 48th Olympiad.” Bünning accordingly found
a new moon on the ecliptic in the year indicated, and the digits, as indicated by
the Prutenic computation, not yet perfectly certain, were eleven and a half, as the
moon was approaching perigee, in 1 Gemini, in that summer from whose middle
the fourth year of the 48th Olympiad began.

The same Herodotus, in Book 7: “When Xerxes set forth from Sardis to
Abydos with the army, the sun, departing its place in the heaven, was not visible,
though there were no clouds, but particularly calm weather; but in place of day,
night came.” Since Xerxes’ passage to Europe fell in that year in whose sum-
mer the 75th Olympiad was celebrated (or some had related that, though Xerxes
had already entered Greece, the Greeks viewed the Olympics in safety), and nev-
ertheless astronomical computation shows no eclipse of the sun in that year, you
might with good reason believe that a monstrous event was brought about in the
sun while the whole army watched (for Bünning wastes his time in producing an
eclipse of barely 3 digits, and that in the previous year). But two years after this
crossing of Xerxes, the Prutenic computation showed a maximum eclipse of the
sun, at the first hour after moon in Asia, and this on the 17th day of February,
in 20 Aquarius, and thus truly the first that agrees with Herodotus’s account. I

Classical Library, 1960), II 35.6, pp. 474-5.
13 Eusius Rehfeld, Prutenicæ tabulae coelestium motuum (Tubingen 1551 and other
editions). These tables were based on Copernicus’s models, parameters, though they
were geocentric in orientation.
14 This is from Book 1 ch. 74, not Book 2. According to David Grene’s note on this
passage, the eclipse took place on May 28, 585 BC. (Herodotus, The History, translated
by David Grene, Chicago [1987, p. 67].
15 Pliny, Natural History II 53 (not ch. 2), pp. 202-3.
16 Book 7 Ch. 37. According to Grene (Herodotus, The History, translated by David
Grene, Chicago [1987, p. 483, this probably happened in April of 480 B.C.}
therefore suspect a lapse in the report, from Xerxes' having perhaps gone forth twice from Sardis. For although Xerxes was everywhere defeated in the first and second year of the 75th Olympiad, nevertheless, in the third year, he was persisting Pausanias about a betrayal, and perhaps for the sake of that he marched to the shore with the army at the time when this eclipse appeared. For in the following year, Pausanias was convicted of this betrayal, and was killed by starvation. 21 Compatible with this is that interpretation of the Magi, taken from the present matter. For the Greeks, it signifies a revolt of the citizens. 22

I have often thought about whether this may be the eclipse that appeared to Cleombrotos sacrificing at the Ismou, really because of this, that it followed closely after the defeat of Mardonius, but the computation does not agree. 23 And you can't tell whether by Herodotos's words, ἐγώ, δὲ καὶ οὐκ ἤγγισεν, an eclipse is understood, or some kind of darkening or obscuring. This, then, is a little more uncertain.

Thucydides, in Book 2, describes a memorable eclipse of the sun in the first year of the Peloponnesian War, which occurred in the second year of the 87th Olympiad. "The sun was eclipsed after moon, and was again made full after it had been made similar to the crescent moon, and a number of stars had gone forth." 24 It was on the third of August, Julian style, 435 B. C. Since often under total eclipses the stars do not show themselves, the author does not seem to be speaking of a partial eclipse, in which the sun was narrowed to a lunar horn, but clearly of a total eclipse, where, after a star has been made, the sun, going next through the shapes of the crescent moon, becomes full again, as in the description which will follow below from the life of St. Louis. For since after seven years an eclipse again occurred, not a slight one, but of seven digits, the magnitude that Maestin found from the Proclus: 25 nevertheless, the author speaks distinctly of it. "Some part of the sun was eclipsed," ἐξελεύθερως ἐκ τοῦ ἐξωθομένου, indicating that it was not totally eclipsed. This is simply ἐξελεύθερως ἐκ τοῦ ἐξωθομένου, further, the reason for his having added ὡς ἐπεξεργασθεὶς ἑπεξεργασθεὶς appears to be the same as that which led him to add this, "It appears [to him that an eclipse of the sun] can happen only at new moon." For, being about to confirm this opinion of his that the moon turns in the way of the sun's light in an eclipse (astronomy not yet being cultivated among the Greeks), he adds what he saw, that the sun, gradually having become full again (as the moon usually does on other occasions), by that very eclipse, and by its concavity, projects the roundness of the moon that stands against it.

Livy, Book 7, sect. 4: "In the consulate of Lucius Cor. Scipio and C. Laelius."
"During those days in which the Consul had set out for war, during the Apollo-
ian Games, on July 9th, the heaven being clear, the light was darkened during the
day, since the moon had gone beneath the orb of the sun."20 The year was the
second of the 147th Olympiad. The moon was rising from perigee. In the Julian
form, March 3, 190 B. C. 21

Two years later, the same Livy (Book 8 Dec. 4) says, "by the light between the
third and fourth hour, darkness having suddenly arisen," 22 at which times
computation shows no eclipse. It was therefore a prodigy. Blunting brings forward
an eclipse of three digits, ill suited to producing darkness, not to mention that it
happened at another hour, namely, the ninth hour of the day (Roman style).

Especially memorable is that eclipse which Hipparchus had introduced in
the book on the magnitudes and distances; it is not certain whether it was ob-
served by him or by Timocharis, since it was described as having been observed
at Alexandria. 23 For, as Cleomedes says about it, "the sun, totally eclipsed at the
Hellespont, was observed at Alexandria with one fifth of the diameter still extant,
the rest eclipsed."

Ptolemy recalled it in Book 5 Ch. 11 of the Great Work, 24 as did Theon in
his commentary on this passage, in noteworthy words: "He makes use," he says,
"of an eclipse in the places that are around the Hellespont, accurately made in the
entire sun, so that nothing of it appeared." 25 He not only asserts that in the Helle-
spont the entire sun was hidden, but also attributes breadth to this phenomenon,
saying: "in the places lying round about." Hence it is that Cleomedes reports, be-
yond doubt from this same passage of Hipparchus. "This shadow takes up more
than four thousand stadia: for every place in which the sun is not seen, when the
moon runs beneath it, is the shadow of the moon." 26 p. 153. For Theon reports
that from this eclipse Hipparchus concluded that the moon’s nearest distance is 71
semidiameters; the farthest, 15: although afterwards, for other reasons, he comes
down to between 62 and 72. Supposing that the moon’s diameter be taken to be
30‘ at apogee, which Hipparchus supposed, it will become 35‘ at perigee by the
Hipparchian eccentricity, exceeding the diameter of the sun, in the quantity
used by Hipparchus, by 5 minutes. Further, in order that, at this elevation of 71

20 Livy, History XXXVII is 4, Vol. X pp. 300-1.
21 In modern terms, this eclipse was on March 14, 190 B. C. E. (+189, in astronomical
reckoning). The Roman calendar was then several months out of adjustment, hence
the change from July 9 to March 3.
22 Livy, History XXXVIII xxxi 4, Vol. XI pp. 118-9. According to the translator’s note,
this darkness was the result of an eclipse on July 17, 188 B. C. E.
23 Hipparchus’s book has not survived, but is known from citations in other authors.>
Kepler’s information comes from Cleomedes, De motu circulari III 3 p. 172, trs. Grolier
(French) p. 153.
24 Almagest, trs. Toomer pp. 243-4. Ptolemy does not mention any particular eclipse, but
describes Hipparchus’s procedure generally.
25 This, as well as the distances given below, appears to be a quotation from Pappus’s
Commentaries, p. 68.
26 Cleomedes, De motu circulari III p. 172, trs. Grolier (French)
semidiameters of earth; the body may change its place by 5 minutes, there is need for it to go through 6 of the terrane circle from the place where the Moon is vertical, and about the Hellespont, at an altitude of the sun (in Cancer) of 71°, by 6°. But in that time Eratosthenes had fixed the circumference of the earth as 250,000 stadia, of which at least 4,000 make up 6 degrees. Therefore, whether by a simple consideration of the extent of lands through which the whole sun was hidden in this eclipse, or, from this line of reasoning just presented, Hipparchus had taken away those 4,000 stadia, of which Chremedes series, under either head it is strongly proven that this eclipse was total, the sun being hidden beyond the moon with some lapse of time.

If the consulate of C. Marcius and C. Flaccus, "in the third hour of the day, an eclipse of the sun darkened the light," Jul. obs. 35

In the consulate of M. Vipsanius and Fonteius, in 59 A. D., Cornelius Tacitus says (in Book 14 of the Annals), 36 "the sun was suddenly darkened." And the not insignificant author counts it among the great prodigies that, to his wonder, were without consequence. That the eclipse was indeed noteworthy is concluded from Pliny, who said (Book 7, Ch. 70) that it was also noted by Ctesibius in Armenia. 37 The day was April 30.

Plutarch, in *On the Face of the Moon*, reminds his interlocutors "Of that coupling of the sun and moon that recently had occurred." (about 100 AD), "upon the beginning of which, immediately at noon, stars shone forth everywhere in many places of the heavens, and the air was cold in such a condition as there is under twilight in unseen light." 38 That it was real is concluded from the preceding as well as the following. For giving causes, "why darkness is not so deep in eclipses, and the air is not taken over by it as it is by night," 39 he is not summoning us towards the view that there is something of the sun remaining, not covered by the moon, but is seeking other motives, of which later, By which it is proved that this eclipse seen by Plutarch was total, not otherwise: what he saw in one case, he believed to occur in all. But the histories give us sufficient confirmation of plain darkness. At least, this eclipse makes for this, that we may believe that the whole sun can be covered.

At the time when Gordionus the Younger began to rule, i.e. in 237 AD, on April 12, "there was such a great eclipse of the sun that it was thought to be night, nor could anything be done without lamps being lit"—Julius Capitolinus 40. This had to have been total; the Plinius calculation shows less than 11 digits.

Ammonius Marcellinus, in Book 20 of the consulate of Constantine X and Julian III: "Over the Eastern lands, the heaven was perceived to be dimmed with

35 Julius Obsequens, *Paradoxia Liber*, pp. 296–7. The second consul's name was Flavius, not Flaccus.
36 Tacitus, *Annales*, 14.12. The consuls' names were Caini Vipsanius and Caini Fonteius. This eclipse occurred on April 30.
39 A.D., pp. 120–1.
40 This is presumably in Julius Capitolinus 8, after 320 C.E. I., *Unter Gordioni*. 
darkness, and from the first rising of dawn until midday." (I understand this to mean the place in the heaven, nor the time; "stars twinkled forth continuously."

To these terrors there was added that when the celestial light was concealed, the light having been utterly snatched away from the view of the world, the terror-struck minds of the people considered that the sun was eclipsed for a long time, at first narrowed into the likeness of a horned moon, then into the enrobed form of half a month, and afterward was restored to its entirety. Consequently, here too the entire sun was eclipsed: the Phrygian computation groundlessly leaves a circle about the moon, the moon being at apogee. For after the explanation of causes, the author also adds these words occasionally: "The heaven is overspread with dense darkness, i.e. are being thickened, so that we are unable to see even the closest things, and those placed in front of us." Whichever thing beyond doubt this eclipse itself teaches, as Plutarch's previously taught him. The year was 380 AD, 28 August.

Since the histories make extremely rare mention of eclipses for a number of centuries, it is believable that only those were noted that struck the eye by turning day into night, and were total. It is credible that Liechtenberger in the time of St. Martin, 41 Jornandus and Marcellinus on 19 July 418: the "Annals Constantinopolitani" on 19 March 592; and Bede on 1 May 664: were reporting such eclipses. This is not much in disagreement with the Phrygian computation, which gives the former as 160, and the latter as almost 12 digits. In 787, "A maximum eclipse of the sun occurred." The computation shows 11\frac{1}{2} digits on 16 September. But that it was total is deduced from the words.\(^13\)

On 5 May 840, an anonymous author said there was an eclipse so great "that even the stars were seen because of the darkness of the sun, and for objects on earth the color was changed." Another author says it was "total". Ammonius, that it was "maximum." The moon was at perigee. The following notable words concerning this eclipse are included in the life of Louis the Pious:

An eclipse of the sun happened on the third day of the greater Lent in an unusual way. For in the withdrawal of light, darkness had prevailed to such an extent that, it seemed to differ not at all from the reality of night. For the established order of stars was so perceived that [none] nor star suffered


\(^{42}\) History XX 1.5, pp. 30-31.

\(^{43}\) In a letter to Hermann von Hohenberg of 16 February 1605 (JGSW XV no. 325, pp. 145) Kepler writes, "The words of Liechtenberg were taken from a small German book: a quarto; without mention of the place, publisher, or year of printing. It is a book of prophecies, where, in Chapter 23, he speaks of the eclipse of 1483, which happened upon him as he was writing."

\(^{44}\) Frisch, JGSW XV no. 422, writes that Kepler's main source for these eclipse reports was Gerhard Mercator, Chronologica, Cologne 1569.

\(^{45}\) Georgii Cedreni Annalis, secundae ad calendas maias ad Isaccum Comes senescope cummentum. Basel 1566. This book lists many eclipses, though they do not seem to tally completely with those mentioned by Kepler.
weakness from the solar light; rather, the moon, which had set itself forth against the sun, restored it to its usual self in procuring for it a gradually rising light [note:] from the western side, first in a horridlike manner, when it was first or second perceived, and thus by increments the whole wheel of the sun received its whole delightful appearance.

The author is worthy of credence, because he had experience of astronomy, knew all the stars exactly, and was accustomed to show them to Louis, as is apparent from the conversation which he had with Louis on the comet of the preceding year and its significance: he seems to emulate Marcellinus. On 29 October 878, the author just mentioned said, "The sun, after the 9th hour, was so darkened, for over half an hour, that the stars appeared in the heaven, and everyone thought that night was impending upon them." Gemma asserts that it was total, which is in agreement with the words. The moon was beyond perige.

In the third year of Lothair, about 957 AD, December 16, "An eclipse of the sun was brought about, such that the stars appeared from the first hour to the third hour."

Compare this with that of Ammianus Marcellinus. For morning eclipses have a special reason why they are more horrible: because of the moon's shadow being above the horizon, as if striking with a certain impact, and dosing the dawn. By which I believe my grandfather the more readily, who used to relate to me how on one occasion (namely, 29 March 1530), the heaven being extremely cold, when it had barely become light, the light of day was suddenly extinguished, and turned into night. The Tychoic computation in fact does not allow this eclipse to be total. Cyprianus also calls it "hilare"; I don't know whether from personal inspection. The moon was near perige.

On 2 August 1133, the historiatus report an eclipse of the sun that was "maximum"; computation allows it to be total.

On 4 September 1187, "An eclipse of the sun so great that the stars were seen by day, as if by night." It was therefore total, as comparison with the preceding shows. The moon was at perige.

In 1191, Alberic; "A total eclipse of the sun was brought about." And yet the moon was at apoge.

On 6 October 1241, a "great" eclipse of the sun. Gemma said it was "total."

This too was in the morning. On 7 June 1415, "there was a horrible eclipse of the sun." (Leonin). That the "whole" sun was covered in is deduced from the Polish Historians. The moon was about perige.

On 16 March 1485, the sun was eclipsed, by Walther's reckoning, 9 1/2 digits, which doubtless the sense of sight get somewhat wrong, for the reason set

44 Lothair I, King of France from 954 to 986.
45 This is Cyriacus Leoviti, of Bohemia (d. 1574 in Lauenin), mathematician to Otto Hahnrich, Elector Palatinate. Kepler's source, according to Fricke: is Leoviti's De communctibus magnis inscriptis, super sedes planetarum, non defectentibus et comens (Lauenin 1564).
46 The 13th century chronicle Albertich the Monk is cited from Mercator.
forth above. For elsewhere, by the testimony of Lycothenes, "It was simply dark, to such an extent that candles had to be lit; the chickens in the towns, and other kinds of animals in the country, bitook themselves to the nighttime places of their accustomed rest." The moon was at perigee.47

On 14 January 1544, Gemma Frisius observed an eclipse of 10 digits (through an opening),48 and thus it is false, as was shown in Ch. 2. And the eclipse was greater. Funk reckoned 11 digits, but here too the eyes were in error, as was said in Ch. 5. In all events it was a little more, and in some places the entire sun was hidden. Hence, Funk said, "they began to become night again, as if at evening twilight, and the flying creatures of the heaven, which at first light were lively, began to become silent when so great a darkness occurred suddenly." The moon was ascending from perigee.49

On 21 August 1560, Clavius attests, "At Coimbra the sun was hidden, being covered up for a not inconsiderable time about noon, and the darkness was a way greater than that of night. For one was not able to see the place where he would set his foot, and the stars appeared in the heaven with great brightness. Even the birds, soundless to tell, fell from the air to earth, in horror of such a terrible darkening."50 The moon was at apogee. Thus, to give credence to this one, which Tycho denied it, one required the accumulation of many antecedents. In Vienna, Austria, as Mercator reports, Tilmann Stella and Paul Fabricius observed 5½ digits at 1st40. The Tychonic cozonization shows the two conjunction at Uraniborg at 1st20.51

Cornelius Gemma at Louvain noted the beginning at just after 11, the end at 1st 23½, 7½ digits, nearly.52

47 Walter’s 1545 observations are included in Recogmontum’s Scritti (Nuremberg 1541); however, this eclipse is not included among them.


49 I have inspected Funk’s Chronologiae, loc. cit. cumiam temporum at omnium ab init: no mundi posse ad hanc praecernam secund Christi M.D.XXX. comparatur, Wittenberg 1570. This is the second edition of a work originally published in 1552. It lists many eclipses, and one would suppose that Kepler is referring to this book, yet no eclipse is listed for 1544 (fol. 162). Fresh, J.K.O.H p. 423, states that this eclipse was also not included in the 1574 and 1601 editions of this work.

50 Clavius, Christoph. In Ephesiam Jasonis de Suctro Bracc Commentarius. Rome 1581, p. 429; Rome 1600, p. 562. Kepler’s quotation differs in many details from the original, though the sense is the same. In the 1581 edition (the second; the year is given as 1550 in the 1606 edition (the fifth)), he corrected the year to 1600. No date or time is given in either place.

51 Paul Fabricius (c. 1500–1588) was a Viennese physician and astronomer. Tilmann (or Tilmannus) Stella is said by Funk to be the author of a small work titled Recoll of Nasccit und Geburth der neuer Lantfyste, Tycho makes a critical remark in the Fraymuisther p. 770 (J.K.O.H. p. 284) apparently directed at this book.

52 The eclipses of 1544 and 1560 are not noted by Kepler in Vol. XV of the Kepler Manuscripts at St. Petersburg, some of which are transcribed by Frisch (J.K.O.H. pp. 423–6).
At all events, from these examples, it is as evident as can be that when the moon is closest to earth, night-like darkness follows, from optical or astronomic causes. But this are not the only causes, but rather that they are very greatly helped or hindered by the conditions of the air, the unequal brightness of ordinary nights shows. For if in pure night the air, because of its whitish color, a sign of thickness (as Aristotle asserts of Ponnus),35 sometimes so imbibes the light of the stars that for the whole night a certain illumination of twilight is emulacred, what can even the slightest particle of sun not accomplish by means of the assistance upon thick air, and illuminating lands of this kind, beyond a few miles from us, and communicatting light to them, which is reflected again to us? Here are arguments of this unequal brightness of night deriving from the air. In 1599 in the eclipse of the moon of January or February, mention of which was made above, the diameter being not yet half covered by darkness, I was able to see the whole edge of the moon. In December of 1601, with the thinnest born remaining, I nevertheless did not see the dark part. It was nevertheless the same season of the year. In the month of May, 1603, when the third part of the diameter remained, some people still saw the dark edge. In the following November, when not a fourth part was in the shadow, the edge was nevertheless seen.

3. Whether it can happen that in a central conjunction of the luminaries, the sun is still not entirely hidden?

And so hitherto we have proved from the histories that the entire sun was hidden more than once, even by the moon at its highest.54 What is more of a wonder, there exists in all history but a single contrary example, which Clavius reports in his Commentariis on Sermone: on 9 April 1567, "the sun at Rome was not all eclipsed, but there remained a certain narrow luminous circle all around.55 The moon was nevertheless behind its highest and lowest. It is a wonder, I say, for the visible diameter of the moon should definitely be smaller, and you could not ascribe this circumstance to the sense of sight, as was done at the beginning of this chapter. For those things that are in the eye, or in a dark cabin, are enlarged because they are luminous; it is necessary that they first project a ray in that place, but those things that do not radiate, prevented by the interposition of opaque bodies, in the sun is by the placing of the moon in the way, will also not be able to spread out. And so this phenomenon is not to be ascribed to the sense of sight.

Therefore, the same thing that Tycho Brahe attacked in the one of Clavius’s eclipses, I have attacked in the other; that is, in calling into question this very thing, whether the remaining circle was completely unbroken, and not instead an

53 Problem 66a6, 938a7-8 4.
54 (To the marginal note: Macatin. Disputatio de eclipsibus. p. 3."
55 Clavius. In Spheriam... commentarium, Rome 1581, p. 425. Again, Kepler’s words are a close paraphrase of Clavius’s. Despite Kepler’s implication, below, that Clavius was reporting someone else’s observations, Clavius wrote, "(aegen)," "I have seen."
exactly this honed edge at one side, the centers still not being fully aligned.

For it can happen that in the beginning these things might really have been recorded in one way, but transcribed differently by Clavius, whether by type of memory or by a somewhat haphazard understanding of the attested document, especially if Clavius was recalling things seen by others. Indeed, even he added, doubtfully, "it perhaps had never before happened."

And in fact, that Clavius committed this phenomenon to memory with sufficient reflection, having considered the circumstances, these very words of the author show. Consequently, let us reason as follows, as about something completely certain.

And first. I do indeed not deny that the hypotheses of certain astronomers are so constructed that the sun at perige can be covered by the moon at apogee in this manner, with the edge left protruding. For him whom we suspected above to have been censured in the plural by Claudiomedes, i.e., Sosigenes, to him this form of hypothesis is openly attributable by Proclus Diadochus, after first recalling Ptolemy's opinion. Ptolemy had said that it was obtained with certainty from Hipparchus's doxa that the sun's diameter does not vary perceptibly from apogee to perige, but is seen under the same angle, but the moon's diameter is only seen to be equal to the sun's diameter when the full moon is situated at the apogee of its epicycle, while at perige the moon appears to be greater. Proclus then adds to this, "If this is true, then what Sosigenes the Perpatic narrator is not true, in what he wrote on the Revolutionaries, that if eclipses brought about at perige the sun is not seen wholly running forth to anterior places," (I understand this to mean beating itself beyond the moon, "but at the edges of its own circumference it goes outside of the moon's circle, and gives light with minimum hindrance." It appears that Hipparchus said the same as what Ptolemy said of the diameters of the luminaries. Sosigenes, however, added that interpretation, if at apogee the luminaries are seen under the same angle, surely the sun will be seen under a greater at perige, even if this is not captured by the dioptra because of the subtilely of the difference. Thus it can happen that the moon at apogee (and thus under a smaller angle) might be set against the sun at perige, and thus the sun might not be covered entirely at all extremities. For that reason, Proclus adds, "If anyone who will admit this, the sun will do a variation of apparent diameters."

Really, however, Sosigenes will reply to Proclus, because it is absolutely necessary that the sun do a variation of apparent diameters, because it approaches and recedes. I myself have therefore considered this as something to be admitted. Thus Proclus teaches us that Sosigenes declared that eclipses such as the single one Clavius presented to us definitely are seen. Indeed, in this passage Proclus proposes for himself an interpretation of that which he had said in the fifth "question of marvels" at the beginning of the Hypotheses. "Also since," he says, "certain differences of the moon are perceived in perfect solar eclipses; for sometimes the whole sun seems to our sight to be covered, sometimes at the very

---

56 See p. 301.
moment at which the two centers and the eye fall upon the same line, the moon is observed within a ring of sun. 598 He shows, I say, in the passage first quoted, that he is speaking here, not from experience, but from the traditions of Sosigenes. For if he had had personal or at least certain, experience of those things he described here, he never would have appended those things which we have added in the first passage: he never would have called those things into doubt which Sosigenes had handed down, since, naturally, they would be confirmed by experience itself.

Could therefore this eclipse related by Clavius be justified by the Ptolemaic hypotheses taken over from Hipparchus and Sosigenes? Absolutely not. First, the moon was right at its mean longitude: the moon’s diameter was greater than the sun’s, even at perigee. Next, the sun itself was going towards apogee, and was seen under nearly its least angle. Therefore, another cause of the phenomenon is to be sought. And I do not know whether Plutarch, in the book on the face of the moon that has by now so often been beat upon might have fully uncovered it when, having begun from the eclipse which he had seen himself, he added a general account, saying, “Although sometimes the moon occults the entire sun, this eclipse is nevertheless lacking in breadth and time,” 599 the makes a rule of his example: you have seen otherwise in the examples cited, “but a certain brightness shines out around the orb, not allowing a deep and excessive shadow to happen.” It is thus clearly the case that although the sun be covered in its entirety, the air surrounding the sun is nevertheless more brilliant the nearer it is to the sun. Consequently, if the air shall be thicker, and some slight particle of it, of a magnitude intercepted by the observed sun’s cone, shall be darkened, the the air in the vicinity will be seen to shine, in a circular form. Other experiences are at hand. Mr. Jessenius, whom I mentioned in Ch. 5 fully confirms for me the eclipse of the sun seen by him through the clouds at Torgau Town Hall on 25 Feb. or 7 March, 1598, clearly ringing the moon with brightness, and says that he followed up that form of the laboring sun with an epigram and an allegory. However, that eclipse in the Brahman computation could not have been total, not even through the legitimate diameter of the moon departing from perigee. Therefore, Jessenius was unable to see the sun everywhere, but what he saw everywhere standing around the moon was the brilliance of the air.

On 8/18 January 1603, at one hour after sunset, when a pillar of a tower was blocking the whole moon, nonetheless a sufficiently bright white orb was apparent, so that you would think the moon to shine through a watery cloud, or that the moon’s brightness was, darkness as if in water. However, it had been apparent end, but vanished into a cloud, and this brightness was easily distinguished from the actual body by moving the head. Something similar happened to me on 6/26 January following, in the evening, the moon being near perigee, with the greatest wonder. I set up a bronze wheel, precisely circular, clasped with a spike to one end of a pole twelve feet long, applied the eye at the other end, and before the eye a very narrow opening, in another bronze sector, so that the eye would have a perpendicular view to the wheel through the opening, and nothing might happen.

598 Ed. 1551 p. 334. The words are altered slightly.
599 On the Face in the Moon XIX, 952B, pp. 120–1.
because of the width of the eye. And since, where the distance between the eye
and the wheel was 10,368, the width of the wheel would be 104, covering an
arc of 34 1/2 minutes, I was hopeful that the moon was going to be completely
covered by this wheel, because of other ways of observing, which I was applying
at the same time. And in fact, the moon was seen to protrude all around. Here I
was troubled by anxiety that perhaps other ways of observing, in which I had the
greatest trust, might be false. But that there was a falsity because immediately
plain when the eye was brought nearer the wheel. For all that brightness still did
not betake itself behind the wheel even when the eye came to a distance of 16
feet. In this way the moon would have had to represent more than 41 minutes,
which everyone knows is false. I was also unable to recognize a determinate dis-
tance from which the moon would be covered, for I always saw something bright
on the circumference, even from a distance of seven feet. Something not much
different was set in my way in 1600 at Graz regarding the observation of the sun's
diameter admitted through an opening. For when two holes were opened, one the
size of millet, the other the size of a pea, and a part of circles were painted on the
opposite wall, one of which exceeded the other by an interval that was as great as
the difference between the larger and smaller holes, the ray of the sun admitted
through the larger hole was indeed equal to the greater circle, but when the larger
hole was blocked, the ray that came through the smaller did not maintain an ev-
ident boundary, and had the edge gradually passing over into a dusky color, and
finally, far exceeded the smaller circle. For the ray of the sun, greatly weakened
through such a small hole, was unable to illuminate the paper much more bright-
ly than the rays from the air standing about the sun, whose continuation with the
solar rays portrayed a breadth greater than the truth, and a brilliant color. Further,
the air on that day was more than usually bright, far from the sun.

It is believable that something like this can also happen around the sun, when the moon is interposed
upon it: that either the air or even the aetherial sub-
stance, which is not just nothing, but also has its own
proportion of density, when illuminated by the sun,
takes on a brightness that, since the sun is hidden, rep-
resents the solar light. Just as above, the moon's red-
ness, in the absence of the solar illumination, seemed
to me to be the aetheriacal light of the moon received
from the sun. For this can be deduced from Prop. 23 of
Ch. 1 in this manner. Let AB be the moon's body, CA,
DB the outermost rays of the solar body bounding
the moon's shadow AE, BF. And let G be the posi-
tion on the earth's surface in the middle of the moon's
shadow. Let GK be the portion of the moon's
substance which is by supposition a partaker of color to same degree something
be seen at G, but anything beyond CE is illuminated by
the sun, therefore, something illuminating by the sun will appear from G. Con-
sequently, by Prop. 23 of the first chapter, there will be communicated to that
substance (which is by supposition a partaker of color to same degree something
of the light of the sun, and it will radiate that into the region away from the sun. Therefore, the particles which are in the space KA will radiate at G more strongly than those which are in the space HF, because the former are of a more direct radiation and are nearer to the actual solar radiation C E, while H G is more oblique to C E.

If those things that Clavius saw in the eclipse of 1567 belong to this kind of effect, it is necessary that to Clavius too there appeared its bright edge, gradually less intense towards the outside, and not notable for a meticulous edge on the outside.

But what if Clavius should also deny this, and what if he clearly saw a cut off and bounded image of light? Are we really ready for this case? In fact, if Clavius could say this most truly, then I both would finally concede that the sun itself was seen round the moon by him, and will assign causes that are probably more geometrical than hitherto. Further, because in Chapter 6, Section 9 above we went along with Plutarch so far as to have dared to ascribe to the moon continents, seas, mountains, and valleys, such as this our earth has, how much more is it also to pour air around the moon, such as is poured around this our earth? For then, and even if it is not poured densely, that which we have demonstrated above in Chapter 7, Section 5 concerning the terrestrial air will happen: the rays, approaching from the edges of the sun, go around the lunar body in a kind of bending through the refractions in the lunar air, and thus are bounded at our vision by a shorter cone. As in the first diagram in Ch. 7 Sect. 5, if the eye were set between the point γ and α. So, in this way, it is not the moon's diameter that would appear less than it should, but the sun's diameter that would appear greater than it should.

In fact, this very thing was suggested to me, in a word, by a certain great prophet of these airs, "the outer parts of the moon appear to be pellicial." But this could not have been as easily defended as it was easily said. For what will help them to be pellicial, if they are of a denser substance than the aethereal air and the moon is round? Now if this should happen, the path for the solar rays is not straight, and nothing certain can be demonstrated without that diagram of Chapter 7. Besides, it might be asked why, if the outer parts of the moon are always pellicial, a circle of the sun does not always remain. I accordingly prefer to give the moon air not of a great depth, so that sometimes, but extremely rarely, it could proceed as far as to this effect. But I much prefer that that cause proposed in the penultimate place be valid, while this last one not be put to use, but to remain in storage.

4. A number of corollaries on eclipses of the sun

Now, in order not to end this chapter without profit, among the rules mentioned previously we shall take note of the following.

603 This was Heurwit von Heuburg; as his letter to Kepler of 16 May 1599 (no. 121; in JG&W XIII, p. 332), containing this remark, shows. Later, in a letter of 11 November 1602 (JG&W XIV, no. 335, p. 312), Heurwit recanted this opinion.

61 Caput, "chapter," can also refer to the "principal" of an investment.
1. As many times as any eclipse of the sun brings darkness similar to that of night upon the lands during the day, it is certain that the whole sun was concealed from view by the moon.

2. However, the astronomer should not as a result predict that there will be darkness even if he sees that the whole sun is going to be covered, unless the moon is also near perigee and the air shall have been pure. The cause of this you have in Ch. 6 Sect. 11, above.

3. It also does not follow that if the stars have appeared the eclipse was total. For we read of stars having been revealed by many partial eclipses.

4. However, this is certain enough: if a total eclipse is going to happen in a calm sky, the stars will be revealed.

5. Morning eclipses are always more inclined towards darkness before the middle, evening ones after the middle, the former particularly at the descending node and on the descending semicircle of the zodiac, the latter at the ascending node and semicircle. For they extinguish the light of that portion of the air illuminated by the sun, which part, being above our heads, brings most of the light to us.

6. Color, however, is changed for things, not so much by eclipses near the horizon as by those near the zenith, in summer, or in a calm sky. For the clearer the air, the most strongly the sudden darkening of the light enters into the eyes, which of course were strongly pervaded by it before, and retain strong images of the light. On the other hand, if rain or clouds occupy the heavens, and bring about a darkening beforehand, nothing unusual will happen to the eyes, even though the darkness is increased by the eclipse. Thus, on 21/31 July 1590, to the reapers in Schwabia, the eclipse, which was not particularly great, showed a yellow color everywhere, and a reddish one in Smyrna. But one much greater than this, on Feb. 25 or March 7, 1598, in a turbid heaven, with rain, in winter, when the sun was lower, gave rise to no disagreement about colors, even though it made the light of day abate to the smallness of evening.

7. There are many that are not completely total that bring upon the lands a darkness greater than total eclipses when the moon is at apogee or when the air is bright. This is because the sun is not deeply hidden all around beyond the moon at apogee. But if the sun be almost fully covered near the horizon by a moon that is slightly higher, the sun nonetheless protruding somewhat below, the air above our heads is the more deprived of a view of the sun the higher it is, so that a sheer night, or the moon’s shadow, jumps over our heads.

5. On the mutual occultations of the other heavenly bodies

Though it is true, there is nonetheless some connection joining this consideration as well with eclipses of the sun and moon. First, in order that any star be eclipsed in the same way as the moon, it must lack its own light, which is denied above in Ch. 6 Sect. 17. If, however, this be granted, the star Mars will seem not to be entirely free of the suspicion that it may run into the earth’s shadow. For, concerning the length and breadth of the shadow belonging to earth, and the sun’s parallax, if you look at the quantity, there is even now grounds for cautious consideration. And since the print of the lunar
shadow falls exactly on the earth, it seems fitting that the point of Venus's shadow come to an end at the moon when nearest the sun, the point of the Mercurian shadow at Venus; likewise, again, the earth's shadow at Mars. Mars's at Jupiter, and Jupiter's at Saturn.

See also whether that darkness which we said in Ch. 6 Section 11 came over the earth in 154762 might have been the more diffuse tail of some comet, a suspicion which I also raised above.

But in the same way as with the sun, we daily see all stars except the moon become eclipsed, with the lower ones always covering the higher, if they intercept them in their path.

In the year 45 according to Dionysius, which was the year 241 before Christ, on the 10th day of Vigo, or September 4, at dawn, the star Jupiter occulted the Southern Ass. Polloon Book 11 ch. 3.63

Aristotle saw the same star come together twice with the other of the two in the feet of Geminus, and occulit — completely hiding63 — it.

In Dionysius's year 13, 25 Capricorn, which is 15 January 272 B.C., in the morning, the star Mars was reckoned to be placed beneath "imposed upon", the northern star of the brow of Scorpius. Polloon Book 10 ch. 9.64

In the year 13 of Philadelphia, between 17 and 18 Moore, which is the same as 272 B.C. 12 October, the star Venus was seen in the morning to have caught up with, or, as Theon expounds it, to have occulted, the star opposite Vindemia, which is located at the end of the southern wing. Polloon Book 10 ch. 4.65

On 16 September 1574, at 1° in the morning, Macranus saw Cor Leonis covered by Venus. I saw the same at Graz on 15/25 Sept. 1596 at hour 3 in the morning, when Venus had barely risen. At the fourth hour, more than one [diameter of] Venus could have fit in between; nevertheless, the line from Venus to Cor fell a little below Jupiter.

That Jupiter should cover Saturn can hardly happen once in twenty centuries; nevertheless, it was seen to have been done in 1561. And events followed in proportion to the greatness of the sign, which even today we perceive before our eyes.66
That some comet was covered by the star Jupiter, certain people conclude from Proclus; see Tycho’s *Programmatum Libri* I fol. 617.\(^{68}\) Marsden, at Tübingen, and I with him, saw Jupiter entirely eclipsed by Mars, on 9 January 1591. The fiery reddish color of Mars argued that Mars was the lower.\(^{69}\)

Proclus said that Venus was observed to run beneath Mars, just as Mercury was observed running beneath Venus.\(^{70}\)

Of Venus and Mars, the same Marsden relates an example on 3 October 1590 at hour 5 in the morning; Mars entirely occulted by Venus, the brilliant color of Venus again indicating that Venus was the lower. Mutual occultations of Venus and Mercury are possible. Now Venus is higher, now Mercury.

When, on 21/31 May, 1599, Mercury was about one degree beyond Venus, it was nearly the same amount farther north, but on the following days it descended, both towards the ecliptic, with decreasing latitude, and towards the earth on its epicycle, with the visible diameter increasing. It nonetheless happened on 29 May or 8 June (for at Greut the whole intervening eight days were rainy and cloudy, as were the following days also, up to [June] 31/32) that, looking with greatest care at Venus, I nevertheless saw no Mercury, while I saw the Twins and Capella. I was in fact persuaded that I saw Cassiopeia rather long and thin rays from the eastern part of Venus; Venus, however, did not change color. The analogy of the diurnal motion and the preceding observations definitely put Mercury very close to Venus.

In this century, Venus cannot cover the sun. It was, however, able to about two hundred years ago, and will be able to at some future date.\(^{71}\)

The nodes of Mercury are at the beginning of Taurus and Scorpio or the end of Aries and Libra, and today, and at almost all times, they can carry this planet westward the sun.\(^{72}\) It is consequently the less to be marvelled at what we read in the life of Charlemagne, noted at the year 807, in these words: “On the 16th day before the Calendars of April, the star of Mercury was seen in the sun like a small black spot. It was, however, a little higher than the middle center of the same heavenly body, which was thus seen by us eight times [as I read barbarously, not

---

\(^{68}\) *TBGO* III p. 134. In this passage, Tycho is quoting Praxeis, who is citing some unknown persons who related something Proclus remarked. Diryfer, commenting on the passage *TBGO* III p. 396, suspects a libitation.

\(^{69}\) Marsden, *Disputation de eclipsibus* p. 18. In this place, according to French *1KBGO* II p. 431, Marsden also described occultations of Regulus by Venus and of Mars by Venus (mentioned below).

\(^{70}\) *Hypotyposes* 7.22-23 (Citation from BAG).

\(^{71}\) Kepler later changed his opinion on this point, announcing a transit in *De natura motu et phaenomenis* (Leipzig 1629), in IKBGO XI.

\(^{72}\) In *Phaenomenon singularia seu Mercurius in Sole visus* (Leipzig 1689), Kepler remarks that he was in error about the location of the nodes: “For today the node of Mercury is not at the beginning of Taurus, but of Gemini.” Cf. KGOW I p. 85.
'eight days'). But when it first entered or departed, was impossible to note, clouds being in the way.

The author was experienced in astronomy, which is evident from placing on record so many eclipses, and that he grasped the connection of the sun and Mercury with a computation, from which he knew that that spot was Mercury; the year, however, was written erroneously, by some chance. For it was on March 17 808 (probably because the year begins with Easter). For at the noon hour of that day in King's-berg, the Potenice computation shows the sun's position as 0° 45' Aries, Mercury's, 0° 31' Aries, with a latitude of 2° 9'. It is certain that this latitude was so arranged by Ptolemy, without guidance of observations, in order not to lead these two planets beneath the sun. But it was already said that the node today is at the end of Aries, so that it is not beyond what is reasonable that at that time it was at the beginning of this sign.

Therefore, we no longer put faith in Averroes alone about this phenomenon, after a professedly Christian man has also added his own computation.33

The encroachments of the moon over the fixed stars occur almost daily, and are usually correctly noted by the astronomers. Thus Agyippus of Byzantia and Timocharis saw the Planudes occupied by the moon: Menelaus the Roman, the Brow of Scorpio; and the same authors, Spica Virginis more than once.34 Tycho Brahe and Copernicus, Paraliticum; 35 i.e. the Heart of Scorpio33 was said above in Ch. 5 Sect. 5.35 Maestlin, very closely spaced fainter stars. Likewise, Walther noted Saturn covered by the moon (see below, Ch. 11 Prob. 30); a monk historian saw Jupiter covered by the moon in January of the year of Christ 807; Aristotle and Maestlin saw Mars, Copernicus (Book 5 Ch. 23) saw Venus.36

Aristotle's words in On the Heavens Book 2 Ch. 12: 'For when the moon was so divided into two parts, that on one part it was darkened, on the other, it shone, we saw it apparently come together with the star that is called Mars, and indeed, when it was covered by the moon's dark part, we saw it emerge from the

33 As a source for this, Franz Hamburger cites "Statikische Reihungsnalnen" (Annales Lau- troenses maioris), Mon. Geom. Hist. Script. I 194. Kepler also mentions this instance in Mecuriae in sole, Later (e.g. Ephemerides of 1615) he thinks what was seen was really a sunspot.
34 Frosch (1600 II p. 431) reports that, according to Mercator, this computation was by a certain Bruxe-schite monk named Antominum (Amiton), who wrote a "Historia Francorum" at the end of the 10th Century. Kepler's source for Averroes' observation is Copernicus On the Revolutions I 10, tr. Rosen p. 19.
35 These observations are reported by Proclus, Hypotyposis, 57-9 (Citation from TLE).
36 Aldebaran, Copernicus refutes this occulation in On the Revolutions IV 27, tr. Rosen p. 218. Brahe's discussion of this passage, and his report of his own observations, is in Progrornmata p. 245. TBOO III p. 245.
37 Antares.
38 See p. 232.
moon’s luminous part. This therefore could not have been at any other time than in the third year of the hundred and fifth Olympiad, on the night of 4 April 357 B.C., the sun being in 10 Taurus, the moon with Mars in 3 Leo, with the same latitude; when Aristotle, a youth of 21 years, would have been studying with Eudoxus, as is known from Laertius. 81

In that piece, Aristotle commends the industry of the Chaldeans, from whom he says many such observations came down to the Greeks. 82


82 Aristotle actually cites the “Egyptians and Babylonians.”
Chapter 9
On Parallaxes

And so the bodies whose appearance and measurements astronomy considers are, as we have said, the sun, the moon, and the stars, to which are added the shadows of the earth and the moon. Aside from that, the foremost thing we seek in these bodies are their motions, so much to be wondered at. But in order that an astronomer be able to fish these out with geometrical demonstrations, one first needs to measure their position with instruments. Now in geometry, when we don’t know how to describe a spiral or a conic section in a single process, we imagine some point which the line passes through, from which the whole trace of the line may be distinguished. So likewise, in astronomy (I speak to those inexperienced in it), we by no means perceive the motions of the heavenly bodies with the eyes, but we compare various positions with each other, and we thence seek the form of the motion, by which all those places previously noted, and similar ones that are going to be, may be set in order. Once this has been put forward, we astronomers have fully carried out our task. And thus the various positions of the stars at certain times are, as it were, like the elementary components, or rather the created objects, of the motions.

And position is in fact in the category of relation, and, insofar as we are now considering it, has reference to the places of the stars. For we are enclosed along with all the stars by the same outermost orb of the world. And since, from the nature of the spherical figure which the world has received, there are three regions of this worldly edifice, the center, the surface, and the in-between; while we occupy the center, whether in reality or in our perception; therefore, there are left to the stars the two remaining regions, according to which their positions are considered in two respects, namely, either in respect to the in-between or the diameter of the world, or in respect to the outermost surface.

1. On the observable position or place of the heavenly bodies, or the reckoning of it beneath the fixed stars

And indeed, the position of a heavenly body with respect to the diameter of the world comes closer to the thing itself, while the position with respect to the surface is almost entirely visual.

For since everything that is seen is comprehended and seen with a certain angle of luminous rays flowing together from the edges of the object seen to the center of the eye, we then finally consider ourselves to have seen, i.e. observed, a heavenly body rightly, when we shall have measured the angle of vision accurately. But the angles are measured by the circle described from the point of the angle. And that enormous circle, or the spherical surface of the outermost world, is described from this point of our visual angles, that is, from the earth, because

1 Kepler uses ‘apolemena’ here, which is a transliterated Greek word meaning ‘things that are brought to completion’, but which can refer to the astrophysical effects of the motions. Cf. L&S p. 222.
the little space by which the surface, entrusted to our care, is distant from the exact center of the world completely vanishes in such vastness. It is established, therefore, that the spherical surface of the outermost heaven is employed by us to measure the visual angles formed at our eye; and conversely, that our visual angles adequately give evidence of the position of the stars with respect to the surface of the world. Now this consideration is also twofold. First, it will be said in what follows that the stars along with their distance from us cannot be perceived with the eyes. Therefore, as regards place, we do not distinguish the stars that are very near us and those that are fixed in the most distant orbit, unless one be covered by another. And so, on this consideration, we note with our eyes the conjunctions of the planets with the fixed stars. Accordingly, the angle by which two fixed stars, in conjunction with a pair of planets, are perceived, is the same angle, and the same arc of a great circle, the measure of the visual angle, by which we say the planets themselves are also distant from each other with respect to the surface. Likewise, the angle by which the outer edges of the sun or the moon are distant from each other is said to be the magnitude of the sun or moon. So much so that this leads to the sun and moon being considered equal, even though the sun exceeds the moon by many thousands of times, just because they are perceived by about the same angle. See Wicelo 4.19. These distances of the fixed stars, as well as of the planets, and even of the edges of the sun and the moon, from one another, the astronomers measure with instruments, such as the astronomical radius or backstaff, the sextant, and others, setting up a comparison of the arc interposed between a pair of stars with the whole circle, or with the fourth part of it. Although this subject is related to the optical faculty, it belongs particularly to the mechanical part of astronomy, which that most noble Tycho cultivated with greatest likelihood, and on the other hand treated with greatest diligence.

Next, since the nature of things does not allow, nor does practice ever permit, that we take note of the approach of the heavenly bodies to the fixed stars by eye (for by day the heavenly bodies are hidden), astronomers had proposed for themselves other marks from which they could number the arc distances of the stars, which do not go out of sight as do the fixed stars, subject to risings and settings, but are absolutely inseparable from their places on the ground. These are the horizon of any place, which, extended as if to infinity, forms the plane surface of the earth, dividing the whole sphere of the world into two hemispheres, equal to the circle, and the pole of that horizon, or the point that stands above the vertices of whatever place at any moment of time, and which is shown by the line of the perpendicular, following which all heavy things are carried downward, and we stand upright. In this way, sea captains take the sun’s altitude with respect to the surface, whether the arc of vision, or the angles which the rays coming both from the sun and from the horizon lying beneath make at the eye. Thus the astronomers take note of the angle which a ray of the sun or a star forms with the line perpendicular to the surface of the horizon, using rules and quadrants constructed for this purpose, with a suspension of weights. But I have begun to bubble about 3

3 Wicelo IV 19, Theonuus II p. 126: “All things seen under the same angle, whose distance from each other is not taken into consideration, are seen as equal.”
industry with ignorant people, as the occasion was given me to exclaim upon a certain Censorus of Tycho, who, while he would have been able to be numbered among the astronomers on account of his happiness of intelligence, preferred to bubble what are more irrelevancies than puerilities along with the crowd of tankers, out of lust for gabbling, such as for example the inexperienced may have, under which myster they may dare to pluck the good arts. He denied that the Tycho-an observations are of the certainty and accuracy that were attributed to them by the author (thinking of these numberings of arcs or visual angles, through the most precise instruments), for the heaven (which likewise before instruments we had made the measure of this visual angle) can be divided into many more parts, and these of fully noticeable magnitude, about which the astronomer must still have some doubt after all the precision of the instruments. And this cunning detector of the art pretends that this position of the stars, with respect to the surface, and these distances, these altitudes, and by whatever other name they come, are not set up primarily on account of the objects or the stars themselves, between the centers of which there are not arcs of this kind, but on account of our vision, and that this whole enterprise in astronomy rests upon optical reasons, and beside, that it is stupid to wish to affect a precision that is different from what can be accomplished by the sense of vision, but proud and barbarous to reject this visual precision, which is for us the first approach to truth. Therefore, neither Tycho nor any other man professes that by such an easy undertaking he lays open the greatest and most true distances of the stars, and the positions on the world’s diameter, through these visual distances. But he does profess this: that he uses the arithmetic numbering and the geometrical division of the visual arc to imitate the precision of the sense of vision, and by these visual distances constructs a road to finding out the truest places of the heavenly bodies on the diameter of the world. And let it be sufficient to repeat and to point out these things in a timely manner concerning the position of the stars, as regards the surface or the visual angle, for the sake of the more inexperienced.

2. On the altitude of the heavenly bodies from the center of the earth and the parallax from the distance of the eyes

Next, I should speak of the position of the stars in regard to the diameter of the world, proceeding on the same track, so that in a place where I cannot propose anything new for the learned, I may assist the learned in language that is popular, in proportion to their abilities, as much as the established brevity allows. I shall begin from what is best known, starting with those things previously established in Chapters 3 and 5. Nature gave two eyes to animals, not just, as is commonly believed, to assist with a loss, should the animal be destined to have occurred such a loss of one eye, but for the grasping of the distance of visible things by the eyes. For since the centers of the two eyes are distant from each other in a certain proportion to the body, that is, by a breadth of about one palm, while no certain vision occurs except when the diameter of the two eyes, which passes

3 This was the Heidelberg professor Jacob Christmann, alluded to above in Ch. 4; see the note on p. 139.
through the centers of the humors and the pupils, is directed towards the object set before the vision, it thus happens that these diameters between the object to be seen are not completely equidistant from themselves, but nod towards each other in proportion as the object seen stands nearer the sense of vision. This nodding together, natural to the eyes, Aristotle describes in Probl. 7 Section 31, when he says that each eye is put in position from the same beginning, implying that each eye sees the same thing, in one vision and image, because the cognitive faculty communicates the same things to the two eyes. And in Probl. 37 he asks by what means it happens, and why, we should see the same image with each eye even when looking at an object from an oblique angle? Let the centers of the eye be A, B, distance AB, object presented for viewing at D. The diameters of the eyes EF, GH will be directed towards it, so that if extended they would come together at the visible D. Now let the object be seen nearer the eyes at C. Again, the diameters of the eyes will be directed towards it, and will be positioned at JK, LM. And because the two isosceles triangles ADB and ACB are set up upon the same base AB, the angle at C will be greater than the angle at D. Therefore, CAB will be less than DAB, and CBA less than DBA. Consequently, the ends of the ocular diameters have passed over from F, H to K, M, approaching each other, and the posterior ends E, G have diverged to J, L. And thus it is necessary to employ a motion and a turning together of the two eyes, when the vision is transferred from the more remote D to the nearer C. And by the use and perception of this motion or animate action, the animal becomes accustomed to distinguishing the longer and shorter distances of visible objects from itself; and this only when there is some perceptible proportion of the distance of the eyes AB to the removal of the seen object from the eyes AD or BD. These things do not in fact have any effect in astronomy. For the distance of the heavenly bodies is so great, and further, so great is the narrowness of the distance of the ocular centers AB, that the eyes, when looking at some heavenly body, and even at a rather distant mountain, extend diameters AF and GH that are parallel within the limits of perception. For the rest, ordinary conceptions are to be elevated to those exceptional and sublime manners by examples of small things. And these are genuine examples of them. I therefore proceed. In the same figures let the visible again be C. The other things as before, And opposite, beyond the visible C let there be a wall, cutting the straight lines AC, BC at N, O.

Now when one eye is closed, once the medium is removed, the function stops. That is, there is no longer any distinguishing between distance, the company and spacing of the two eyes being hindered. Consequently, the eye A will consider the visible C and the point of the wall O to be in conjunction, because

---

they are situated upon the same line $ACD$. So where the eye $A$ is closed, the visible $C$ will be seen by the eye $B$ joined to the point of the wall $N$, because there is no distinguishing between the distance $BC$ and the distance $BN$. In this manner, when the eyes $A, B$ are alternately opened and closed, the visible $C$ will change its seen place from $O$ to $N$ and vice versa, as if by a leap. And the optical writers, imitating the words of the astronomers, will call this "concentration of vision," or, using the Greek word parallax. The demonstration is almost the same as the one I brought in above in Ch. 5 Prop. 7 on Aristotle's problem on the double image. And so I would say that something similar happens to us in the contemplation of the heavenly bodies, so that it may become completely clear what it is that astronomers call parallax.

3. On the diurnal parallax resulting from the distance of the earth's surface from the center

And it was said shortly before that the diameters of the two eyes proceed parallel within the limits of perception, if the sense of vision be directed to some heavenly body. Thus, with the inclination of the diameters removed, the discovery of the distance of the heavenly body by the eye, or its position with respect to the diameter of the world, is taken away. And if two heavenly bodies shall have been by our vision in the same line, our vision, although making use of both eyes, will be unable to distinguish the different distance of the two, and will consider them in conjunction.

In this way it happens that we reckon the moon and all the planets to be located under the sphere of the fixed stars, since the distance of the eyes (from each other) does not help us. And in this way, our vision may be completely in error in judging the positions of the planets with respect to the diameter of the world.

This defect in the sense of vision Nature removes through a wonderful device. For it was by all means the will of God the Creator that the human being, His image, should lift up His eye from these earthly things to those heavenly ones, and should contemplate such great monuments of His wisdom. Hence, the entire arrangement of the fabric of the world tends to bear witness to us of this will of the Creator, as if by a voice sent forth. For that reason, the ratio of the earth's globe to the orb of the moon has been made perceptible, so that what has deserved the eyes of individual humans, the attentiveness of all of them living on the whole surface of the earth, assisted by its magnitude, might comportment, and might in this way teach the position of the planets on the diameter of the world by those prior positions with respect to the surface, i.e., the distances of the angles. In the above diagrams, let $A, B$ now not be the two eyes of the same person, but two places on the earth's surface. $A$ in Europe, $B$ at the extreme promontory of Africa; let the moon be at $C$, and let it be seen at the same moment by people at the two places. And so, since the ratio of the distance of the places to the distance of the moon from earth with respect to the diameter of the world, is perceptible, which is now our supposition, the inclination of the lines $AC$ and $BC$ will also be perceptible. Further, let $NO$ be the sphere of the fixed stars at night, or the body of the sun by day at the moment of the new moon. Therefore, since those who are at $A$ cannot...
distinguish between the distance of the moon and the distance of the sun or of the fixed stars with respect to the diameter of the world. The moon C will seem to be in conjunction with the fixed star O, or the edge of the sun. But to those who are at B at the same time, the moon C will seem in conjunction with the fixed star N, or the edge of the sun, so that the former observers have the portion NO at the right side of the moon, the latter, at the left side. Therefore, the position of the moon imagined by the people at the two places will be switched from O to N. The description is exactly as before, concerning the conjoined vision of the two eyes. And in this way in general the parallax of the heavenly bodies is taken, that the arc NO of the outermost sphere, or the angle which that arc measures, is the parallax of the moon. But because it is not essential on every occasion, that people have been set out upon the earth’s surface who may look at the moon at one and the same moment of time, astronomers proceed by a somewhat different road. For by another method, about which this is not the place to speak, they compute the fixed stars among which the moon seems to appear, or the distance from the zenith under which it ought to appear, at any given moment to an eye set at the center of the earth. At the same moment that distance from the zenith is measured, which makes its way into an eye set at the surface of the earth. The difference of the two angles they call by the special name of Parallax or the commutation of vision, so that the parallax is in fact sought in this way in order to learn the position of the heavenly body on the diameter of the world, although in itself it is comprehended under the genus of visual angles, of place with regard to the surface.

About center A let there be described a great circle BC on the earth’s surface, and another DEF on the surface of the outermost sphere of the fixed stars, and let A, B, D be in the same straight line, passing through the zenith of the place. And in an intermediate place let there be a heavenly body at G not on the vertical line AD. Next, from center A and the place B of the earth’s surface let straight lines be drawn out to G and extended to E, F. Finally, about center A with radius AG let the arc GH be drawn in the orb of the heavenly body. Astronomers, then, as I have said, first find the angle DAG or the arc DE, the measure of that angle, that is, the arc of the body’s apparent distance from the zenith, which is DE, by what was said at the beginning of this chapter. Next, they find the angle DBF or the arc DE, the measure of that angle (since B is imperceptibly distant from center A, when BA is compared with BD), that is, the arc of the heavenly body’s apparent distance from the zenith from the place B of the surface. The reason for this is that if the body were to fall right on the vertical line DA, such as if it were at point H, beneath the zenith D of the place, it would clearly not change its visual place at all. For if the body H, the sense of vision B, and the center of the earth A were in the same straight line, BH and AH extended would fall upon the same point D. And AH is the visual line from the center, while BH is the visual line from the surface. And so from either place of vision, the same place D beneath the fixed stars would be indicated.
And so astronomers choose this terminus of the arc on which the parallaxes are numbered, because from this common terminus the parallaxes first begin to exist in all directions. For example, the heavenly body now at \( G \) will appear from \( A \) at \( E \), from \( B \) at \( F \), so that \( DE \) or \( DF \) is less than \( DF \) or \( DA \), or the place \( F \) seen from the surface \( B \) is elongated from the zenith \( D \) more than the place \( E \) from the visible center \( A \), which otherwise they call the true place. For it is an optical axiom that those things that are more distant appear smaller; this is, are perceived by a smaller angle of vision.\(^3\) If \( HG \) be the arc of the heavenly body's distance from the zenith in its own orbit, which, since it is perceptibly closer seen from \( B \) than from \( A \), becomes the semidiameter \( AB \) is given as having a perceptible distance [in relation] to the distance of the body \( AG \); therefore, angle \( HAG \) will be perceptibly less than angle \( HBG \). Consequently, to measure \( DF \) will also be perceptibly smaller than \( DF \), the measure of the latter angle. And therefore, their difference \( EF \) is called by astronomers the parallax of this heavenly body placed at \( G \).

Further, \( EF \) is the measure of the angle \( EGF \) or \( BGA \) because the ratio of \( BA \) to \( AE \) is imperceptible, while the ratio of the same \( BA \) to \( AG \) is perceptible by supposition. Therefore, the ratio of \( CA \) to \( AE \) is also imperceptible, and further, the angle at \( G \) is sensibly equivalent to the angle which can be set up above \( EF \) at the center \( A \). The same is also evident thus: since \( DF \) is the measure of angle \( DBF \) within the limits of sense perception, but angle \( DBG \) is equal to the opposite internal angles \( BAG, AGB \) taken together, therefore, \( DF \) is the measure of the angles \( BAG, AGB \) together, within the limits of sense perception. But \( DE \) is readily the measure of angle \( DAG \) by itself. Therefore, the remainder \( EF \) is the measure of the remainder \( AGB \), or \( EGF \), equal to it in the sense.

Consequently, it follows from these things that the angle \( BGA \) is likewise said by astronomers to be the parallax of the heavenly body at \( G \).

Now the parallax is of godlike use in astronomy. For whenever the arc \( DE \) of the heavenly body's true distance from the zenith at some certain moment can be obtained from astronomical theory (which is done in various ways), and the arc \( DF \) of the visible distance from the zenith is taken with suitable instruments and with the required accuracy at that moment, in order that the parallax \( EP \) be thus obtained, the ratio of \( AG \) the most true position or removal of the heavenly body with respect to the diameter of the world, to the semidiameter of the earth \( AB \), has then become known. And indeed, this theory of the parallaxes of the sun and the moon gives support, both to all the rest of astronomy, and particularly to the theory of eclipses of the sun. And so that parallax which is considered on the vertical circle is distributed in different ways: either in longitude and latitude through the ecliptic and the circles of latitude, by which name all of this kind of parallax is called latitudinal/longitudinal,\(^6\) and the rest are called by some other name; or it is distributed through the equator and the circles of declination, into

---

3 See e.g. Wittelo N° 7, in Th. Campana III p. 120.
4 aliquinctur.
the parallax of right ascension and of declination, according to what serve the uses of astronomers.

4. A most easy and succinct derivation of the diurnal parallaxes in longitude and latitude, using a new parallactic table

The theory of parallaxes makes its main trouble for astronomers in the eclipses of the sun. So much so, that just on account of this labor alone, it is no surprise if astronomy is neglected. And thus theorists have always led themselves to be praised for this: to assist everyone with a succinct method. Ptolemy produced an outstanding piece of work, with a lucidly described delineation of the varieties that exist in parallaxes. Reinhold, with tables very laboriously worked out at many elevations of the pole, considered that he had come to the support of the less expert. In fact, he opposed the zeal of the intelligent with the tediousness of seeking the proportional part. And so Tycho Brahe called astronomers back to the triangles, by showing a number of ready ways of solving them. However, even this labor is enormous. And so I think that if, having imitated the examples of these authors, I at last shall roll thisSysyphus stone on across the ridge, so that it may not again fall back, I may hope to deserve a certain amount of gratitude from astronomers.

I say, first, that all parallaxes of latitude in whatever degree of the ecliptic the sun or moon happens to be are equal, provided only that the same point of the ecliptic is at rising in the same altitude of the pole, and the moon is equally distant from the vision. About center A, representing the center of the earth, let the great circle of the earth BF be drawn, on which let B be the place of the observer; but let F be the point lying beneath the pole of the ecliptic, so that both this circle and all the lines cutting it are thus in the plane of the circle of latitude, which is the same as the vertical circle, because it passes through the observer’s place B. Further, let the line AB be drawn stretching to the zenith point at L, again, line AF stretching to the pole of the ecliptic, and in the same plane let AG be erected at right angles to it, which will stretch towards the nonagesimal degree, because GA is right and AF stretches toward the pole.

7 In its general sense, this word can mean ‘sketch’ or ‘outline,’ but also had a technical meaning in astronomy very much like our word ‘model.’ Cf. L&S p. 1900.
8 Cf. Almagest V11. 12, 17, 18, and 19.
9 Cf. Reinhold, Practicae tabulae cosinorum notorum (Tübingen 1551) and other editions, pp. 99–121.
11 The line of latitude is the circle along which celestial latitude is measured, perpendicular to the ecliptic.
12 The nonagesimal is the nineteenth degree on the ecliptic measured upward from the rising point, i.e., the point where the ecliptic and the eastern horizon intersect. A great
But because the proportion of $AB$ to the sphere of the fixed stars is imperceptible, let $BF$ be drawn through the place $B$ parallel to $AG$, and let $GA$ be extended to $E$. $EB$ to $K$. Therefore, $JK$ will now be visibly on the ecliptic. Further, let the moon be right under the ecliptic at point $G$, which is closest to the zenith.

It will be seen from $B$ on the line $BG$, and for this reason $GJ$ will be its parallax on the vertical. For $AB$ to $BG$ is perceptible, and is the cause of the parallax. I say that however many lines are drawn from $B$ in the plane of the ecliptic, equal to $BG$, represented by the line $GH$, make equal angles with the plane of the ecliptic. For from $B$ let a perpendicular be drawn to $GH$; let it be $BD$. Then, because $BD$ is in the plane of the circle of latitude, and this is at right angles to the ecliptic, $BD$ is therefore in a plane perpendicular to the plane of the ecliptic. And because $GH$ is the common intersection of the planes, and $BD$ is perpendicular to it, $BD$ is therefore universally perpendicular to all the lines in the plane of the ecliptic. But it is supposed that all lines $BG$ all around are equal, and $BD$ is common to all $BG$, and $D$ is everywhere right. Therefore, it is necessary that all the remaining angles $DGB$ be equal. But $DGB$ are the angles of parallax of latitude, because $B$ is in a plane tending towards the pole of the ecliptic, and $DGB$, $GBJ$, which formerly measured the vertical parallax of the highest point of the ecliptic, are equal. Therefore, what was proposed is evident: the demonstration of this could in fact have taken their occasions from Copernicus and Alfraganus on parallaxes.\[13\]

circle drawn from the nominateal to the zenith will make right angles at the ecliptic, and therefore, extended as necessary, will pass through the poles of the ecliptic. Although this may at first seem implausible, it is readily shown as follows.

The nominateal and the zenith are both $90°$ from the rising point, the former by definition, and the latter because the rising point is on the horizon. Therefore, a great circle drawn through the nominateal and the zenith will have the rising point as its pole, and will consequently be perpendicular to all great circles drawn through the rising point. The ecliptic is one such circle, and the vertical circle through the rising point is another.

\[13\] Copernicus, De revolutionibus IV 26, tr. Rosen pp. 216–218; Alfraganus, Alfragani astronomorum periculiis et comprehend., ad quos et ad Astronomiae rudimenta spectat comment., Latin Translation by John of Seville (in 1135) (Paris, 1566), Cap. 7 pp. 98–101. This book is often referred to as the Rudimenta, or as Elementa astronomicae. For Alfraganus, or (correctly) Al-Farghanī, see A. I. Sabra's article in DSB. The Rudimenta was also translated by the 'Tycho-whippers,' Jacob Chrys komment (Frankfurt, 1600).
I say again: Even if the moon is not equally distant from the vision, provided that it is equally distant from the earth's center, again the parallaxes of latitude are approximately equal. For if the place B be at M right under the ecliptic, all parallax of latitude is absorbed; and if at the pole of the ecliptic F, then if the moon G be equally distant from the center of the earth A all around, it will also be equally distant from the place F, and this case reverts to the preceding one. For all GA are by supposition equal, and AF remains everywhere the same, and GAF is everywhere right, because AF is the axis, therefore all GF are also equal. And thus if there is some inequality, it ought to be at a maximum where B is the middle place between M and F at degree 45. It will be useful to look into this. So, where AF is 1, let AG or AH be 54, as much as is the moon's least distance, when it has the greatest parallax. And because MB is 45, DB or BE, that is, DA, will be 70711 where AF is 100,000, and AG 5,400,000. And thus GD is 5,329,289, and DH, 5,470,711. Hence BGD comes out to 45° 38'. But BHD is 44° 26'. The difference is not greater than 1° 12', but we have not used BHD, the deepest parallax beneath the earth, for we do not go below the horizon in seeking into parallaxes. Therefore, from point D let DC be erected at right angles to the plane DBA, and let AC be extended equal to AG, determining the length of DC, and let C, B be joined. Therefore, because AD, DB are equal, and DC is the same, and ADC, BDC are right, ACD, BCD will also be equal, representing the horizontal parallax of latitude. For because CDG is right, the moon C will be at the horizon, or a little below. And because CR or CA is 5,000,000, where it is 70711, angle BCD will be 45° 1'. And thus the difference from BGD is 37', which is the greatest of all possible.

I say third, at the same altitude of the pole, when the same point of the ecliptic is rising, and the moon is at the same distance from the center of the earth, when it is seen on the ecliptic, all parallaxes of latitude are really equal at any degree of the ecliptic.

For, other things staying the same, and GH representing the true ecliptic from the earth's center A, and JK the visible ecliptic from the place B on the surface, let the moon be at I or K so it may be seen from B on the lines BI, BK on the ecliptic, visibly, at any rate. Now since MI, AK are supposed equal, and IBK is one straight line, AIK, AKI, and all these angles set up with the upright

---

14 The horizontal parallax is the parallax at the horizon; the horizontal parallax of latitude is the latitudinal component of the horizontal parallax.
plane through $IK$ will be equal to each other. But these represent the parallax of latitude, because the place $B$ is in the plane of the circle of latitude.

Fourth, as regard the parallaxes of longitude, let $C$, $E$ be joined. And let $CBI$, now be right. $CBI$ will also be right, because $CB$ is in the plane perpendicular to the plane $ABE$. Therefore, $ACB$ will be the parallax where it is the greatest of all, when the moon is visually setting, and it will be the parallax on the vertical circle, or the circle of latitude/longitude. And because $CEA$ is right (for $C$ is now in the plane $ICK$ perpendicular to the plane $ABE$), and $ACE$ is the parallax of latitude, $ECB$ will be the parallax of longitude on the horizon, because $BEC$ is right. But as the whole sine is to the sine of the distance of the zenith from the pole of the ecliptic, so is the sine $AB$ of the total parallax at the horizon to the sine $BE$, of the parallaxes of longitude at the horizon. But also, as $CE$ is to the sine of angle $CBE$ or $CBI$, the visible distance of the moon from the nonagesimal, however much it may be, so is $BE$ to the sine of angle $BCE$ or the angle corresponding to the parallax of longitude, and that from trigonometry.

Next, as for the procedure, from the hypothesis of the lunar motion, the ratio of $AC$ to $AB$ is given. But from the altitude of the nonagesimal, by the theory of the primum mobile or the table on fol. 42 of Copernicus, the $AE$ is given, and $AIE$, by operating either by the sine of or by Tycho's parallactic table on fol. 120 of the Prognosticatia, and because $AEC$ is right, as is $ABE$ also, and $AEC$, $AIE$ are equal, therefore $EC$ and $EB$ are given. And because $CBE$ is right, from this and through $CBI$ the parallax of longitude $BCE$ is given at any altitude. But because $AC$ is slightly longer than $EC$, it is worthwhile to see again how much error is incurred if $AC$ be taken instead of $EC$. And again, when $BF$ is $45\degree$, the error is greatest. For if $B$ be at $M$, then $EC$ and $AC$ coincide. If, on the other hand, it be at $F$, then there is no parallax of longitude. Let $AF$ be 54 semidiameters: $AIE$ will be $45\degree$ 1'. Consequently, $E$ or $EC$ will be 5,399,552.

This line, with $EB$, lends not half a second to angle $ECB$ compared with the case where we would have used $AC$ and angle $ACD$ instead of $ECB$.

And because Tycho’s table has few colations, and does not operate down to the greatest first minutes, but has appendices of the seconds, even at the top, then too, because the entry into it is properly made through the true altitudes, while we shall need the observed ones; and finally, because in the margin it has, not the distances from the zenith, but the altitudes, which fact made confusion for us; for these reasons, I have added here a more universal table of parallaxes. Hence, the following instructions.

---

17. Copernicus, De revolutionibus (Nürnberg, 1543). II, 10, fol. 42, tr., Wessel p. 76. The altitude of the nonagesimal is the same as the angle the ecliptic makes with the horizon, which is the title of Copernicus’s table.

18. TACO II pp. 132-4.

19. The parallax table compares the products of sines, allowing entry and extraction through the angles themselves. The number in the body of the table is the angle whose sine is the product of the sines of the angles at the top and the margin. As Kepler remarks below, the table can be used for all kinds of sines, not just parallaxes, and in fact serves it for other computations in his Astronomia nova.
By the theory of the primum mobile, compute the distance of the nonagesimnal from the zenith, and its complement, that is, the altitude of the nonagesimal or the angle between the ecliptic and the horizon. In fact, Copernicus's table on fol. 42 shows the latter in a rough way, while Reinhold's parallactic tables on fol. 99 and following of the Prutenica shows the former at the beginnings of the signs and a certain number of altitudes of the poles. Next, enter in to one parallactic table the argument through the distance of the nonagesimal from the zenith, but from the top through the greatest parallax of the heavenly body, which applies at the horizon (and this is located at the top immediately below the distance of the body from the earth's center), first, by the whole number of minutes, then by the seconds, if there are any, and the body of the table will show the parallaxes of latitude, in the former case in minutes and seconds, and in the latter, in seconds and third parts, as is usual in these tables. And these are obtained most correctly when the heavenly body is visibly on the ecliptic, as the moon is in an eclipse of the sun. The parallax of longitude is exacted by a double marginal entry in this way. First, enter from the margin the altitude of the nonagesimal, from the top in the greatest horizontal parallax; the body of the table will show the greatest parallax of longitude. Then enter by this, opus, from the top, but from the margin to the visible distance of the heavenly body from the nonagesimal; the body of the table will show the corresponding parallax of longitude at your moment. You thus see that this table of mine is chiefly useful when the visual place of the moon is known from observation, while the Tychoic is chiefly useful when the moon's true position is had from computation. Nevertheless, each can be obtained approximately from the other, by adding or subtracting the parallaxes, first taken roughly with precision of half a minute, next, by the transformed place, extract the perfectly accurate parallaxes, from whichever table you please. How little this procedure may be in error, even if the moon has latitude, and how it is to be remedied, and how the same table is to be accommodated to the equator and the circles of declination, and thus to the first motion, and the problems that arise thence of finding the altitudes of celestial bodies from the earth, would be too lengthy to state here, the reader himself may imagine. I think, however, that to prepare for future use, that the column which has at the top the horizontal parallax of 60 minutes, should be marked in red.

5. On the parallax resulting from the distance between the sun and the earth, or the annual parallax

But in fact the most wise Architect of the world does not cease to edify humankind with this. For just as, when the distance of the eyes was not sufficient

16 Prutenica tabular. Württemberg 1585. In this edition which is the one Kepler was using, there are three sizes of page numbers. First there are eight immovable foot pages together with folia 1-68, comprising the "Logistica spirularum astronomica". Then there is a new title page, inscribed, "Initium canorum Pruteniorum," followed by folia 1-1. Finally, there is yet a third title page, "Sequence spatium miner canum," etc. It is fol. 99 of this third series that Kepler means.
to a person for knowing the true distance of the moon (which is the lowest heavenly body) from earth, it was fitting that the breadth of the orb of the earth come to the aid of the narrowness of the sense of sight. Likewise, when even this remoteness of the surface of the earth from its center has vanished compared to the incredible altitude of the higher planets, in order that those heavenly bodies might not be spread out in vain and unobserved through the circumference of the heavens, but that instead the human mind might even carry on through to them, God has constructed another, much broader class of parallaxes, in consequence there might be destined to exist among humans one who might want to follow this line of reasoning without offending piety, and disregarding false criticisms of his works. Copernicus, and Reinhold in the Proteus, call this the parallax of the annual orb. However, Tycho Brahe carried that line of reasoning over from the earth’s mobility to the sun’s mobility so that the optical theorist has nothing by which to choose the former or the latter reasoning. I shall enumerate them both in the diagram in Sect. 3 above, arising from the Copernican reasoning.

Let A be the sun’s body, the common center of the annual orb BC, which carries the earth, and of the sphere of the fixed stars EF, and let the ratio of BA to AB be imperceptible. Now let the heavenly body be at G and the ratio of EA to AG be perceptible. Therefore, when the earth is placed at C, at the midpoint between the sun A and the body G, the line AG and CG will coincide, and, both being extended, will arrive at one point E among the fixed stars. Therefore, whether the eye be placed at the sun A or at the earth C, the heavenly body will be seen in the same place under the fixed stars. Here, then, namely at opposition of the sun and the heavenly body, there is no parallax of the body from the annual orb. With the other thing—remaining the same, let the earth be at B outside the line GA. Therefore, the heavenly body’s place will appear from the earth to be at F beneath the fixed stars, but from the sun A at F, because BA to AG is perceptible, and consequently the inclination of the lines BG, AG is also perceptible. Therefore, since the center of DF is equally B, to the sense, and A in truth, the arc EF will again be the measure of the angle EGF or BGA, and the former as well as the latter will be the parallax of the annual orb, when the earth is located at B. And so, when the heavenly body’s place E, which a line projected from the sun determines beneath the fixed stars, is known by other astronomical principles, while that place F of the heavenly body, which the vision coming from earth determines beneath the fixed stars, is known, and so also the parallax E F or BGA, it will be impossible for the ratio of AG, the distance of the heavenly body from the sun with respect to the diameter of the world to AB, the distance of the sun from earth, to be unknown, however much it may be exceedingly great. And so it was evidently not fitting that the human being, destined to be the inhabitant and watchman of this world, should reside in its middle, as if in a closed cubicle, under which circumstances he would never have made his way through to the contemplation of heavenly bodies that are so remote, but rather, by the annual translatory motion of the earth, his domicile, he circumambulates and struts around in this most ample building, so as to be
able more rightly to perceive and measure the individual members of the house. The geometrical art imitates something similar in measuring inaccessible spaces. For unless the measure passes from one station to another, and turns his eyes sideways at each, he is unable to arrive at the measure sought.

In the Typhonic reasoning, let $A$ be the earth, the center of the sun's orb $BC$ and of the sphere of the fixed stars $DF$. Let the heavenly body lie at $G$, and let the ratio of the three lines $BA$, $AG$, $AE$ be perceptible. First let the earth $A$, the sun $C$, and the heavenly body $G$ be in the same straight line; there will be no parallax, because lines $CG$ and $AG$ drawn from the sun and earth to the body coincide. Now let the sun be, not at $C$, but at $B$, and let the line $AB$ show at $D$ the place of the sun beneath the fixed stars. And because the sun at $B$ is that point to which Tycho refers the eccentricities and apogee, and the simple motions of the planets' orbs, it may therefore come to be known through astronomy now great is the angle $DBG$ between the line from the sun through the earth, and the line from the sun through the heavenly body. And at that very moment, the place $E$ of the body $G$ beneath the fixed stars that appears from the earth $A$ may become known through instruments. Therefore, the arc between the sun's place $D$ and the observed place of the heavenly body $E$ will be obtained, which is the measure of angle $DAE$, and because the ratio of $BA$ to $AG$ is perceptible, while $B$ is outside the line $AG$, the lines $BG$ and $AG$ will therefore be inclined, and the angle $DBG$ will be equal to angles $BAG$, $AGB$ together. Thus the parallax $BGA$ will be known, and again as before, the ratio of $BG$, the distance of the sun from the heavenly body, or $AG$, the distance of the earth from the heavenly body, to $BA$, the distance of the sun from the earth, will come to be known. The only difference is this, that here $EF$ is not parallax, because it is not the measure of the angle $BGA$ or $EFG$. For because the ratio of the lines $BA$, $AD$ is perceptible, and $A$ is the center of $DF$, $B$ will therefore be perceptibly distant from the center of $DF$, and consequently $DF$ will not measure the angle $DAB$ within the limits of sensor perception, nor are $BAG$, $AGB$ together equal to it. But $DE$ measures the angle $DAE$ or $BAG$ by itself; therefore, the remainder $EF$ does not measure the remainder $AGB$, but there is a perceptible difference between them, which needed to be said in place of a warning lest this ratio escape someone who is straying over from one form to the other.

The optical principles upon which rests the theory of parallaxes, that is, the distance of the stars, have I think been explained sufficiently in accord with the brevity that has been established. It remains for us to speak as well of the motions of bodies, so that in this book nothing of those things in astronomy may be overlooked that are to some extent to be decided from optics.

6. Short appendix on the curved tail of comets

Those who argue about the curvature of the tail in the comet of 1577 from the theory of parallaxes as if this illusion of curvature was based on the different
parallaxes of different parts of the tail, and were not really in the tail itself, do not sufficiently consider the subject of parallaxes. Nor do they rightly bring in the optical theorists as witnesses, for in those passages they are dealing, not with the aberration of vision about curvature, but with the true and legitimate vision of obliquity. If it really were true that parallaxes could show a curve from straight lines, it would not now be true that every straight line, no matter how extended, viewed from the perceptible center of the world, corresponds in all its part with some great circle. As a result, the ways of observing the places of the heavenly bodies with a thread and with rulers would be false. Therefore, as was said above in chapter 6, another occasion for this curvatures must be sought, or if one cannot be found, this phenomenon must be left among nature's mysteries.19

19 See Ch. 6 Sect. 13, p. 274
Chapter 10

Optical foundations of the motions of the heavenly bodies

Since in astronomy, our aim is the contemplation of the motions of the heavens, while everything that we know previously came into our sense, therefore, it is worth our while to consider whether the motions of the heavens come immediately into the perception of the eyes, and what kinds of optical illusions occur in the celestial motions. Let us begin thus.

Everything that is moved is moved in place, for motion is change of place. And place is attributed to the surface with contains the movable body. But the container is greater than the contained, and makes room for places because it occupies places. For the whole is greater than the part, while that which contains and encloses together with that which it encloses is a kind of whole.

It therefore follows, conversely, that, of two objects which are separated from each other by motion, that which appears to be greater performs the visual role of the place, while the remaining one performs the role of that which is placed. For as motion is in a place, so is the motioned a visible in a visible place. For this reason, since rest belongs to place, what is viewed to be greater will be thought to be at rest, while what is viewed to be less will be thought to move, even if it might happen to be the contrary in actual fact. For there exists no way to grasp motion visually except by comparison to some things at rest.

But the cause of this fact is obtained more evidently from the form of vision. For since the eye is spherical, and in addition makes use of multiple refractions, it happens that in one and the same glance the image of more than a hemisphere flows into the eye at once, and nonetheless, from this entire hemisphere only the slightest little part is perceived directly and distinctly, namely, that which is in the middle of the hemisphere, while the parts lying all around are all perceived more and more obliquely and confusedly. On these points, see chapter 5. Hence it happens that what is perceived to be greater takes up more of the eye, while that which is perceived to be less takes up a lesser part of the ocular surface. Therefore, when a separation occurs, such as, for example, that of some cloud from a star shining in between, then the object that is smaller to the sight, the star, more conspicuous through the effect of the separation, attracts the eye's vision to itself. Thus the star, since it is perceived under a small angle, is observed directly by the eye, while the cloud which is seen under a greater angle, and which takes up nearly the whole eye, is observed obliquely by the same eye. It therefore ascribes the act of separation to that object which is perceived directly, namely, the star. In this way, the sense of vision is in error about the movable.

2 Here Kepler departs from Aristotle, who held that there are four kinds of motion, corresponding to the four kinds of being (thinghood, amount, quality, and place. See *Physics* III 1, 206b 35, trs. Sachs p. 73.
For if a cloud tends from east to west with a swift motion, the star, even though it tends to the west, albeit slowly, will appear to be carried to the east, in the path of the cloud.

But the cause of this aberration that is the most obvious of all lies in this, that the eye, being attached to the posterior part of the head, sees nothing under an angle greater than the projecting parts of the face. These, however, maintain the same place with respect to the eye, with the result that all the rest will appear to move, if by some device one might turn it about without its knowledge, so that it might not be the assessor of the motion which it creates itself. And just as the eyes are attached to the head, so, through the head, they are attached to the body; through the body, to the ship or the house, or to the entire region and its perceptible horizon. Since these are nearby, and appear large, and are seen under a large angle, and hold the same place with the eye, it is necessary that all the remaining things, whose place changes with respect to that which contains the viewer (be it a ship or the surface of the earth), appear through themselves to be in motion. For they play the role of the located, while those things that lie nearby about the place and the container.

From these things it follows that even if someone were to carry us across to the moon or to another of the wandering stars, and the moon’s motion is most highly perceptible because of its swiftness, of which more later, nonetheless the moon is going to appear to be at rest along with us, while the sun and whatever heavenly bodies are at the right distance are all going to be thought to be moved with those motions which were proper to the moon itself alone, in addition to their own motions. Therefore, the optical writers do not have anything to produce from their arsenal against Copernicus when he maintained that our home, the earth, moves.

And, to add this too in passing, the absurdity of the triple motion is slanderously exaggerated. For the motion which Copernicus attributes to the earth has no other form than that of a wheel on a curb moving in a straight line. First, the wheel rotates, then by the rotation itself the axis is carried along, and third, the axis points in the same direction, and remains parallel to itself. Who is it that conceives the opinion of a third motion, from the fact that it is really rest?

For the fact that in the passing of the ages the earth’s axis finally also inclines, does not come into this reckoning, and would instead be a fourth motion, if a completion of the wheel were desired. Astronomers know it to be of that kind of motion by which the appearances and modes of all the planets are moved along, not by a true motion, but by a kind of difference, so to speak, of two other acknowledged motions.

And now the optical writers will have much less against Copernicus if I should remind them that the speed of the stars is not even perceptible: Aristotle, Sect. 15 q. [i.e., problem] 12, takes that as acknowledged, saying, “the motion of

2 Kepler later elaborated upon this theme in his science fiction novel Somnium (Dream), which describes a journey to the moon and gives an account of the astronomy of its inhabitants. See Kepler’s Somnium, rev. Roger Hudson, 1965.)
the sun is not evident.” 15 And Cleomedes, Book 2: “since it appears to be standing still.” 16 For it is in conformity with reason that, between two motions, the one that is slower is thought to be more like rest. Therefore, since those animate motions and rotations of the body, of the neck, of the eyes, are perceptually many times faster than the celestial motions, it is evident that the motions of the heavens are to be thought instead to be like rest by our sense of sight. This all the more so from the fact that any visible object, just as it has a quantity, ought also to have a certain speed, and a perceptible proportion to the eye, and so ought its motion.

Whatever there is besides this cannot by grasped by the sense of vision, as Optics attests. See Wietelo IV, 3 and 110.17 Now the speed of the stars has no proportion to sense perception, as will be apparent if you consider that your sense of vision is carried around by the sun about the center, either of the eyes, or of the head, in the space of twelve hours, by not more than 180 degrees, through which space we are in other circumstances accustomed to turning the eyes faster than in one second part of hourly time. And in one hour there are 3600 seconds, in 12 hours the sum is 43,200 seconds. And what proportion is there between one and forty thousand? Quite imperceptible. The same is derived from Wietelo IV 112, when the object seen stays in the same perceptible place over a perceptible time, it is thought to be at rest.18 But this is true of the heavenly bodies. For a moment, or one second part of an hour (which is usually set about equal to the arterial pulse) is a perceptible time, but the place of the eye with a size of one degree cannot be sensed without the use of an instrument. And a star stays for four first minutes, i.e. 240 second minutes,19 in any one degree, that is, in some space that hardly has the ratio of a point. And so whatever is in our senses concerning the motion of the heavens, we have absorbed thanks to the intervention of reasoning. The sun was there before, now it is here. It therefore moved from there to here.

Moreover, just as the vision generally attributes motion erroneously to things at rest, so it also contrives for itself the forms of motion. Hence, the vision attributes rising and setting, that is, ascent and descent, to the stars and Ovid,20 imitating vision, attributes it to Phaethon, in the most delightful fable of book II of the Metamorphoses, because the vision finds these differences of places within a person, and in that person’s upright position with respect to the visible horizon, since there is nothing of the sort in the heavens themselves. Hence, when Wietelo showed in Book IV Prop. 10 that bodies that are set in order in a continuous

---

15 ὅτι τὸ ἐνεπήφανον Ἀλεξ. This is actually from Proclus, XV 13, 913A 7-8. Kepler has changed the Greek slightly, and omitted a few words.
16 ἐνυφοτα γενέσε ξενίτε, De natura coeli II 1 p. 130, trs. Guiter (French) p. 137.
17 Theonar H pp. 119 and 167. The former establishes the limits of the angle under which an object can be perceived; that later describes how visual perception of motion takes place.
18 Theonar II p. 168.
19 The hour is divided into 60 “minute parts”, which are the “first minutes”, each of which is divided into 60 parts, the “second minute parts”, or seconds.
20 Ovid, Metamorphoses II 63 et seq.
line with the \textit{vision} are evidence for a distance of the last that is greater than if they had not been set in order in a continuous line, he showed in prop. 13 why the earth's horizon appears to be in contact with the heavens; and why that part of the heavens appears more distant from us, and the distances of the heavenly bodies greater, than those that are at the head's zenith.\textsuperscript{11} These are accordingly the necessary terms of vision, which we would not be able to do without even if we were really carried about in the globe of the moon.\textsuperscript{12} We ought all the less to be amazed that Copernicus dared draw a distinction between those things said in Holy Scripture, rightly indeed, to give an account of vision, and those things which, examined astronomically, are understood to hold otherwise. For scripture does not speak falsely, but affirms with perfect truth that the sense of vision says, this, or better, that it accommodates to its own purpose this thing suggested by the sense of vision. The astronomer, on the other hand, or rather, the Optician, convicts the sense of vision of error without any affair. Indeed, when we read in a thousand places the mention of the ends of heaven, to which the people Jeddah are scattered and hence called back, there is no one who doesn't see that these are the things explained by Witelo IV 13.

But since by Witelo IV 11, motion itself is judged by the space over which the visible is moved,\textsuperscript{13} while all motion takes place in a line, and this is either straight or circular, it is therefore evident that if some error befalls the vision in the lines of motion, the same happens in the positions themselves. Errors of this kind, or rather, phantasms,\textsuperscript{14} are of two kinds among astronomers, following their attribution of two main circles to one planet, one the eccentric and the other the epicycle; but Ptolemy does not in one way. Copernicus, whom I follow, in another, and Tycho Brahe, most recently, in another. For the planet itself traverses the eccentric circle with its own body for Copernicus and Tycho, while for Ptolemy it is not the body of the planet, but the center of the epicycle; on the other hand, for Ptolemy the planet itself traverses the epicycle with its body, east on its own epicycle; for Copernicus, the single circuit of the earth removes all the epicycles; for Tycho, both the sun and the whole planetary system have one and the same circuit, common in all parts, which likewise removes all the epicycles.

For that reason, for Ptolemy and Tycho, the planet's motion is not simply about some immobile point, epicyclic for Ptolemy and eccentric for Tycho, but it is

\textsuperscript{11} \textit{Prior} II pp. 12-4. Kepler's sense here is that in proving the latter proposition, Witelo cites the former. His restatement of Wittelo IV 10 is somewhat different from what Witelo wrote. He said, for example, \"therefore, if we are lacking the comprehension of the bodies set in continuous order, or if we are lacking an intermediate amonters, we will never comprehend the remoteness of those bodies by a true comprehension; only following guesses.\" (p. 121).

\textsuperscript{12} This is, terms such as \"rising,\" \"setting,\" \"moving ahead,\" \"falling behind,\" are indispensable to the way we see things, applying only to our perception and not to how things really are.

\textsuperscript{13} \textit{Prior II} p. 167. Witelo actually says that the \textit{quanta} of motion is comprehended by the space.

\textsuperscript{14} \textit{Prior I}.
truly spiral and compounded of eccentric and epicycle: for Copernicus alone, that eccentric which he hypothesizes is truly the simple motion of the planet itself about a motionless point or the sun (except insofar as I am going to be introducing a small correction to this opinion, applicable to all the authors, in the Commentaries on the motions of Mars), not is it varied by an epicyclic motion, since according to this author that is nothing but an illusion of motion, in that the earth’s motion is not noticed on the earth and is described instead to the individual planets. However, this difference of hypotheses be compared, there exists, as I said, a twofold perception of motion, vulnerable to error. One, which is from the eccentric, is common to all the authors; the other, arising from the epicycle, in Copernicus is a more hallucination of vision about the planet, in the other authors has something joined together from the true motion of the planet. Because of the eccentric, the planets appear either slow or fast. The cause is partly physical, partly optical. The physical part of the cause does not give the sense of vision a reason for error, but also represents to the vision that which in fact occurs, (an account) of which is in the Commentaries on the motions of Mars. The ancient used to represent it through the equant circle. Part of the optical cause is located in this, that since the planet’s motion is eccentric with respect to the vision, some parts of this circle are accordingly farther away from the vision than the rest. Therefore, through those things that were said in ch. 9, and through Wecel IV 7 and 131, equal arcs of the planet’s, circle will appear unequal, small above the appogee, large about the perigee. Therefore, even if the planet itself were equally fast in all arcs, it will nonetheless appear slow where the arcs appear small, fast where they appear large.

15 The Commentaries on the motions of Mars appeared in 1609 as the Astronomia nova (translated into English by W. H. Donahue under the title New Astronomy, Cambridge 1992). At the time of writing the Opus, Kepler was in the midst of his work on Mars. Although he had not yet decided that the orbit must be elliptical, he had known since April of 1602 that it must be some kind of oval, not a circle. This is the “convincing-lan” to which he refers. See W. H. Donahue, “Kepler’s First Thoughts on Oval Orbits: Text, Translation, and Commentary,” Journal for the History of Astronomy 75 (1993) pp. 77–100, and “Kepler’s Approach to the Oval of 1602, from the Mars Notebook,” Journal for the History of Astronomy 89 (1996) pp. 281–295.

16 Kepler’s account here contains another simplification. As Copernicus stated in On the Revolutions V. 4, fol. 142c, the planet’s actual path in Copernicus’s theory is not a perfect circle, but “differs imperceptibly” from a circle. The irregularity is not based upon observations, but is created by an extra epicycle which Copernicus used to replace the Ptolemaic equant. In chapter 1 of the New Astronomy, Kepler states that the resulting path bulges out slightly at the video, a change from circularity contrary to that which Kepler’s theory required.

Kepler evidently did not count this departure from circularity as a true epicyclic motion, even though Copernicus used an epicycle to produce it, because it did not result in the loops and reversals characteristic of the motions produced by the much larger Ptolemaic epicycles.
This optical magnification or diminution of arcs is best evaluated in semicircles, in the following way. Let A be the center of vision, as well as the center of the world. B the center of the eccentric; C, D, E, F being extended to C and E, C will be the apogee, F the perigee. At the points A and B let perpendiculars GA and DE be set up. And let the planet now be constant in its strength of motion, throughout all the arcs of the eccentric. It will be on GHC and HFG for an equal time, and will consequently be on DCE for a greater time than on DFE. But at D and E, it will appear from center A to be at opposite places on the sphere of the fixed stars, whose center is A; therefore, it delays more on DCE than on DFE. Furthermore, because the vision does not know that DCE is greater than DFE, because it does not distinguish the remoteness of the parts of the two circles, but considers them equally distant, it therefore considers the planet to be nearer above the line DAE, than below it.

But you ask, by what argument is it known that the planets seen at opposite parts of the circle? I answer, first, from the center A, we imagine for ourselves a great circle, which is called the equinoctial. Next, we know from observations that the same center of our vision A also lies in that plane in which any of the planets you please carries out its eccentric path. We also know that this plane is even with itself, not twisted, and is inclined to the former circle, that is, that it cuts it. And when two circles cut each other, the line of common intersection is straight, by Euclid XI.3. Since this goes through the center of the equinoctial, as through A, which by supposition is in both planes, it will therefore cut the equinoctial at D and E, places opposite when seen from A. Therefore, whatever contravance is used to ascertain that the planet is on the equinoctial, is also used to ascertain that it falls at opposite places. And that is taught in astronomy and in the theory of the quantum mobile.

Further, from the occasion of turning their attention to this phantasy of optical speed and slowness, the astronomers run into a different and contrary error, not so much of vision as of reasoning. For when astronomy teaches us this axiom (in Aristotle's words, "the more distant moving bodies appear to be moved more slowly,"17 the astronomers convert it with great plausibility to say "the slower moving bodies seem to be carried along farther out."20 To the extent that a planet delays for a longer time in some arc or semicircle than in

17 The verb "more" could perhaps more accurately be translated "speeds time." However, the root of "more," "delay," plays a crucial role in the New Astronomy, so the present translation has been chosen accordingly. For Kepler's use of this term, see Peter Barker and Bernard R. Goldstone, "Distance and Velocity in Kepler's Astronomy," Annals of Science 51 (1994) pp. 56–73, and W. H. E.薏eed, "Keples' invention of the second planetary law," Proceedings of the History of Science Society 92 (1944), 39-102.

18 This is another name for the equinoctial.

19 See the passage in which the same author, E. H. E.薏eed, states that it appears to be a paraphrase of On the Heavens II 13, 290, 32–34.
the rest, they argue that that arc has receded that much farther from the vision. But the conversion is not necessary, and is in part false. For here are also other causes of retardation mixed in with these optical ones. Thus, in the beginning, Ptolemy was deceived when he read the planetary epicycles too high on the one side and pushed them down on the other, because the slowness of the one place, and the speed of the other, seemed to require as much. But the error was immediately obvious from the apparent magnitude, for the epicycles increased less at prorogue than accorded with such a close approach, for which reason another cause for the slowing down was seized upon, which, as I have just said, Ptolemy ascribed to the circle of the equant. 21 In the sun, no epicycle was needed, and as a consequence this error has remained to this day. It was, however, first discovered by me, through an exact observation of the visible diameter, as I shall say below, and then by Tycho's most precise observations taken of the star Mars, as I shall make plain at the proper time and place. 22 By both arguments I've established that the sun recedes from us by only half of the eccentricity which was attributed to it by Alhagreinuis 23 and Tycho, and thus that an equant circle governs the motion in the sun, too.

Because of the epicycle, or, in Copernicus, because of the circuit of the earth, and of the vision along with it, the planets do not always go forward for us, but sometimes are seen to stop, sometimes even to go back. They stop when they are perceived to stay in the neighborhood of the same fixed stars; go back, when they are at first perceived in the neighborhood of most easterly fixed stars, then after a number of days they are perceived in the neighborhood of more westerly ones, just as, on the contrary, they are seen to go forward, when the contrary occurs. But, as I have said, in Ptolemy the planets really go back on their own epicycles when they traverse the epicycles' lower semicircle; since, as Aristotle teaches in the Mechanics, 24 a circle moves with one and the same motion in opposite parts, but in different directions, and this retrograde motion of parts surpasses in speed the slower forward motion of the center. The same happens in Tycho, when the planets does in fact go forward somewhat on the eccentric, but, being driven back by the motion of the sun itself along with its entire eccentric, it is carried far faster in the opposite direction. But since, by Wito IV 4.4, a straight line appears to be a point when it is extended directly away

21 The procedure sketched here appears in the Almagest X 7.8. For Ptolemy's determinations of the eccentricities and sizes of the epicycles of the outer planets, see Otto Neugebauer, HSMO pp. 172–80, and Olaf Pedersen, A Survey of the Almagest pp. 273–86.

22 Kepler's observations of the apparent diameter of the sun appear below, ch. 11 problems 2 and 3. The demonstrations to which he alludes here establish that the earth (for Copernicus) or the sun (for Ptolemy and Tycho) does not traverse an eccentric orbit with uniform speed; but instead moves on an orbit with a smaller eccentricity, and the outer planets, moves faster when closer to the "center of vision." The demonstration from Tycho's observations was later (1609) to appear in Part 3 of the New Astronomy, chapters 22–30.

23 See Ch. 4, footnote 172.

24 Mechanics 847b 16–848a 37.
from the vision, the judgement will also be very nearly the same concerning the apsides or points of tangency of the circle of the epicycle, which arcs, directed nearly straight upwards from our vision into the depth of heaven, appear, if not in points, at least with the image of a very small quantity, as a consequence of which they are perceived as slowest about those parts. Thus it can happen, despite the epicycle's being faster than is the eccentric, that nonetheless the forward motion of the eccentric and the backward motion of the epicycle become equal to the senses, so that when one is subtracted from the other, the planet appears to stand still, and stays in the same longitudinal position in the heaven itself, with respect to the sphere of the fixed stars, even though meanwhile it either labors at an ascent into the aether away from the vision, or sends itself downwards from the deep aether to the earth, in a nearly straight line.

As regards Copernicus, however, this whole illusion of standing still and retracing of steps is demonstrated most beautifully from optics. And although these things are more appropriately learned from the author himself, nevertheless, so that nothing might here be said which would affect the reader negatively, I shall repeat the fundamentals in three words from Euclid himself. It is indeed my judgement thA if we had not had other arguments by which ansynuity had tested this Copernican opinion, this passage alone would have been sufficient to vindicate Copernicus from the truth of Pythagoras. First, it is evident, not only in itself but also from the commentary of Proclus,27 that all of Euclid's geometry is Pythagorean and aims at the knowledge of the five regular figures which are called "cosmic."28 Euclid was therefore a Pythagorean. Next, consider for me the bundle of Euclidean propositions in his Optics, namely, 53, 54, 55, 56, 57, 58, which filo carried over into his Book IV Propositions 134, 135, 136, 128, 132, 133, 129.29 In these propositions, Euclid propounded pure, undiluted Copernican astronomy.

And in fact, Proposition 53 appears to seek an exemplar of celestial objects in nearby objects, and calls that into consideration. For it states,

> Of those things which are carried with equal speed and are in the same straight line, that which is nearest the eye appears to follow; that which is farthest, to go ahead. But where the line of the movable objects gets more distant from the right of the vision to the left, that which had gone ahead before now appears to follow; the one which has followed, to go ahead.

This appears to have in view a cart going across in front of the eyes, in order to show that it is not absurd for many things to happen in the heavens which, though not completely the same, are similar or of the same kind. One might, however, use this for his own ends, to demonstrate that even if for Saturn, Jupiter, and

26 geocentric.
28 Mundane: I have, however, reverted to Proclus's original word.
29 Theorica II pp. 176–8.
Mars, the epicycles were so adjusted as to be exactly the same size (which one would do if he were to correct the Ptolemaic form from the Tychoic observations); nonetheless, the epicycle of Mars would turn out to appear greater, and that of Saturn less, than Jupiter.

And now, in Proposition 54, he smacks of nothing but Copernicus. "If, it says, "some things be carried with unequal speed, and the eye also be among them, those which are carried with the same speed as the eye will be thought to stand still; those which are carried more slowly than the eye, to be carried in the opposite direction; those which are carried more swiftly, to go ahead." I shall change nothing but the words. If the planets and the earth, the lookout post of our vision (which, moreover, takes place on the semicircle of the terrestrial orb that faces the planets), are carried forward, and it should happen than the earth and a planet are moved forward equally (with respect to some identical straight line), the planet will seem to stand still;[30] but if the planet be slower, it will seem to be carried backwards: and if it be faster, it will seem to be carried forwards.

If there be anyone so nitpicking so particular, as not to be able to hear this, let him substitute the moon in place of the earth, and locate upon it some viewer of celestial objects, then the same things would follow in the moon: this earth of ours, even if it really be at rest, will appear to move, but the moon will appear to be at rest, although it moves, and those things will not be able to be overturned by any solution.

Proposition 55 appears to make noises about the diurnal motion. "If, it says, "some number of things be carried along together while some one thing is at rest, it will seem to move in the opposite direction." The eye may indeed be supposed to be in the middle of the world, because of the earth's evanescent ratio to the world, and the earth may be rotated from west to east, with the diurnal motion; therefore, the mountains, which seem large and continuous, being thus carried along and the stars, which appear small and scattered, being at rest, the stars will seem to move in the opposite direction, that is, from east to west.

Again, Prop. 56 distinctly expresses the spirit of Copernicus. It says, "When the eye approaches the thing observed, the thing will be thought to grow larger." I accordingly bring in under this, that when the earth carries our vision towards the bodies of the planets, both the lines of motion and the bodies of the planets themselves will be seen as large. Thus this phenomenon is explained, not only by the approach of the star to the eye, which is imagined to occur through an epicycle, but also by the eye's coming closer to the object. And that is powerfully obvious. Melanchthon states that in July and August of 1559, Mars was seen with such a prodigious image that it was believed to be a new star. The same ought to have occurred in August, 1561, and nearly the same in August, 1593; and it will happen in July, 1608. And in the month of February and March of this year 1603, we saw the star of Venus, setting through clouds, unusually large, and many were

30 Here Kepler refers to the following endnote.

To p. 332. This is therefore the definition of a station in Copernicus, because it is made to happen when a line through the earth and the planet, being moved along, does not incline, but remains parallel to itself.
asserting that they had seen a new star. For at these points of time, the perigees of the eccentric and the epicycle came together, which in these two planets is of the greatest importance; it has a lesser effect in the others.

Proposition 57 is greatly accommodated to that illusion of motion that is ascribed to the eccentric. For it states that "Of things that are carried with equal speed, those which are farther away appear the he carried more slowly."

Finally, Proposition 58 states, "When the eye is carried along, those things seen from farther away appear to be left behind," where he openly uses the astronomical term ἀπομείωσις. Further, in astronomy, ἀπομείωσις is the same as to move forwards, "in consequence": I believe the term is relative to the other contrary, ἐπιμείωσις, to go ahead. If for some things go forwards, it is necessary that the other be left behind. And thus he is say obviously speaking of astronomical matters. The experience drawn from mountains and fences is properly subsumed here: to walk walking by a fence, the nearby fence appears to go in the opposite direction, and the distant mountains appear to accompany him. You might be able to accommodate an example drawn from this to the sun's motion among the fixed stars, but with terms bent slightly from astronomy to everyday usage. For let B, A be fixed stars C the sun, the earth at D, at which place let the earth be moved in the direction of B, just as when it is opposite it is moved in the direction of A. Therefore, when the earth D is going towards B, the sun C, being at rest, will seem to move towards A, but the fixed stars in conjunction with the sun, as E, will seem to be left behind by the sun and to accompany the earth in this position in the same direction, which in this place comes to be indicated by the word ἀπομείωσις in common usage, although

31 [The Pythagoreans, when they shared out musical tnes among the stars, gave the lowest the legeon among the strings of the lyre to the moon, because the motions of both are slowest. Hence have originated the words ὑπομείωσις and ἐπιμείωσις. The former of these terms originally corresponded to a star which, on the next day, comes to its setting before another star, which is said to be ὑπομείωσις with respect to the moon. . . . The latter term corresponded to a star that is slower in the first motion (such as the moon here), which is, as it were, abandoned and left behind . . . by the other ones . . . for more on this subject see our Optics, ch. X. — Kepler, New Astronomy, Chapter 1, nos. 25-28, pp. 195-7.

32 Here Kepler refers to the following echô: To p. 333. The argument is new. Euclid uses an astronomical term; therefore, he is presenting astronomy. Further, he speaks of eye being carried along. Therefore, he is presenting the kind of astronomy in which the vision is so moved that the heavenly bodies appear centrifugal; that is, of the Copernican. I nevertheless add in the present account the following by the term ἀπομείωσις. Euclid appears to want to present a reason why the mountains do not just appear to accompany one running in the plains (which is the only thing I am considering), but even seem to run on ahead. Thus the origin of the term ἀπομείωσις will become more popular.
subsequent people used it more strictly in astronomy, of motion B "in consequence", without respect to the earth’s position, by which procedure it does not accord with the fixed stars. They would be able to offer this justification of the so- termed terms: that in the combination of the first and second motions, those things that are moved with a retrograde motion outrun their position of the previous day among the fixed stars by an earlier approach to the meridian, and thus they go ahead. But those which move with a direct motion, their places beneath the fixed stars which they held the previous day come to the meridian the horizon first. They are accordingly thought to be left behind, just as if two runners ran for the very same goal, but one, being slower, complains that he is being left behind. And so an optical cause has given birth to these terms. For even though, as was just said, not even the first and daily motion of the stars comes under the bare sense, nevertheless, by an easy process of reasoning the eyes, being raised up, now easily take notice of this first and daily motion from east to west; but of the second motions by no other means than the difference of the diurnal motions.

The fallacy giving birth to these terms is nothing other than if one should see, by the shore, a boat going downstream, following the river, and in it two people, one of whom stands still in the bow, and the other walks against the current from the bow to the stern; the spectator, however, ignorant of the facts, would consider those two people to be carried in two different boats, one slow. For he will be in error, and the motion which is in the person on the same boat, he would attribute instead erroneously, to the boat itself, in the opposite direction. And those who first introduced these terms into astronomy erred similarly, thinking that the same first motion is weaker, for example, in the moon than in the sun, not knowing that the moon itself, by its own motion, strives against the common first motion to present for now the commonly accepted hypothesis.

And these are about all the things that the sense of vision, and incantations reason following it, attaches to the stars, contrary to the truth, and that must be freed from obscurity optically.

Appendix on the Motion of Comets

Those who demonstrated the motions of the comet of 1577 with circles took on an extremely difficult task, nor did it entirely succeed, because they did not think they had to investigate more carefully. They will encounter much greater difficulties if they take up the demonstration of the same thing in other comets. I met with success more easily in many for which I acquired observational records, in this way: if, as the nature of things urges, I should attribute to them straight lines, which they mostly traverse uniformly in equal times, only a little slower at the beginning and end, and closer to rest, as is usual in other trajectories. For the earth’s motion, assimilating itself, easily obtains circularity for them. For example, the one in 1577: if it had ascended in a straight line originating from

\[33\] Thid is, the diurnal rotation (parallel to the equator) and the various planetary motions (along the ecliptic), respectively.

\[34\] ἐγνώσαντος.

\[35\] ἑνώδους.

\[36\] ἀναλυτικά.
the plane of the Tropic of Capricorn towards the north pole, or slightly more inclined, but still in a straight line, then, just as in going around the motionless sun, the earth induces it with an image of circular motion, so, by the same process, in going around the comet which is practically at rest (for by this supposition it tends almost entirely crosswise), the earth will obtain for it an image of circular motion. Thus that comet of Regiomontanus, carried from the deepest aether close to the earth in a straight line, and passing by it rather close, will find a most beautiful occasion for completing a great circle in one day in the middle of its appearance, a very small amount before and after. Here too, the cause will be evident why, at that section of time when the comet was so fast, the tail equalled 50 degrees in length, B. Wiele IV 22.

Using this support, some have asserted of the year of 1572 that it was taken up into the depth of the ether with a rectilinear motion, using the evidence of its decreasing magnitude, for whom Wiele IV 4 and 132 was of service. In fact, they are aptly correct, if you grant their assumptions; as for the rest, those things which Echel Brute argued to the contrary in Book 1 of the Prognosticorum from other sciences, soundly and with great judgment, see in that author. 56

57 Theoraro II pp. 119 and 177.
58 Brute did not give much consideration to comets in the Prognosticorum, which was largely concerned with the 1572 nova. Kefer may have had in mind De mundi aeterno reconvintibus, obnovis novis (Uraniburg, 1585), in TBOO IV.
Chapter 11

On the observation of the diameters of the sun and moon and eclipses of the two, following the principles of the art

Problem 1

To construct an eclipic instrument

Let a curtain 1 be set up in the open, with so many layers of black cloths that no light can break in. If this convenience is lacking, let a chamber be chosen facing the region from which the eclipse of the sun will be viewed. Let this chamber have a wall that is not thick, that offers a window, and let it be possible to block off this window as well as all cracks against the entry of light. Next, let a rule (A) be made of as great a length as can be, all lines of which are straight, whose thickness is as much as the planked lumber provides; and whose breadth is half a foot. And let it be so adjusted that, because the wood is flexible, it may lie on its back, and at a place intermediate between the ends, so that it may be less bent. Moreover, let it not have any hole through the middle of its breadth, lest, being weakened, it break under its weight. Instead to the line of its back, upon which it is to rest, let a board (B) be joined, so that a hole or socket 2 can be set on the line of the joint. Let the socket have its own axle (C). Next, let there be a post (E) capable of being turned above an axis (D), divided at the top, so as to receive the thickness of the rule in the slot, drilled so that it can be transversed by the same axle as that of the rule. From a board (F) on which is a socket (H) receiving the post, let there set up the post in its (the board's) perpendicular with equal pieces of wood (G) arising crosswise on both sides, surrounding the rounded post in a fixed position with their hollow embrace. Let three other boards (HI) be joined with this one, so that a right angle parallelogram is made of all of them, in the place of the azimuthal circle. Now, upon the head of the column.

---

1 scena: This can also mean "platform"; however, in the first example of Problem 32 below, Kepler refers to the pole from which the scena is hung. I therefore believe he is describing a sort of tent, much like the one whose use by Kepler is claimed by Sir Henry Wotton in a letter to Sir Francis Bacon on 26 September 1620 (no. 392, in AGR XVII, p. 121).

2 The bracketed letters refer to the diagram on p. 348, below, I have inserted the into both the diagram and the text, to help clarify the relation between the two. The diagram is otherwise Kepler's...
from which the axis goes out into the board, let there be attached and joined a crosspiece \( K \), in one of their planes.\(^3\) of the post which has a slot on top, and let this crosspiece likewise be beveled to a transverse post \( E \), so that the post may attach to the crosspiece at a right angle, and so that if the crosspiece, resting upon the parallelogram, be moved, it rotate the post. Let the crosspiece be of a suitable length, round out in the middle, so that it may hold the thickness of the rule in this slot, and so this the rule may be carried around together with the crosspiece and the post, and at the same time, the rule can be turned towards the zenith, or lowered towards the horizon, as much as the sun's altitude at the beginning and end of an eclipse requires. For this purpose, it is appropriate both that the post be tall enough that the parallelogram never get in the way of the rule, and that the crosspiece, and the rule itself, be long enough that they do not come apart as the sun is setting, and that the slots be configured in both directions to the same end, and that the square \([1, \text{ the parallelogram } H]\) be high, so that when the back end of the rule is lowered it may not hit the floor, and in the plate of the horizon, which a perpendicular suspended on the post will easily indicate. Now this form cannot indeed be universal, unless either a geometrical square be made of the crosspiece \( K \) and the post \( E \), or a full quadrant is available. However, this construction was adequate for me for impromptu use.\(^5\)

Now to the rule, whose use is the main thing here. And so, measure so as it is a certain space of length, until the point of attachment downwards, no more than is the height of the post, and there you should make grooves on both surfaces of the breadth, perpendicular to the length; and similar ones at the head of the rule, which will be above, beyond the axis, the upper parts being about 12 feet away from the lowest. Next, you should prepare two wooden guards, a palm or a little more in breadth, of a length that is its own width and that of the rule combined, of a thickness that fits the grooves of the rule. They should be routed

\(^3\) The Latin is puzzling here. "Triec." I take to refer to the north \( H \) forming the base. I believe he says, "one of their planes," because they have two common plane surfaces, one on the top and one on the bottom. I am uncertain, however, about how the next phrase fits into the sentence.

\(^5\) As Kepler's description may be hard to understand, the following supplementary description is included in the hope that it might be helpful. Bear in mind that it is based only on the translator's understanding of the text, not on inspection of any existing instrument.

In the base board \( F \) is a socket that is the same size as the bottom end of the post \( E \), which has a round peg in axis \( D \) that can turn in the socket. The crosspiece \( K \) is rigidly attached to \( E \), so that as \( K \) is swung around on the other base board \( H \), \( K \) rotates on the socket. There is a cross brace \( E \), joining \( K \) diagonally upwards to \( E \), to which it is fixed so as to hold \( E \) perpendicular to \( K \). Above the joint where \( L \) meets \( E \), there is a kind of bearing consisting of concave surfaces hollowed out of the two braces \( G \). These braces are attached to \( E \) and to each other at the top, while \( E \) is free inside in their "bellow embrace." The top of post \( E \) has a slot cut in it that receives the axle \( A \), and an axle peg \( C \) goes through the two prongs of \( E \) and through the socket made at the bottom edge of \( A \). The small piece of wood \( B \), which is hard to see in the diagram because it is easily inside the slot, attaches to the bottom of the rule, \( A \), and serves to hold the axle \( E \) in place.
out in the middle of the breadth from one part of the length, so that the height of the slots may equal the breadth of the rule, and the opening remaining may pinch the thickness of the rule which it has after the grooves that were made. And so, when the panels have been set in the grooves, they will be both parallel to each other and perpendicular to the rule in length and breadth. On each panel, let a line be drawn lengthwise, beginning at the middle of the slot and from the thickness of the rule, and let another line be set up perpendicular to it. Inserted is the point of intersection, drawn through the full width of the panel. Afterwards, you should cut out the middle parts of the panel that is going to be upright, with a rectangular opening, about two fingers wide; in which the intersection of the straight lines occurred: but let the residual part of the straight lines remain at the edge of the opening. Now, in a very thin sheet of bronze, well smoothed and not unyielding, let a pair of lines intersect each other at right angles, and, with the intersection as center, let a small circle be made, the size of a pea, so that this diameter may be less in proportion to the last: one between the panels than the diameters of the luminaries are to their distances, by 6 of the second chapter; and let it be drilled through, so that the hole may be accurately circular, and be at the middle between the crossed lines, the width of the sheet being a little greater than the opening in the panel. Let this be attached to the panel with the cut-out, so that the lines match, and so that the hole is in the center of the opening. On the other panel, which is going to be lower, and upon its upward facing surface, describe a circle about the center of the intersection of the lines, as great as the width of the panel allows, divided from the top to the middle parts into 90 individual degrees, and the same number from below on both sides. Or, if you prefer, use another numbering sequence. Next, let the panel hold a very short style in the middle, at the center or intersection. About this let a volvelle be attached, upon which we are afterwards going to describe small circles from the point of attachment. From one side of the volvelle let a pointer project, whose edge line should go out from the center of the volvelle, of such a length that it may reach as far as the circle of the panel, and by rotation of the volvelle the pointer may be carried around on this circle. Once these things have thus been built, measure everything very exactly, as I have done on my instrument. When it is going to be used, cover the way lying between the panels with a tube that is black on the inside, so that nowhere is entry allowed to light other than by the opening in the upper panel. Moreover, you should position the instrument so that the part of the rule that is above the post, along with the tube and the panel, may be outside, beyond the curtain in the open air, the rest in darkness inside; and let it be able to be turned freely. And let the window, which lets the rule out into the air, be well sealed all around against the light.

Problem 2

To measure the sun's diameter with the instrument

Really, this might in general be done; by 8 and 9 of the second chapter, or any one let in through any opening, even a square one. Besides, the senses here do
not follow mathematical precision, but by 29 of the first chapter they do not grasp the edge of the image, to which few particles from the sun radiate, in the face of the brightness of the parts of the image in between, to which all the particles of the sun radiate. This happened to the Tychoic observers, when they did make use of the ray, but with an opening that was quadrangular, and too wide for a complete intersection to occur, by 6 of the second chapter.

So let the instrument be set in order, and the rule be brought to bear on the sun’s body, which is done by a double rotation, one of the rule upon the pos, the other of the post and the crosspiece upon the azimuthal quadrangle. And thus the sun, striking the upper panel directly, will send rays down through the hole in the sheet and the tube, onto the curtain and onto the lower panel, and will create a round image of the illuminated surface, by the eighth proposition of the second chapter. Where you shall see this image we shall henceforth call it the ray), first use a compass to take its diameter, as carefully as you can. But because unsteadiness will hinder you, with half of what you took with the compass, from the center of the panel (which takes the place of the wall), draw a circle, and one slightly smaller than that, and again one slightly larger, as many times as you think necessary. Then investigate again which of the described circles the ray equals.

So, let \( AB \) be the radius of the opening, \( AK \) the radius of the ray, and let \( KC, KA \) be equal. Therefore (by 6 of the second chapter), \( AC \) is the size of the image that will come down through a single point of the opening. But since the edges of the ray and of the sun’s body are touched by the same straight lines (for by 4 of the first chapter, the lines of light are straight), therefore, the angles of the imagined point of the opening are at the same vertex, and are equal. Therefore, by an eye located at the place of the opening, both \( AC \) below, and the radius of the sun above, will be seen under the same angle.

And so, when the radius \( AB \) or \( CK \) is subtracted from the radius of the ray \( AK \), the remainder will be \( AC \), which, together with the distance of the panels, shows the angle of vision. For as the distance of the panels is to \( AC \), so is the whole sine to the tangent of the angle under which the body of the luminary is viewed.

---

6 The term, *pleuraxis interi vix*, did not appear in Ch. 2 Prop. 6. The sense of it appears to be that, under the ideal conditions described in that proposition, the rays coming from the midpoint of the luminous surface would spread out exactly as much as would the rays coming from the entire luminous surface and passing through only the midpoint of the opening. Those rays intersect at the wall, at the points \( K \) and \( L \), in the diagram for that proposition.
On 1601 13/23 December, the diameter of the ray was 72 digits or units, and 36 units in addition, that is, 110 in total, the half being 55, the radius of the opposite side. Therefore, $\frac{1}{2}C = 45\frac{1}{2}$°. Consequently, as the distance of the planets, 10,368, is to 441, so is 100,000 to 511, the tangent of the arc 15° 30'°. Twice this is 31°. This is the diameter of the sun at perigee, with which a repeated examination in December 1602 agrees.

In 1602, in the month of June, using the same opening and rule in an equally dark place, the sun’s radius on the panel very obviously fell short of the quantity in winter. And when the quantity in winter was divided into its 12 digits, the ray in summer fell short by about 2/3 of one digit, as far as could be judged in this small breadth. Since, therefore, the whole diameter amounts to 51 minutes, that is, 31,800 of one degree, therefore, 1/12 of 31,800 is 31720.25 of this made 62/560 of one degree, or 62/60 of one minute, that is, about one minute. And the diameter in summer is 30 minutes, it was, of course, able to proceed as before in the winter view, but this way is the safest of all, because it connects the summer radius in a ratio with the winter one. 3

Indeed, in 1600, in the month of June, at Graz in Styria, through the same opening and the same distance, the units of the ray appeared to me to be 101\frac{1}{2}. Half of this is 52\frac{1}{2}. The radius of the opening, 8\frac{1}{2}, subtracted from this leaves 44\frac{1}{2}. And as 10,368 is to 44\frac{1}{2}, so is 100,000 to the tangent 429, whose arc is 14° 47'°, its double, 29° 34'. This quantity falls short of the previous examination by half a minute, or \frac{1}{2} of one small part of which there are 73 in a digit. On the preceding days, with a brighter horizon, the ray, coming through an opening 40 small parts in diameter, exceeded a circle of 129\frac{1}{2} small parts, by about 1 or 2 parts. When 40 is subtracted from 129\frac{1}{2}, the remainder is 89\frac{1}{2}, its half 44\frac{1}{2}, which indicates an arc of 14° 55'°. But by the addition of two parts, 15° 10'°. And thus the mean of these is 15° 25'°, double 30°. But there is no reason for me to trust the year later 1602, and my observers were expert in astronomy, while of the earlier year 1600, I wonder whether I was so precise in drawing the digit that I did not commit a greater error. For in addition, the certain that I had set up did not show as much darkness as I wished, so I was unable to draw the line accurately enough at the edges of the top. 4

Tycho found nearly the same quantity in 1591. 5 The tube was Ad, the opening Md 10 units and was square. Therefore, AE was 5, AB 3000, CG 18 \frac{1}{2}. Consequently, BG was 9\frac{1}{2}, and MD (after Ad and AE was subtracted) was 8\frac{1}{2}.

1 Kepler also used a foot divided into 12 inches, each of which is divided into 72 digits. The distance between the panels is 12 feet, or 12 x 72 = 10,368. Note that below, in dealing with the diameter of the image, the digits change meaning, becoming the Ptolemaic unit of the sixth planet.

2 Furst first notes that, while Kepler's value for the difference is correct, his value for the planet's diameter is too small. It should have been 32° 32', the corresponding angular diameter being 31° 28'.

3 It is a note to this passage. Frew (1960 II pp. 455-56) includes an extensive series of Keppler's observations of this eclipse. From Vol. XI of the Kepler Manuscripts in St. Petersburg.

4 These observations are in 1600 VIII pp. 151 and 152.
4 4. Hence, the radius was 14° 37'1. But it came out variously from 14° 20' to 15° 40'. But on December 5, and with different tubes, he thrice found 15° 30' (he did this with me), as I in fact computed from his observations. For he had not drawn anything from this, and it is known that, by persuasion of the hypothesis of eccentricity, he makes z perigal diameter of 32 minutes and more.

Now as for there being no more than one minute’s difference between the summer and winter diameter, and it is wonderful how well this fits with the true and geometrical eccentricity of the sun (if you separate out a fallacy that arises from a physical cause, on which see ch. 10). For Tycho Brate and the Landgrave’s observers, by unanimous computation, show the eccentricity to be 3,600 out of 100,000. But through a physical cause, the half pushes in to reduce this, as will be shown geometrically in the Commentary on Mars. Therefore, the real eccentricity is 1,800. And when the sun is at apogee, in the month of June, its distance is 101,800; but at perigee, in the month of December, its distance is 98,200 of the same units, of which, the mean distance is 100,000. And as 101,800 is to 98,200, so conversely is 31 to 30, approximately. For as regards Theorem 8 of Euclid’s Optics, it does not apply to such small arcs.

**Problem 3**

*To observe the sun’s diameter through a slit*

I found this in Tycho’s observations, with an appended Exconium of the rectangle as the master of the universe of mathematics. Let AB be a plane equidistant from the horizon. AE a wall perpendicular to AB, in which GE is the slit, DF the luminary line. Then, from its highest part F, a ray descends through the lowest edge G of the slit, and is extended to the point C, closest to the perpendicular A. On the other hand, from the luminary’s bottom edge D through the top E of the slit, the ray DE is extended to the most distant point 9. Therefore, as CA is to the whole sine, so is AG to the

---

1 This is the subject of Part III of the *Astronomiae nova* (chapters 22-40), and especially of chapters 22-29.
2 Euclid, Optics, 8. Equal magnitudes at unequal distances from the eye do not have the same ratio of angles as of distances.
3 TycIo X, 56-7. If the “Exconium” of the rectangle was anything other than the words, “per rectangulam instrumentum instrumentum,” it was not included among these observations. Drever’s numbers are in part different from Keplcr’s. For 15 March, he has: (1) 30° 48’, (2) 30° 6’, (3) 30° 44’. For 14 June, he has: (1) 30° 4’, (2) 30° 8’, (3) 29° 30’.
tangent of the angle $GCA$, which measures the height of the highest edge $F$ above the horizon. Again, as $BA$ is to the whole size, so is $AE$ to the tangent of angle $EBA$, which measures the height of the lowest edge $D$ above the horizon. Then by subtraction of the smaller arc from the larger, the remainder is the angle that the body of the luminary fills here on earth. This procedure is good, too, if one can make sure that in some building $EA$ is exactly perpendicular to $AB$, and is sufficiently high. On 1578 March 15, Tycho observed the diameter in this way, and the results were $30'\ 40''\ 30'\ 6''\ (30'\ 44''\ 30'\ 50'')$, and on 14 July, $30'\ 4''$ twice $29'\ 30''$ once.

There exists among Tycho's astronomical papers a letter of my teacher Maestlin to a councillor of Augsburg, if I am not mistaken, in which he makes the sun's diameter about the same, except that, being an adherent of the hypothesis of simple eccentricity, he enlarges it, and in fact says that at apogee he caught it at $29'\ 36''$, at the middle longitude $30'\ 11''$, at perigee $31'\ 45''$.

Gemma, too, does not differ much from this method in his Radius, if it is possible to find out anything certain using so coarse an instrument.

The ancients, however, fully go along with me. Archimedes reports of Aristarchus that he said that the sun's diameter is 1720 part of four right angles, that is, 30 minutes. Hipparchus, however (Alhacenius complains about Ptolomy's following him in his computation), denied that the sun perceptible changes its magnitude from apogee to perigee. This is undoubtedly because it does not vary by more than a minute, and one gathers that he himself also supposed it to be 30 minutes (chapter 8, above), so far as is clear from Ptolomy and his expositor Theon. Proclus seems to relate the same thing about Sosigens.

Ptolomy was the first to revolve simultaneously from the ancients' ways of observing and from the perfectly true quantity itself of the diameter of the luminaries, with a procedure that is absurd in measuring, and as well as impossible in practice, which the authorities up to the present, and even Copernicus himself, followed. For this reason, I have used that measure in Chapter 7 above, though certainly with no danger. But it is a certain thing, and evident to anyone to investigate, that the sun's diameter at apogee is 30', at perigee 31'.

---

14 These were made in June, at the solution, as Tycho noted.
15 The letter, which according to Franz Hammer, in JGWH II p. 455, was addressed to Hieronymus Wolf of Augsburg, seems not to have survived.
16 Gemmae Frisii medici et mathematici, de radio astronomica et geometricis liber (Antwerp and Louvain, 1545). The radius is a cross staff, whose construction is described in chapters 1-4, folia 5v-12v.
18 Ptolomy, Almagest V 14, trs. Toomer p. 252; Alhacenius, Opus astronomicum, ed. Natalino, Vol. I p. 56; For Alhacenius (i.e., Al-Battani), see ch. 4, note 172.
19 See p. 374.
20 Proclus, Hypothesis IV 3, Latin trs. by G. Valla, ed. 1541 p. 490, ed. 1551 p. 352. What Proclus says here is that Sosigens believed it possible for the moon not to cover the sun completely in a central eclipse of the sun (an annular eclipse).
Problem 4

To observe the moon's diameter through the instrument

A difficult task. For the moon is not so bright as to give our eyes adequate

344

sharpness of vision. And if it shine: in through the instrument into the chamber

and the darkness, the difference between the the ray and the nearly edges of the

paper is made out with great difficulty. Nevertheless, ye shall make the attempt

as follows. You shall draw on the paper some number of circles rather close to

each other, but increasing somewhat in sequence, each by itself, and you shall fill

the enclosed area with black, especially near the edges. so that the blackness may

extend over the whole surface, or at least for some width from the edges towards

the center. You shall then attach them in order to the panel on the instrument,

considering which particular one of them the ray of the moon encircles in such

a way that the illuminated whiteness of the paper around the black circles enters

into the eyes to some extent. For whichever one may be the first to be perceived

to be smaller than the ray, is soon establishes itself as the greater measure of the ray.

Nevertheless, after all your carefulness, there will still remain some uncertainty

in this method. What you get from it is this alone, that you will be able to make

judgements about huge errors.

On 15/25 July 1600, at Graz, I found the ray to be more than $10^5$ small

units, less than 116. Therefore, the apparent diameter was greater than $28^\circ 30^\prime$

less than $31^\circ 12^\prime$. It is always presumed to be less than it should be, because

the edges of the ray, having very weak light, are not well discerned by the eyes.

The moon was full, at its mean distance.

On 4/26 January 1603, in the evening at Prague, the ray held a quantity

intermediate between two black circles, one of which, the enclosed one, had

113 small units, the other, enclosing the ray, had 120 small parts. Therefore,

the apparent diameter also appeared definitely greater than $32^\circ$, but less than

$34^\circ 18^\prime$. The moon was closest to earth: every precaution was taken, especially

the darkness of the chamber.

But in the previous month, when it was gibbous, and revolving through

the same perigee, with light diffused through a turbulent sky, and an insufficiently

sealed chamber, it was considered by this method barely to surpass 30 minutes,

even over the whole illuminated area, to such an extent that it easily eluded the

younger observer. And so other ways of observing the diameter of this luminary

must be tried.

Problem 5

To pass judgement on or the ratio of the apparent diameter

of the moon to the diameter of the sun from the horn of the moon

in the first phase

In chapter 6 section 11 above. I recalled the moonrise that I saw on

4/14 March 1603, at Prague, at the 6th tour, when the sun was in $23^\circ 49^\prime$ Paces.

In the evening. See p. 269. Perhaps by “extenu.” Kepler means the moon’s first appear-

345

ance in the evening.
the moon in 14:41 Aries, by its motion on the ecliptic, however. The arc through the moon's apparent position and through the sun cut the horizon at an angle of about 78°. For the moon's horns clearly stood closest to the left of the zenith. And thus the apparent distance of the center of the moon from the center of the sun was 20°10'.

So, then, the moon in this position, when, avoiding itself of the earth's light (as was said above in Chapter 6 Section 10), it was seen most clearly in its entire body, it was surrounded by the shining horns on less than half its circumference. For this was plainly deficient from a semicircle to the vertices. I say that from this it is shown that the moon's apparent diameter was notably greater than the sun's apparent diameter, although the sun was moving towards its mean distance, nearer to perigee, while the moon had descended, not two signs from apogee. This works towards me end that we may here as well fight for Chapter 8 and may believe with all the more certainly that the moon's diameter is decisively greater than the sun's diameter, and consequently that the entire sun can be covered by the moon.

Demonstration

About center A let the great circle of the moon's body BCD be drawn, in it the diameter BAC, and EA at right angles to it. Let L be the center of the sun; therefore, the circle of illumination will be parallel to BAC: let it be FG. Now let a point H be taken not on the line EA: let this be a place on the earth's surface, Prague, say. And since HA is the axis of the circle of vision, this will be at right angles to HA: let it be KML, cutting off from the moon the small part illuminated by the sun, or the horn, whose true breadth is LG. And since this horn has fallen short of a semicircle, half the horn will fall short of a quadrant, and it therefore will not reach all the way to M. Therefore, let the circle of illumination FG cut the circle of vision KL at a point beyond M towards L: let this be I. Let the axis of vision HA, on the other hand, cut FG at N, and the circle of illumination cut the same at R. Therefore, if the horn had come all the way to N, the circle that passes through N, equidistant from KL, and representing the circle of vision, will come cut to be less than the circle of illumination FG. For AN, subtending the right angle ARN, is longer than AR. Therefore, a circle through N would be more distant from the center A than is the circle FG, as a result of which it would be smaller, since that which is through the center is alone the greatest of all.
Now, however, the half horn did not go as far as \( N \), so it is to be perceived with the length of a quadrant, but fell short at \( F \). As a result, \( KML \) was at a greater distance from \( A \) than \( N \). It was therefore much smaller than the circle through \( N \). But the one through \( N \) was also smaller than the illuminatory circle \( F G \), so the visual circle \( KL \) is much smaller than the illuminatory circle. But by Theorem 24 of Euclid's Optics, the less of a globe is seen, the greater its diameter appears to be. And above, in Ch. 6 Sect. 3, it was shown that if the visual circle \( KL \) and the illuminatory circle \( FG \) had coincided, the two luminaries would have been seen under the same angle. Therefore, because \( KTL \) is now smaller than \( FG \), the moon’s apparent diameter is therefore greater than the sun’s, and quite perceptibly so, even though it is near apogee.

Now, so that the theorem may be perfect, let it be that the ratio of the horn to the remaining circumference of the body be known without error from estimation by eye, which is indeed quite difficult; I too was unable to estimate anything more exact than that the horn is between a third and a half of the visual circle, that is, between 120 and 180 degrees. Let it be proposed for us to find by computation from this the moon’s apparent diameter. Therefore, the horn \( LI \) will be half of the estimate. Let \( D \) be the pole of vision, and let the arc \( DI \) descend, and let it cut \( BC \) at \( Q \). Thus the angular length \( QDC \) of the half horn is given on \( DQ \). But \( DCQ \) is right. And the side \( DC \) is given. For because the sun is at its middle distance, and the moon is 56 degrees past apogee, the ratio of \( HE \) to \( HA \) will thus be given from the hypotheses of the astronomical authorities. But the angle \( AME \) comes to be known from the visible position of the two luminaries, so when \( A \) is produced to \( T \), \( TAI \) will be given, whose measure is \( TD \), and its remainder in the quadrant is \( 2Q \), which is what is required. So, in \( DQC \) let there be found from the given \( DQ \), then \( DQC \). Next from \( I \) let an arc fall perpendicular upon \( BC \), and let it be \( IS \). Thus an triangle \( I QS \) the angle \( S \) is right, \( Q \) is given, and side \( IS \) is 15 minutes, that is, as much as is the distance of the circle of illumination from the greatest circle, Ch. 6 Sect. 3 above. Therefore, \( IQ \) will be given, which is to be subtracted from \( QD \), so as to obtain the arc between the pole and the circle of vision, whose complement shows the apparent semidiameter, by what was shown in Ch. 6 Sect. 3. Oue should not pretend not to notice that the moon’s apparent diameter is made enormous here, but if you say thus even the least perceptible amount is wanting from the semicircle of the waxing horn. And so another cause has to contribute, namely, the points of the horns disappear in the sense of sight because of their narrowness against the brightness of the parts in between.

Appendix

Other ways of measuring the moon’s diameter are compared with the preceding.

Even though the most certain and reliable measure of all will finally be introduced later through observations of solar eclipses, nevertheless, for the remaining
methods should meanwhile be set down for examination here. And indeed, the
dioptre of Hipparchus is known from Ptolomy, which Ptolomy himself also used,
though he despised of eliciting a reliable magnitude from it.21 He found only
this, that the moon at apogee shows a diameter equal to the solar, and increases it
at perigee, which Hipparchus also had said.

On 29 March or 8 April 1598, at hour 8 of the evening in Graz, I saw the
moon in conjunction with the western stars in the quadrilateral of the Pleiades, in
such a way that the moon’s edge was removed from the nearest by no more than
a sixth part of the lunar diameter. At the farther edge its distance from the shining
star of the third magnitude was as great as was the width of its body.24 Thus
the two stars there on the western side of the quadrilateral were too far apart to
allow them both simultaneously to cover the moon, had it happened to pass over
them. The moon was three days old, its mean distance surpassed, tending towards
apogee; and close to its greatest northern latitude: it was seen most clearly, in its
entire body.

On 17/27 July following, between hours 2 and 3 in the morning, the moon,
ascending again towards apogee, and closer to it, and near the northern limit,
was standing at the Pleiades with the horns reversed, so that a line drawn from
the lower horn to the perpendicular of the intersection would graze the Pleiades
from above. It had passed through the Pleiades, and was distant from the brightest
by more than Pamphilium is from the neighboring star of the Hyades towards the
nose of Taurus, less than the latter is from the lowest in the nostrils.25 Thus the
distance of the two bright transverse stars of the Pleiades appeared to be equal to
the diameter. Comparisons of this kind to nearby fixed stars with each other ought
to be more vigorously sought out, because this measure is universal to all people
and is permanent.

The same thing can be tried, either by the fixed stars, or by the radius, or
by the dioptre, when the moon is passing through the earth’s shadow, while still
bright with its reddish glow. But when immersed in the middle of the shadow, as in
March of 1588, it is seen poorly, and the observation is unrewarding.

Let us see, however, what it is observed simply by full light with in-
struments, whether that be done with the radius, or by comparing the edges with
stars on opposite sides, or by the altitude of the two edges found with quadrants.

On 14 June 1592, when an eclipse was impending, the Bruehan observers,
having measured the diameter of the moon with the astronomical radius, follow-
ing the teaching of Gemma, recorded 32 or 31 1/2 minutes.26 The moon was at
apogee. The method is both unreliable and presumably gives results greater than
the truth.

21 See note on p. 231.
24 The “shining star” is η Tauri, described by Tycho Brahe (TROO III 345) as “Media et
facida pleiudum.”
25 Pamphilium is another name for Aldebaran (α Tauri).
26 This was Gemma Frisius. See the note on p. 354, above. The observation is recorded in
TROO XII p. 196.
in the same year, on February 12, when the moon was not at its lowest, they recorded 35°. 27

On 6 January 1587, the elevation of the moon’s top and bottom edges above the horizon, observed by day when the viewing of the moon is more reliable (for it was bisected, at quadrature with the sun), showed a diameter of 30°.28 The moon was at apogee.

On 2 March 1588, before an eclipse in the evening, the difference of declination of the edges, by armillary spheres, repeated many times, was 33°, plus or minus one half.29 On the meridian, the difference of altitude of the edges was 31°, 32° 1', 30° 1'.30 The same on the preceding day was 33°.9 Evidently, this method is somewhat more uncertain. The moon, ascending from perigee, was approaching its mean distance.

Observed in this way on 22 February 1591, at its mean distance, it appeared to be 31° twice, 32° six times, 33° seven times, 34° six times.31 This variety came about partly because of different eyes, partly because of the sights, partly because of the abundance of light seen at night. For all these means make the procedure of this observation a little less reliable. And although my instrument the moon’s diameter may perhaps appear a little less than the truth, there is no greater consistency in it, than in this procedure of observing through sights. I would nonetheless not deny that if anyone should apply the dioptres of Hipparchus skilfully, he will sight more surely.32 All the more so that since Hipparchus’s dioptre, Tycho’s daytime observation, and my instrument agree closely in this quantity, which I will present below through the moon’s shadow, I therefore conclude more easily, at least now, that the moon’s diameter at apogee is 30° 2 minutes. But how much it is at perigee is not as easily deduced from this as it was in the sun. For since the moon has a twofold maximum equation, one of 5 degrees, the other, at quadratures, of 7° degrees, the quantity of these, considered physically, is 4336 for the one, 6520 the other.33 It postulates an eccentricity.

27 TBOO XII p. 90. The record indicates that this diameter was supposed rather than measured. On the following page, it was recorded as 32° once and 30° twice.

28 TBOO XI p. 142. However, there are three such measurements on that page, the resulting diameters being 30°, 29° 1', and 31°.

29 TBOO XI p. 237. The actual recorded differences on the armillary were 33° 1', 27°, 27°, 27° 1', and 27°.

30 Kepler omitted the first measurement, taken with the rotating quadrant, recorded as 36°. The others are reported correctly.

31 On the previous day, these three measurements were recorded, one of 33°, a second of 30° 1', and a third of 25° 1'. TBOO XI pp. 256–7.

32 TBOO XII pp. 129–1. Although the number of observations relied by Kepler tally with the number in the observation book, the values are differ considerably. However, Kepler’s point that this method produces inconsistent results remains valid.

33 For the verb “equisimilior” translated into English as “equisimilar,” the note on p. 226.

34 Kepler divided the whole equation, or correction to be applied to a celestial body’s mean motion, into two parts: the physical part and the optical part. The latter represents
you would not know whether you ought to follow the one or the other, or the mean, 5428, so that the two physical causes thus harmonize. And thus the moon's diameter at perigee is either 33° 20', or 34° 0', or 34° 40'. And the mean is either 31° 55', or 32° 15', or 32° 35'. You see that the observations made with sights in the years 1587, 88, 91, 92, being reviewed, for the most part play around these; but about the selection of one of these three, we are left in uncertainty. However, if you trust my instrument, from the observation of 1626 January 1603, the last perigee quantity will be set aside. And surely, physical arguments also do not particularly favor the greatest eccentricity of these three.

Problem 6

To estimate the quantity of the defect in an eclipse of the moon, or also of the sun

This is commonly done without an instrument, by an imaginary division of the diameter into 12 parts.

We make use of this method most safely when the quantities in the defect and in the light [i.e., the bright part] are about equal. Thus, on 25 June 1572, Maestlin estimated the maximum defect to be exactly half the diameter. (However, Gemma Frisius, in Book II fol. 233 of the Cosmographia, wrote that the defect at Louvain was 8 digits.) Further, it is more helpful for that eclipse for which the shadowed circumference is in view at the same time. Outside these cases, the procedure is variable and risky.

On 29 November or 9 December 1601, although there was a good part of the moon's body remaining, it was not possible for anyone to discern its quantity reliably. Ambrosius Rhodius, from a Tycho's computation for a considerable time, estimated the defect to be 10 digits at Wittenberg.

On 21 November 1603, some argued that it was more than one fourth deficient, while I judged that somewhat less than this was missing. And nevertheless, the bright part of the circumference was darkened.

the apparent increase and decrease of speed resulting from the observer's eccentric position alone, and the former expresses the real variations in speed along the orbit. Each part is responsible for about half of the whole equation, the exact proportion being determined by the form of hypothesis adopted and the parameters used. Kepler does not tell us what geometrical model he is using, but the numbers are approximately the tangents of \( 2^{\frac{1}{3}} \) and \( 3^{\frac{1}{2}} \), respectively, the radius of the moon's deferent being taken as 100,000 units, as was usual at the time. These numbers represent the eccentricities (or radii of the epicycles) that would be required to produce the two equations. The exact numbers are not important in the present instance: Kepler is arguing that the equations imply real eccentricities of about the sizes given, and that these in turn imply observable variations in the moon's apparent diameter.

Michael Maestlin, Epitome Astronomiae (Heidelberg 1588), p. 460 (Tübingen 1610) p. 494. The "Cosmographia" was written by Cornelius Gemma, not his father, Rainer Gemma Frisius.

The computation is in TBEO II 141-3. Tycho predicted an eclipse of 11 digits.
Since, however, it is of great interest to Astronomy to take note of partial eclipses correctly, Tycho Brahe, from the precepts of Cornelius Gemma, was accustomed to measure with the radius, both the moon’s diameter before the eclipse, and the remaining part at the greatest defect.\(^{35}\)

By the way, the difficulties by which this method is encumbered are recounted at great length in the preceding.

On 14 June 1592, Tycho Brahe observed the diameter of the moon by the radius, at the beginning and still whole, to be 2 minutes.\(^{36}\) Then, around the middle of the eclipse, he found 14 minutes remaining, so that the defect was 18 of these. But this quantity diverged greatly from his own computation, for he adopts 26\(^{3/4}\) minutes for the shadow of the lunar body. And so I attribute this to the difficulty of observing. For although someone might call into doubt the Tychoantic computation or the diameter of the shadow (on which more elsewhere), he will nonetheless never make this eclipse, with regard to this quantity, square with the others in one month.

And so, in order that here too I might rest on more sound defenses, I make a practice of estimating the missing are of the lunar circumference.

For once this is given, and the ratio of both the moon’s diameter and the shadow are known approximately, the quantity of the defect is itself also given. About center \(D\) let the circle of the shadow \(F E G\) be described, and about center \(B\), the circumference of the lunar disk \(F A G\), cutting the shadow at \(F\) and \(G\). Let the centers be connected with each other and with \(F\), and likewise also the points of intersection, with the lines \(FB\), \(FD\), \(BD\), and \(FG\), which will cut each other orthogonally at \(C\). Let \(F A G\) be for example, one sixth of the circumference; \(FA\) will be half, that is, 30°; namely, the angle \(FBC\). Consequently, \(BFC\) will be 60° and the secant \(BF\) will be 200,000 where \(FC\) is 100,000. But let the ratio of \(BF\) to \(FD\) be given, which is 1 to 3; \(FD\) will be 600,000, the secant of angle 80° 24'. But also given are the tangents of the same angles. \(BC\), 173,205; \(CD\), 591,236. Which, subtracted from \(BA\), 200,000, and \(DE\), 600,000, leave \(CA\), 26,795, and \(CE\), 8764. And so, where \(BF\) is 200,000, the deficient part \(FA\) is 35,559, made up of \(CE\) and \(CA\). Once this ratio is given, the digits or minutes of deficiency are easily had afterwards. For the semidiameter of the moon is 16 minutes; there will be 2° 51' in the defect. You will have instances below, in the eclipses of the year 1601.

Here, although both the sense of vision and the estimation of the sense of

\(^{35}\) This time, it is Germa Frisio’s precepts, in the De radio astronomica that Kepler means here. It is not clear why this phrase is, in Italian, possibly it is a quotation, though I have been unable to trace its source.

\(^{36}\) TBOO XII pp. 195-7. In the midst of these observations, Tycho remarks, “one must not trust the radius too much.” I have not succeeded in locating the computation referred to later in the paragraph.
vision can be in error to some extent, only a slight portion of that error figures into the estimating of the moon's diameter.

I want astronomers to work at setting up some more reliable ways of making this observation. For dependent upon this one thing is what is commonly made the chief point in astronomy, the measurement of the solar altitude and the solar body. That is, if the sun's maximum parallax is 3 minutes, and in the estimation of the lunar defect an error of one third of a digit is made, this is a matter of 600 semidiameters of the earth; but if the sun's maximum parallax is 2", we shall be in error by 1700 semidiameters of the earth, by omission of one minute in the moon's defect, as is to be seen at the head of our Parallactic Table. 39

Problem 7

The true image of the eclipsed sun being set forth, to find the true ratio of the diameters of the sun and moon, and the true [number of] digits of the eclipse

This problem is from my teacher Maestini. 40 We went up under the roof of the church, and, the doors being shut against the light, someone climbed up into the highest beams, in order to remove a roof tile in a suitable place so that a very slight crack could make an allowance for light, now one tile, now another, according to the way the beams were interceping one or another ray. Thus the opportunity of the docking provided us a much larger ray than my instrument or rule, which is no longer than 12 feet. He received this ray, eclipsed along with the sun, on paper (by Chapter 2 Sect. 9). And because the whole radiation is a right cone, whose vertex is about at the opening, it is obvious that the ray formed on the paper does not come out circular, unless the paper be perpendiculary opposed to the radiation, by Apolloniais 1.9. Therefore, he drew upon the paper a number of circles of different sizes, about as many as he saw that the ray was going to take up, and, after drawing the diameters, divided them into 12 equal parts or digits. Next, he received the ray with the marked circles set opposite, in such a way that the edge of the ray should coincide everywhere with the circumference of one of the circles, by changing the circles or by putting them nearer or farther from the opening until this should happen. This was evidence that the cone of radiation was cut perpendicularly by the paper. Now he directed the divided diameter by rotating the paper so that it should bisect the horns of the sun. The interior arc of the deficit ray, where it cut the divided diameter, thus then showed the digits of the eclipse. This teaching, presented by Reinhold, he refined with greater care.

39 One third of a digit is about one minute. As the head of the Parallactic Table (Jorgin II following p. 240), Keplar has the distance in earth radius corresponding to that amount of horizontal parallax. To a parallax of 3" there corresponds a distance of 1143, to 2° corresponds 1718, and to 1° corresponds 3337. The effect of an error of one minute could therefore have an effect as great as that described by Keplar.

40 See Problem 28 below. This observation took place in Tbingen, while Kepler was a student.
following the advice of the author. 4) For he extracted the ratio of the diameters at the same time, in this way. When the edges of the ray precisely coincided with the circle, he marked the interior circumference of the ray with three or four points. From these the circle continued through these points easily showed what ratio it had to the former one, which represented the sun.

Let A B C D be the ray of the eclipsed sun, and the true image of it, let that square with the outer surface A B C D, in a circle described about E. Now let any three points be marked on the interior circumference A D C, and let them be A, D, C. Therefore, by Euclid III. 24, let a circle with center F be continued through A D C. And because the cause of the defect in the sun is nothing but the interposition of the moon between the sun and the source of vision, therefore, the interior arc of the horned sun A D C is a small part of the circle by which the moon is viewed. Consequently, the circle A D C with center F represents the lunar body, and the ratio between F D and E B, which is found mechanically, is the ratio of the visible diameter of the sun to the visible diameter of the sun.

Incidentally, the moon’s diameter, observed in this way, appeared much smaller than the solar diameter. But by Chapter 2 Sect. 11 it is clear that unless the opening is very tiny indeed, almost like a point, the ratio of the diameters is always faulty and the sun appears smaller than the truth, and by Sect. 22 the digits appear to be fewer. Consequently, this method is worthless, applied simply in this way. In addition, the shaking of the hands, and the rapid motion of the sun, greatly disturb the marking of the points. Therefore, to my instrument.

Problem 8

To capture in points the image or ray of the eclipsed sun, as is formed in the instrument

4) is done as in the preceding, and with somewhat greater certainty. For the instrument made there be no need for me to change the circles, nor for moving in and out. For on the day on which the eclipse is to happen, the quantity of the sun’s ray is noted accurately on the instrument, and a circle of such a quantity is described, which is carried all around attached to the panel, and for that whole day certainly equals the ray of the sun. In the marking of the points, however, there is the same objection as before. For at every moment the sun’s ray is being carried across, and so both the crosspiece and the eye on the crosspiece have to be carried across between the marking. Nevertheless, take this precaution: mark one point when the inside circumference of the ray falls exactly on the circle. Then you should get ahead of the ray by moving the instrument, and when the ray catches up, you shall note the place with your eyes for marking the other point. And it will be advantageous that a spectator be at hand, who, while you

are intent upon the other edge, can indicate when the edge on the other side again falls on the point already marked. You shall again apply the procedure as a test, and do it quickly, before the defect grows or shrinks perceptibly. Besides, this image is adulterated. There follows therefore:

Problem 9

From the image on the instrument or the ray, however much adulterated, to extract the true image of the eclipsed sun

For let the circle in which the three points have been marked (namely, $H$, $B$, $F$) be extended, its center $L$ being established. This will be the center of the lunar body. Next, to $LB$ let there be applied the radius of the opening $BA$, and about center $L$ with radius $LA$ let the circle $DAC$ be drawn, representing the moon's true body. In the same way, from $AG$ let there be subtracted the radius $GE$ of the opening, and about center $A$ with radius $AE$ let the circle $DECA$ be drawn, representing the sun. Therefore, $DECA$ will be the true image of the eclipsed sun, by Chapter 2 Sect. 10, 11, and 12. See the computational procedure below, in the examples.

Problem 10

To extract the image of the eclipse more skilfully using the instrument’s transverse rule

Because there is difficulty in marking points, as was said, and because many things have to be taken care of at once, especially if the observer is alone, I thought up another, easier procedure. Above the volvelle mentioned in the description of the instrument, I placed a pointer of solid parts, by which it might project slightly above the plane of the paper wheel, in such a way, however, that it should not cover the diameter divided into degrees, but should enclose it with a pair of pieces of wood equal distant from the diameter. Above this solid pointer let a transverse rule rotate, perpendicular to the pointer and to the marked diameter, with the length of the diameter. Specifically, let the groove in this rule be equal in its depth to the thickness of the wood of the pointer, and let it attach to the wooden pieces with a clamp, so that one may push it forward. And let the rule have a sharp edge. Now, with the wall and the opening correctly oriented to the sun, let the pointer with the volvelle be rotated on its axle, as well as the transverse rule up and down above the pointer, until the sharp edge comes into contact with both, upon which. And then let the point be noted at which the sharp edge of the transverse cuts the marked diameter, and let the point also be noted at which
the exterior circumference of the sun, or the moon’s shadow, cuts the same diameter. If this will be managed correctly, there will be no need to mark the point. For there follows:

**Problem 11**

*From the transverse rule, to show the true image of the eclipse*

Let \( C D E F \) be the image of the eclipsed ray, whose center is \( G \), the marked diameter \( D H \), and with it, the line of the point \( I D H \). And let there be the line \( CH \) touching the oblique points \( C \), \( E \), and \( F \), with the marked diameter at \( H \), and let the segment \( C G \) at the same diameter at \( F \). Next, let the radius of the opening be extended at the diameter \( D H \), and let this be \( DL \) from \( D \) to \( L \), as well as from \( F \) to \( M \) and from \( M \) to \( P \), and through \( P \) let the straight line \( NPO \) be perpendicular to the diameter and equidistant from the rule. Then, with center \( G \) and radius \( GL \), let the circle \( VLO \) be drawn, which will represent
the true image of the sun, as in 9 of this chapter. This will cut the line \(NPQ\); let it cut at \(N, O\). Next, through the three points \(N, M, O\), let a circle be described, and let its center be \(E\). I say that \(NLOM\) is the true image of the eclipsed sun. For let the perpendiculars \(NC, OE\) be drawn. So, since \(NC, OE\) are perpendicular to the same \(EC\), they will be equal. And because \(DK\), that is, \(PH\), is also perpendicular to the same \(CE\), from the structure of the instrument, therefore, \(NC, PH, OE\) will be equal. But \(PH\) is the radius of the opening. Therefore, \(NC\) and \(OE\) are also. But since, by 10 of Chapter 2, the small circle of the opening is also drawn around the points of the ends of the horns, and since \(CHE\) is tangent to those small circles (for they are the base horns of the ray), it is necessary that \(NO\) be those points of the ends of the horns. For from any point of the circumference \(NLO\) above \(N\) or \(O\), a longer line would be dropped, and from others below \(N, O\), a shorter line. Thus \(NC\) \(OE\), the radius of the hole in the opening. Therefore, if \(N, C, E\) are the points of the horns, the circle of the moon cuts the solar circle there, but \(M\) is also on the moon’s circle, as in 9. Therefore, \(NMQ\) is the moon’s circle. And \(NLO\) is the sun’s. Therefore, the true image is \(NLQM\).

Problem 12

From the ray or image of the eclipsed sun marked by the instrument, to find the ratio of the diameters, the visible distance of the centers, and the quantity of the eclipse

First, from the adulterated ray, let the true be extracted, by 11 or 9 of this chapter. Next, let it be done as in problem 7; that is, let the true circumferences of the image be extended, and the diameters be compared mathematically, as well as the distance of ciphers and the digits. But if you have proceeded by 11, all these things are different in the observation itself and in the computation. In the preceding diagram the points \(F\) and \(H\) are given. Therefore, \(FH\) is given from the observation itself. I say that this is equal to the line \(MP\). For if \(F\) and \(P\) are equal, and \(FP\) is common. Further, \(ML\) is obtained at the same time by subtracting \(FM, LD\) from \(FD\). Consequently, \(PQ\) is found. But \(Q\) is found from 2 of this chapter. And this is the diameter of the circle \(QNL\); therefore, both \(QP\) and their [i.e., \(PL\) and \(QP\)] mean proportional \(PN\) are found. Moreover, \(PN\) is also the mean proportional between \(MP\) and \(PR\), and \(PM\) was given from observation, i.e., \(FH\). Therefore, as \(FH\) or \(MP\) is to \(PN\), so is \(PN\) to \(PR\); which, with \(PM\), makes up the visible diameter of the moon. However, the quantity of the eclipse is also given more simply. For because \(SQ\) and \(PM\) make up the radius of the opening, \(SF\) and \(QM\) are equal. Further, as \(Q\) is to 12 digits, so is \(SF\) or \(QM\) to the digits of the defect or if you prefer you shall say that as \(LQ\) is to 12 digits, so is the shining part \(LM\) to the digits not
covered, which, subtracted from 12, leave the digits of the eclipse. Examples of the computation are in Prob. 32 below.

Further, these things presented in Prob. 10 are a little more uncertain to apply if one should examine them using the [procedure of] 11 and 12. For it is with difficulty that one simultaneously attempts to the simultaneous of the wheel, and of that of the ring and of the shadow, with the marked diameter. And the small circles of the points on the horns are very eluded, since they are spread out from a point, and in estimating the intersections we err in the small things. As on 30 June or 10 July 1600, when 3 digits were eclipsed on the ray, the transverse rule cut off 1 1/2 digits. Therefore $HF$ is 1 where $FP$ is 9, and in the dimensions of the opening, where the whole radius had 10 1/2, but 9 with the shell removed to allow this quantity for now to be true, which, as was noted in Prob. 2, is a little less than the truth: three digits are 26 1/2 of these units. And $HF$ or $PM$ are $11 1/2$. But $PB$, 9 digits, is 90 1/2 units. Therefore, the whole $HD$ is about 95 1/2 units, and with $HP, IL, LD, 16$' units, subtracted, $PL$ will be 85 1/2, and $PQ$ 14 1/2. These multiplied by each other make 999 1/2, $NP$ squared. But $PM$ was 11 1/2. Therefore, division of $NP$ squared by $PM$ gives the remaining part $PR$, about 93, to which the segment $PM$ being added makes the moon's whole diameter 106 1/2, where the sun's is 91. Therefore, the moon's diameter would have been 82 1/2 minutes, even though it was a little below its mean distance. Consequently, so that you may see the fallacy, let the moon's diameter have been 32, and 3 digits obscured on the ray: it is asked, what ought the transversal to have cut off, that we reckoned the digits to be 1? If then, 29 1/2 minutes give 89 units, 32 minutes will give a little more than 96 1/2 small units of the moon. And because 3 digits on the ray are 26 1/2, it is also deficient by so many small units from the ray (89). So, since as $PL$ is to $PN$ so is $PM$ to $PQ$, and at the same time, as $PR$ is to $PN$ so is $FN$ to $PM$, therefore, as $PL$ is to $PR$, so is $PM$ to $PQ$. And, separately, as $ML$ is to $QR$, so is $PM$ to $PQ$ and componentally, as $ML$ to $QR$ together are to $ML$, so are $PM$, $PQ$ together, i.e., $MQ$, to $PM$. Therefore, since $QL$ is 89 and $QM$ that is $XF$, is 26 1/2, $ML$ will be 62 1/2. And since $PM$ is more than 96, while $QM$ is 26 1/2, $RQ$ will be 70 1/2. The sun is 132 1/2. Therefore, as 132 1/2 is in 62 1/2, so is 26 1/2 in $PM$, 12 1/2, approximately. But $ML$ is 62 1/2, and $PH, LD$ are two semidiameters of the opening, that is, 56 1/2. The sun, $HD$, is about 91 1/2.

Since, therefore, 105 1/2 makes 12 digits, the remainder, 13 1/2, makes 1 1/2: approximately, but how does this differ from $1 1/2$ in reckoning by eye? So, since here too the senses have left me nearly bereft, I thought of a third procedure for measuring the moon's diameter in eclipses of the sun, which now follows.

**Problem 13**

In an eclipse of the sun, to estimate the moon's diameter civilly and surely using prefabricated small moons

The procedure is clearly the most reliable of all that can be thought up, which even in itself is a reason for my bringing this publication to completion. I want mathematicians, in whose care astronomy is, to observe the eclipse that
will occur on 2/12 October 1605, near perigee, in this way.\textsuperscript{42} For in 1601, right at apogee, a most beautiful calm shone upon me, and it can happen that some people, no matter how much their desire are hindered by clouds or by health. It is therefore expedient that everyone be ready. For by comparison of the diameter at apogee and perigee, it will be possible to conclude something certain about the true and geometrical eccentricity of the moon at conjunction, which occupies a fundamental place in the discussion of physical causes.

To the point. And so let our paper volvelle be taken in place of the pieces of wood and the somewhat solid pointer, and in place of the transverse rule; two folded back handles, likewise parallel to the marked diameter, but outside the circumference of the circle, which measures the ray on that day. Between these handles, in place of the transverse, let there be fastened small squared off sheets, in the middle of which are little circles, as great as we suspect the diameter of the moon will be, diminished all around by the radius of our opening. Also, let the volvelle nonethless bear its own pointer from its center, made of paper, in place of the previous wooden one. For I shall show the use of this later. But let the small sheets be enclosed in a grid, with parts of the paper cut back from the circumference of the small moons, except at the sides, where little arms are to be left for keeping the small moons in the center of the squared off sheet. It will thus happen that you may nonetheless see the diameter marked on the wheel, unhindered by the squared off sheet. With these things thus prepared, when the eclipse of the sun is beginning and the rule of the instrument along with the panels pointed at the sun, let the volvelle be turned around its axle, while the sheet is moved up and down between the handles of the

\textsuperscript{42} To this end, Kepler published a small book, Ad astram coelestium amatorum univer- sus... de soleis infatis, quod hoc anno 1605 in museo Octobi context, opuscula (Prague 1608) in JKGW IV pp. 27-53. The contents of this book are closely related to the present chapter.
volvelle in a straight line, until the top edge of the small moon sits upon the deepest apex of the shadow. Thus, in a rather large eclipse, if your small moon is larger than it should be, it will cover the horns of the ray from the sides, before it can touch the inner circumference of the eclipsed ray from the front. If, on the other hand, it is less than it should be, when it has been applied to the edge of the moon’s shadow from the front, it does not reach to the sides to the circumference which it touches in the middle. And so, change the small moons, until some one of them fully matches the interior circumference of the ray. That one will give you the moon’s apparent diameter.

On 14/24 December 1601, at Prague in Bohemia, by my usual instrument, without changing the opening at all, I studied an eclipse of the sun in the way just described. The quantity of the circles were exactly as those you see here. For I had taken for the small moon a radius of 37 small units, which with the 85/3 of the opening made 45, which is, so as to make 15 minutes, or 30 minutes for the whole diameter. For the sun also at apogee has 30 in its diameter, but the moon is said by Ptolemy and Hipparchus to be equal to the sun, and Tycho himself assigns a minimum diameter of 30 to the moon at opposition. However, those things he had said about the diminution of this diameter in eclipses of the sun, I have held suspect for many reasons. And so, with the small moon set at that size, to the Brabian observers, when the eclipse had now grown to its middle, it appeared most evidently to all of them that my small moon not only was not greater than the shadow, but still fell short of the sides of the shadow, if it was tangent to it at the front. And so the apparent diameter of the moon is greater than 30. It would also be much greater if it were to appear at that place on the epicycle in Cancer and at mid-heaven, because of the greater nearness of the observer, for which cause it can appear almost half a minute larger at the zenith than at the horizon.

For the rest, I only had a single small moon, and so I was unable to proceed exactly in the way prescribed. I did, however, manage to pull the moon somewhat away from contact with the interior circle of the ray, so that the circumferences were at a uniform distance. And thus there were then estimated to be about 2 small units between the circumferences of the small moon and the ray. And so the radius of the moon would be 71/2, in minutes, 155/2. Therefore, the apparent diameter was 31 1/2. And this was at apogee. But to avoid my attributing too much to the sense of vision in determining the edges of the shadow, let there have been just a single small unit in between, and the diameter at apogee be 30 1/2, as great as is the sun’s mean diameter; so that thus, following the teaching of Sosigenes, the moon at apogee cannot cover the whole sun at perigee. And so let there be this most certain axiom: the diameter of the moon at its most remote in the eclipse of 1601 did not appear to be greater than 30 1/2 minutes. Now supply the rest, of course. Some of matters celestial, and size; all of you, the occasion which the year 1605 will offer, with the moon at perigee.

12 The adjacent diagrams are copied from the woodcuts in the 1604 edition, and are the same size as those woodcuts (as the figures in 1600 I II, p. 196). 13 In chapter 5 section 3, above Kepler, arguing against Sossigenes, denied the possibility of an annular eclipse of the sun.
Problem 14

To extract the inclinations of solar eclipses: the method of Maestlin, and a caution

Of the same sort it is proverbial that there is not part that isn’t useful. The same thing is true in general about the phases of eclipses, especially of the inclinations, which Ptolemy, in the final chapters of Book VI, calls **producers**, as if they were librations. If these are observed with greatest certainty, they provide us evidence of the most important things in the moon’s motion, and serve as a short cut, as will be said in the other part on use. But also in this category there exist phenomena worthy of admiration and display, not adequately explained by our predecessors, and which had kept me tied in knots for a long time.

Now my teacher Maestlin uses a small quadrant in this way. Let the eclipsed ray fall perpendicularly on the paper in a chamber, with an assistant supporting the paper. Next, let the side of a small quadrant be set perpendicularly in the way of the ray, so that it cuts the horn of the ray through the middle, for which the estimation of the eyes is to be believed. From that side, thus directed, let the quadrant be hung, and a plummet line on the quadrant; and the plummet line on the edge will show the degree or angle by which the vertical circle (represented by the cord) cuts the circle through the center of the luminaries, represented by the side bisecting the horns. But immediately, and practically at the same moment, the sun’s altitude must be taken, without which the inclination is almost useless. Let $GHI$ be the eclipsed ray, let the side $AC$ of the quadrant $AOC$ bisect this through its shadow, and let $AF$ be the plummet line: $FAC$ or $FC$ will be approximately the angle sought. For it is beyond controversy that $AF$ is on the vertical, but $AC$ passes through the centers $D$, $E$. For it is possible that it bisects the horns $GI$, that is, the straight line $GI$ described in the two circles, since it intersects both arcs. It is therefore necessary that $AC$ pass through the centers. For the rest, in the example of the eclipse of 31 July 1590, in the observation of which eclipse Maestlin used this method, the sun was low, and the angle of inclination was either fully a right angle or almost none. And so that method was certainly of value then, and was satisfactory for the accuracy of sense perception, but if the sun had been higher and the inclinations in between, there is no doubt that I would have been advised by Maestlin to be cautious. For although $AF$ represents the vertical, $AC$ the circle through the centers, $FAC$ nonetheless does not measure the angle of those circles, but is smaller. This is readily evident, if the point $A$ be right at the intersection of these circles on the sphere, and from the same point there go out two lines, one tangent at the same point to the vertical and the other tangent to the circle through the centers. For the former will enclose an angle the same as that of the circles, while the latter will enclose one that is clearly smaller. The demonstration and diagram and the computation of the necessary precaution you will find below in Prob. 29.
Problem 15

To extract the inclinations using the eclipse instrument

The method is easy, and not troubled by the trembling of the free hands, as the former one is, and is demonstrative. For with the rule pointed at the sun’s center, the wheel is turned with the pointer, until the transverse rule other touches the horns, or is parallel to the topmost, and the pointer will show on the outside circle the degrees of inclination with the vertical, and also immediately given is the sun’s altitude by the intersection or the rule with the crosspiece, by the same distinct mark made on the rule and the crosspiece, so that you may measure after the observation, if the crosspiece and rule are not divided beforehand according to the sun’s degree of altitude. If you like to use the small moons instead of the transverse, as in Problem 13, the volvelle is again named until the eclipsed ray embraces the small moon on all sides equally with its horns, which both the divided diameter and the small arms holding the small moon, which can take the place of the transversal, easily indicate.

The demonstration is in this, that the two slots, both of the crosspiece and the post, locate the rule in the plane of the vertical circle, and the line of length on the panel by the same means also follows the rule, from which line there begins the division of the outer circle on the panel.

If the whole instrument is not always at hand, as, if there be nothing present but the rule with the pins, you shall point the rule in the plane of the vertical thus: you attack to the rule at its base a transversal two feet in length, or longer, and at right angles to the rule, and afterwards you allow the rule with the transversal to lie out on a floor that is even and parallel to the horizon. For the transversal will set up the rule on its back, so that it lies in the vertical plane, exactly as it was kept before by the slots in the crosspiece and in the post.

Problem 16

To mark out the inclinations even on the floor

It was said in Chapter 2 that a ray of this kind in a camera obtusum is a right cone, if the opening be circular and perpendicular, whose vertex is above the opening, and the base on the surface illuminated. If this surface be directly prescribed to the opening, the base of the cone, or portion illuminated, will be a circle, by 8 of Chapter 2. But if the surface be obliquely presented to the ray, the base of the cone, or the portion of the oblique surface illuminated, will be a conic section, and in fact an ellipse, if the surface intercepts the whole ray, by Apollonius I 13. Since in our latitude the plane of the horizon or the floors always receive the oblique ray of the sun, the figure of the ray will therefore always be elliptical, and this even in nearly all shapes of openings, provided only that there be a full intersection. For by 6 of Chapter 2, the opening then communicates the least of its shape to the cone, the sun the most. Therefore, since the sun is circular in form, it nevertheless will bring about a cone that is

43 The Latin word, ‘climina’, used here, refers to the division of the globe into bands of latitude each of which is called a ‘clime’.
approximately right, even if the shape of the opening be something very different from circular.

Therefore, if you are without instrument, as happened to me on 25 February or 7 March 1598, make out ellipses upon a paper laid out on the floor in the extent to which the sun may shape them; and since they leave their place quickly, let three or four points be made, at the top apex $A$ and the bottom apex $B$ of the ellipse, as well as at the points of the two horns. Now the length of the ellipse $AB$ extends itself into the vertical circle. Consequently, by the mediation of the knowledge of the ellipse, the inclination of the ellipse $CD$ to the vertical $AB$ will be given in position. This is most of all useful at the beginning and end of the eclipse, where the points $C, D$ become one. Likewise, for determining whether the shadow stands exactly at $A$ or at $B$, or at the position at right angles. Outside these cases, it is a little more risky.

Problem 17

From the marking out of the ellipse, to learn the inclination of the ellipse.

First, mechanically as follows. In the preceding diagram, draw the longer diameter $AB$, and, it being bisected at $E$, about center $E$ with radius $EA$ or $EB$, let a circle be drawn, while from the horns $C$ and $D$ the perpendiculars be dropped to $AB$, extended to that part of the circumference of the circle, and let them be $CG, DF$. And let the circumference $GF$ be bisected at $H$, and let $EH$ be drawn. Therefore, $H$ is the angle of inclination, to be learned from the arc $HH$ if $AB$ be divided into 180. For since that cone is extremely acute, its angle being no more than half a degree, the middle of the ellipse is distant from the axis of the cone by a completely imperceptible amount, and for that reason, no matter from what point of the ellipse’s circumference the perpendicular to the longer diameter is dropped, it cuts off the versed sine of that arc on the circle whose diameter is $BA$, beginning from the vertex.

If you have also marked a pair of points for the shorter diameter, or if you have used the sun’s altitude to derive the ratio of the shorter diameter to the longer, as a test you shall describe about $E$, a circle with the size of the shorter diameter, and from $C, D$, you shall draw lines parallel to $AB$, so that they may cut the small circle at $I, K$: if the middle of this arc falls on the line $EH$, the marking out is certain. For again, the shorter diameter of the ellipse is imperceptibly longer than the line perpendicular to the axis at the point of the axis, which is in the plane of the base; consequently, no matter from what point of the elliptical circumference the perpendicular is drawn to the shorter diameter,
it cuts off the sine, between itself and the center, of that arc which, on the circle having the size of the shorter diameter, lies between the vertical point (or the one on the line $AB$) and the given point.

Indeed, so that those who see experts in the elements of cosines may find nothing lacking in me, I also add a raffinement to Apollonius. Let $N$ be the point in which the axis of the cone falls. Therefore, $NA$ and $NB$ will be obtained in this way. Increase and decrease the sine's distance from the zenith by $15$ minutes or the amount of the sun's radius, and take the tangents of all three arcs. The differences of the tangents will show the required portions in proportion, where the whole size is $100,000$.\textsuperscript{27} The shorter radius, however, going out from the point $N$ (applied ordinately, as in Apollonius) is obtained thus.

As the whole sine is in the center of the sun's distance from the zenith, so is the tangent of fifteen minutes to the quantity of the shorter radius, in the meanest in which $AS$, $NB$ is given from the tangents. This, then, applied ordinately, is the most exact size of the smaller circle. And because there is a danger that you might not have marked the points $A$ and $B$ exactly in the middle of the circular vertices, especially as, owing to the intervening delay of time, the vertices of the ray depart from $A$ and $B$ before you have punched the points $C$ and $D$; thus relating the quantities $CL$ and $DM$, therefore, you will note, by Corollary 4.23, that as the larger diameter $AB$ (Apollonius calls it the axis of the figure) is to the shorter in general, as the transverse side of the figure is to the upright side (hence rectum), so is the rectangle $AL$, $LD$, to the square on $EC$, and the rectangle $AM$, $AB$, to the square on $MD$. And in this way, $CL$ and $DM$ are restored, if they have an error in the marking; provided only that $AB$ be correct, and the place of the points $C$, $D$ between $A$, $B$ be correct with respect to longitude, where there is less danger in the marking. There is an example below in Prob. 23.

Problem 18

To take lunar inclinations with the instrument

On the perpendicular of the lunar rays, set up a small circle such as is on the outer panel, of solid parts, and cut back on all sides, divided from the top by any procedure into $90$ parts, so that the beginning of the division is exactly at the highest place, at the moment $\alpha$ which you wish to take note of the inclination $\gamma$ of a lunar eclipse. Let a rule rotate on the circle, longer than the circle's diameter, and on the rule let there be a transversal, perpendicular to the rule. And so, with the small circle positioned as you were instructed, transfer the eye to the rule, and turn it and move they eye until the transversal subtends the moon's horns equally. The rule will show on the circle the degree of inclination required. The calibrations which make instruments of metal are used to attaching a quadrant, fitted to the plane of the vertical circle; to a tripod, and to set the quadrant into a circle.

\textsuperscript{26} Appendix.

\textsuperscript{27} Let the center of the opening be $O$, and from $O$ drop a perpendicular $OP$ to the plane of the floor intersecting $AB$ (assumed at $P$. The angles $AOP$, $NOP$, and $BOP$, are the distances from the zenith, and the lines $MP$, $NP$, and $BP$, are proportional to the tangents of those angles.
with an open trough, but to attach the circle to the same tripod from two opposite points on a line parallel to the horizon. This instrument is extremely useful.

It should be used in this way in an eclipse of the moon, where our ecliptic instrument does not clearly enough present the ray of the eclipsed moon in the darkness, and the ray itself does not allow the presence of a lighted taper for getting the numbers of the degrees of inclination on the panel, or if, in the often repeated switching of the tapers with the darkness, leads to an impediment for the sense of vision in perceiving the ray. In this way I observed the eclipse of the moon which happened on 14/21 May 1903, the series of observations of which I will append to Problem 21.

Problem 19

*To take note of certain inclinations of both kinds of eclipses as is ordinarily done*

Look carefully at those moments when the shadow is either exactly at the zenith or exactly at the lowest part of the luminary, or on either side; that is, when the horns are turned exactly supine upwards, or prone downwards, or stand erect on the perpendicular. For the estimation does not easily err here. Take note of these moments carefully in those methods that follow. Many examples of this precept are supplied from the observations of Tycho, and also from those which, observed by me, I shall now directly add.

*To take note of the inclinations of a lunar eclipse without an instrument in another way*

Almost as before, note the times when the moon's horns coincide in the same straight line with one of the fixed stars or of the planets, or are turned perpendicular to it. But then in the extraction you should remember the moon's parallels, especially if it was compared to a nearby star. For the line was going to be applied differently to the horns if it had been viewed from the center of the earth. If you withhold this one thing, you have an easy summary of the rest of the method here. For up to now the inclinations of the circle through the centers were noted with respect to the vertical, not because of the vertical itself, but so that through the vertical the inclinations of this circle through the centers might be known with respect to the eclipse itself. Here, however, the inclinations are referred directly to the sphere of the fixed stars itself. The foundation of these problems, as regards the horns falling on a straight line with a fixed star, is this, that the line joining the ends of the horns, that is, the intersections of the pair of circles, is perpendicular to the line passing through the centers. Examples are in Prob. 21 and in a few places in the preceding problems.

Problem 20

*To take note of the phases as is usually done*

Regiomontanus once taught, from Alhazenians, 20 how to take note of the

---

20 The Arab astronomer Al-Battani. See Chapter 4, note 172.
san's altitude. The altitude of some bright fixed star is taken, most correctly, at night. If the sky be cloudy, and the fixed stars are hidden, the altitude of the moon itself is noted. The notations of the azimuths of all of these is also helpful. But how the time may be obtained from these, is taught thoroughly in the theory of the primum mobile. I hear that the illustrious gentleman Melchior Iostellus has in hand an excellent work, of forty problems of the primum mobile through the bare addition and subtraction of arcs and chords, a kind of computation familiar to Tycho for many years, and developed to a considerable extent by Clavius, now finally perfected by Iostellus. Therefore I refer the reader to it. For the present problems are observational, while those which will follow in the other part will be used for the second movable, specifically, the moon's motion, not the primum mobile. Tycho Brahe teaches how to use clocks more efficiently. First, a test of the clock is made over a number of days preceding. Next, once the beginning of the course has been made, from any mark, there are noted on it the indicators, not just of the phases, but also of the conjunction of the sun or moon or of the fixed stars with the meridian, or with another circle that is the same and certain. Thus the phases, with the moments of the day through determined time intervals indicated by the clock are commensurate. There are examples in Probl. 21 and the following.

I shall, moreover, state a number of ways by examples. On 13/23 April 1595, at Graz in Styria, when the beginning of an eclipse was noticed, the altitude of Aucturus was 44°, approximately, with a latitude of 1° by a paper astrolabe and a pendulum, while the moon's altitude was 15°. When now no more light appeared through the thick air, the moon evidently being deeply immersed in the shadow, the altitude of Aucturus was 34°, of the moon, 6; approximately. Therefore, the beginning would have been seen by me at 14° 50', from the altitude of Aucturus. From the observations of Tycho at Hven, the beginning was concluded to be at 14° 51'. And so my position was 2 degrees farther to the east. But it is certain on other grounds that the difference is a little more. And so my observation was not perfectly certain.

On 11/21 February 1598, at the time when the town bell was striking three, Spica was 56° from the zenith, with a wooden quadrant of 1° of a foot. Then, at the town time of 4°10', I was not yet able to notice the defect, but it could

Although this work by the Wittenberg astronomer Melchior Iostellus was completed in manuscript, it was never published. Kepler learned about it from Iostellus' colleague Ambrosius Rhodus, with whom Kepler carried on a correspondence from 1601 to 1603. See Franz Hauser's note in Iochl III p. 356.

The technique to which Kepler refers was, in modern terms, the trigonometric identity

\[ \sin \alpha \cdot \sin \beta = \frac{1}{2} \cos(\alpha - \beta) - \frac{1}{2} \cos(\alpha + \beta). \]

Since the cosine had not been named as a separate function, there was no such simple way to state this relation then. It appears to have been an invention of Paul Wittisch of Wroclaw; cf. Frisch's note in Iochl II pp. 438-9, and O. Gingerich and Robert Westman, The Wittich Connection, esp. p. 12 and the articles referred to on that page.
have happened that it had begun then, because of the clouds (this is what my note says). For less than a quarter later, the moon, not yet half obscured, stood fully erect, and a little after the fifth hour, half the diameter was eclipsed, the moon now being inclined. A little after the sixth hour, it withdrew behind the clouds with an extremely weak light, so much so that it seemed to diminish further, and about to lose all its light. The sun was at 2° 26′. Therefore, this altitude of Spica would add 24 minutes to the time. The beginning would fall a little before 5, the last phase at about 20 minutes past six hours. And this was about the middle, for it was observed at Wundesburg nor to be deeply eclipsed, of which you shall judge from Ch. 5 and 7. Further, the Tychoitic observers refer the middle of this eclipse (with the help of computation of the observation of the beginning, for the end was beneath the earth) to 6° 30′ at Hven. Therefore, the difference of meridians is about 15 minutes, or 4 degrees, probably.

On 25 May or 4 June 1602, at Prague in Bohemia, I observed the end of a lunar eclipse in this way. Tycho’s clock was at hand, indicating the minutes and seconds. Once the beginning of the course was taken, by whatever means, I accommodated the clock to the setting sun’s altitude of 42 degrees.

<table>
<thead>
<tr>
<th>Tycho's clock</th>
<th>From clock</th>
<th>Altitude of the sun, going behind the mountain</th>
<th>Resulting time</th>
</tr>
</thead>
<tbody>
<tr>
<td>6:43</td>
<td>4:44</td>
<td>7:24</td>
<td></td>
</tr>
<tr>
<td>6:52</td>
<td>Hilo still says</td>
<td>7:33</td>
<td></td>
</tr>
<tr>
<td>7:00</td>
<td>Trace of cloud</td>
<td>7:41</td>
<td></td>
</tr>
<tr>
<td>7:01</td>
<td>24:00</td>
<td>7:42</td>
<td></td>
</tr>
<tr>
<td>7:17</td>
<td>0:15</td>
<td>7:58</td>
<td></td>
</tr>
</tbody>
</table>

8:02
8:13
8:19
8:21

Trace of the moon’s light first seen
About 2 of the moon’s upper circumference was shown; the shadow was directed below Jupiter.
All had not yet come to an end
The circumference was seen, but was pulled.

It came to a complete end one minute later, that is, 9′ 30″. The computation of Tycho showed the end as 9° 19′, and that by the hourly motion, the equation of time, the radii of the moon and the shadow, and the difference of meridians of Prague and Hven established by Tycho, which can change, retaining the hypothesis she represents the actual moment of eclipsions.
To compare the moon’s visible place in longitude and latitude to the fixed stars, in the principal phases.

The instruction is chiefly about those things that are usefully observed in eclipses of the moon, so that they be not accidently omitted: it has much in common with 19 above. Now this can be done in various ways. It is best if the eclipse is at the beginning of Cancer or Capricorn, and its middle coincides with the actual approach of the moon to the meridian; or in any sign, when at the middle the moon stands at the nonagesimal degree, and is not far removed in longitude from some fixed star. But if it was outside of those moments, provided that the middle of the eclipse is attended to the remaining advantages will be able to be provided by consideration of the lunar parallaxes.

Direct a plane perpendicular to the horizon, into the meridian, or the nonagesimal, or right at the moon’s region at the middle of the eclipse. Then carefully note on the clock’s indicator of minutes and seconds for the moments at which the moon’s western edge, eastern edge, and the nearby fixed star approaches the meridian. For from the interval of times the difference of the intermediate spaces of the heavens is obtained. If it be the case that right at the middle of the eclipse the moon stood at the nonagesimal, the business will not require any further meandering; and the right ascension of the centers of both the moon and the shadow is obtained, numbered from the nearby fixed star. Consequently the distance of the fixed star from the sun’s center as well. Nevertheless, other supports, lacking these conveniences, should not be spurned.

Tycho took the distances of the unbroken edge from nearby fixed stars frequently over the whole duration, before and after the middle, with sextants and armillaries. It will also be helpful to take note of the track of lines, as Maclain used to do in observations of the other celestial bodies. It is plausible that Hipparchus used one of these methods. For, as Ptolemy relates in Book 3 Ch 1, when he had ascertained that the spring equinox had occurred, on the morning of the 27th day of the month of Mechir in the year 32 of the third Calippic period, afterwards, in the observation of a certain lunar eclipse that coincided with the equinox, he related that Spica Virginis was at 24° 45′ Virgo. In his eclipse, the sun’s position from the equator, and therefore on the zodiac, was given from the time, and from the sun’s position, the opposite position of the shadow, from the shadow’s position at the beginning and end of the eclipse, the moon’s position.

The same observer first ascertained that the spring equinox had occurred on the 29th of the month of Mechir, after midnight on the 30th, following, in the

50 When the moon is at the nonagesimal point on the ecliptic 90° from the eastern horizon its parallax is at a minimum, and affects the latitude only.
51 The date of the observation and the following one correspond to 24 March – 154 and 25/24 March – 154, respectively. See Ptolemy. Almagest, iv. Toomer pp. 138-9.
year 43 of the third Calippic period. And again, a coinciding eclipse of the moon, when the moon’s position was compared to Spica, seemed to him to bring Spica back to 24° 45’ Virgo.

Regiomontanus and Peurbach recorded an eclipse of the moon at Vienna in these words. On 27 December 1460, “at the beginning of the eclipse, the star that they call Almach”52 had a pre-meridian altitude of 7 degrees. At the beginning of the time interval, 17’; at the end of the interval, 29’. At the beginning of the eclipse, the moon was visually on the great circle passing through the head of the preceding of the Gemini and the brighter star of Canis Minor; at the end, on the other hand, it was above one circle passing through the brad of the following of the Gemini and Canis Minor.53

From this, we conclude that the beginning was at 11° 42’; the beginning of the time interval was 12° 47’, and the end of the time interval 13° 55’. To this record Tycho Brahe appended the note that if these things were exact, the positions of the stars could be verified from them.

On 29 November or 9 December 1601, we observed an eclipse of the moon here at Prague in this way. We used the Tychonic clock; the beginning of the course was arbitrary. We also used large quadrants for noting the arrival of stars at the meridian. The sun’s position was 17° 48’ Aquarius.

<table>
<thead>
<tr>
<th>Our clock True time</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.25' 5.00</td>
</tr>
<tr>
<td>5.33' 5.21</td>
</tr>
<tr>
<td>5.33' 5.23</td>
</tr>
<tr>
<td>5.37  5.25</td>
</tr>
<tr>
<td>5.50' 5.38</td>
</tr>
<tr>
<td>6.20  6.08</td>
</tr>
<tr>
<td>6.29</td>
</tr>
<tr>
<td>6.56  6.46</td>
</tr>
<tr>
<td>7.07  6.53'</td>
</tr>
</tbody>
</table>

53 Regiomontanus, De Temporum (Nürnberg 1544) 37°–38°. Although Kepler does not indicate it, this is a direct quotation, only slightly altered (except that he corrects the accusative “anteecedentem” replacing it with the genitive “anteecedentis.”).
8.08 | 7.34 | The crescent moon stood upright.
8.48 | 8.34 | End. The moon’s shadow was about 90 degrees from the zenith to the right.
8.51 | 8.38 | Transit of the bright star of Arietis.

a. Matthias Sandford had been an assistant to Tycho Brahe before assisting Kepler. He also helped observe the lunar eclipse of 14/24 May 1603, described in C. J. OBS III p. 318.
b. The right ascensions of the two stars differed by only six minutes of arc, so for timekeeping purposes they calibrate together.
c. Geminus Regum in Bayer’s catalog.
d. Eta or Beta Tauri.
e. Frigidus is an alternative name for Arietis. The two doubles are Alpha Arietis (Capella) and Beta Arietis.
f. Beta Tauri (4th) and Zeta Tauri. The former is in the thorax of the eagle, while the latter is to the south.
g. Alpha Arietis (Marsa).

The duration was 3h 1s. In Tycho’s lunar writings, there is in fact exists a computed duration of 3h 56m. But in error in the computation slips in. For the minutes of the incidence, prescribed by rule and computed from the tables, come to more than 1h 34m 39s. Therefore, the whole duration was 3h 56m 18s.
A minimum difference from the observations. The middle falls at 6h 56m, again not impressively close. The eclipse nevertheless appeared somewhat smaller than that which is depicted in the lunar writings.
On 14/24 May 1603, the moon was seen with faint light from the south side, not far from bright fixed stars and Saturn. I append the observation of this, but I shall first give warning that we were occupied with the altitudes of Saturn and Jupiter, and did not note their arrival at the meridian with the degree of precision that would serve for the fixed stars. I used a clock with a pointer for the seconds, again Tycho’s:

| Zytenschloß | German clock | Roketens clock | Time for
| 9.12 | 10:00 | 9.53 | 1st. The moon was not yet eclipsed, stood it precisely the same with the Heart of Scorpions. from what it receded again to the west because of the first (i.e., diurnal) motion and the eastward parallel and the whole duration of time.
| 9.92 | 10:04 | 3:00 |

54 Tycho included the computation as an example in the Prognosticon: see TROY II pp. 141–143. The computed duration is exp. 143.
55 In a letter to David Fabricius of 19 December 1604 (KGVW XV no. 308, p. 790, Kepler corrects this to 3h 33m 22s, the correct duration being 3h 56m 44s, but added, “by a correct clock, the minutes of time were 1h 29m, the duration 3h 56m.” Similar remarks are found in a letter to Herwart von Hoheneburg of 10 December 1604 (KGVW XV no. 302, pp. 86–90). The “rule” to which Kepler refers is the use of computed diurnal motions of the two bodies.
10.10 10.49
10.14 11.00
10.18 10.59
10.23 11.08
10.29 11.18
10.42 11.24
10.48 3.48
10.51 11.32
10.57 11.49
11.02 11.55
11.09 11.34
11.13 12.01
11.19 12.06
11.24 12.13
11.31 12.25
11.33 12.44
12.02 5.00
12.03 0.05
12.23 1.29
12.47 1.52
1.03 6.00
1.10 1.56
1.14 2.09
1.26 2.09

Jupiter on the meridian with altitude 24° 32'.
And now, the emission of the moon into my own
with flashes removed, was broken.

To me it seemed the beginning. The lunar
momentum grew on inclination of the shadow of
22 1/2° below and in the cos. with 27 1/2° standing
at the bottom.

To others, too, something seemed to be eclipsed.
The inclination being 22 1/2°. The moon left the
central of the heart of Scopius that rose with its
entire body, as if it were progressing.

Inclination 23° 51'. The line from Saturn passed
through the center.

Jupiter is on the meridian. Inclination 21° 57'.
Next, the circle through the centers passed above
Saturn.

Heliocentric.

Nature on the horizon, altitude 21° 47'.
Half the circumstance was gone.

Inclination 24° 51'.

The moon's center on the meridian, the middle
of its remaining bright part was 19° 7' at altitude
Inclination 255.'

The horns were pointed at Cor Scorpia.

The moon was standing spin, the horns
directed equally downward.
The horns were in a straight line with Jupiter,
which was now raised up somewhat more
towards the west.

Putative middle.

Inclination 28° 51'.

Inclination 30°. About 1/2 of the diameter was
gone.

Inclination 302°.

Still something missing.

[Inclination 302° or 300°. Some people thought it
was not. It was considered by me to be not yet
ruined.]

[Broken] (continue for me) as at the beginning at
11° 3°.

a. Intent.
b. There is no 'Caput Serpentis' in Bradley's star catalog; however, in his short list of the hundred
brightest stars, listed by right ascension and declination, the one star from Serpens is 'Ludica (Colli)
Serpentis' (the 'bright star of the neck'), Alpha Serpentis in Bayer's catalog. This is probably the
star meant here. See 1900/21 p. 376.
Chapter 11

381

In the meantime, Sauron was also observed from the fixed stars, so that there might once be certainty about its place, because inclinations were described using it. Between Saturn and the Northern Pan of Libra\(^{56}\) were 17° 22'. Between this and the Knee of Ophiuchus,\(^{57}\) 13° 20'. From this and the meridian altitude, its place comes out to be 0° 23°4 Aquarius, with latitude 2° 10' N.\(^{58}\)

The duration, from Tycho’s computation, comes out to be 3° 20'; it was seen by others to be 3° 53', by one, suffering from night blindness, 3° 56'. And with the waviering that preceded and followed the bate moments, 3° 19'.

The agreement of the three clocks is evidence of their constancy in number ing time intervals.

The middle was at 12° 30'. Tycho’s computation shows it to be at 12° 27' on the meridian of Hven, 12° 32' at Prague.

Since, therefore, it was half an hour past the hour of 12 at the middle, the moon’s center preceding the middle of the shadow by 15 minutes, which amounts to 1 minute in transit, it came out beautifully that the center of the moon was on the meridian at 12° 15'.

In Tycho Brahe’s observations, the most fitting of all for this procedure, both in the opportunity of the meridian, and the care of observation, is the eclipse of the moon on 3 March 1588.\(^{59}\) For in the whole time of duration, especially at the nonagesimal, the distance of the edges from Cor Leonis and Spica Virginis were taken very frequently. However, this is more appropriately referred to the other part, which contains demonstrations of the moon’s returns from eclipses.\(^{60}\)

**Problem 22**

*To note the times expeditiously with the ecliptic instrument*

Nothing new is demonstrated, except that I explain the use of the instrument, and how one observer may carry out all the things, whatever they are, that have so far come up to be observed.

So, let the observer stand at a table, moving the crosspiece around to the sun’s position with his arm, while either lifting the rule up or pushing it down, and where he wishes to note some moment with all its circumstances, and the pointer on the volvelle is correctly oriented to the ray, he makes a mark, both on the azimuthal quadrant, and on the crosspiece, and on the rule, where they

---

56 Bibliotheca.
57 ζ Ophiuchi.
58 Franz Hammer corrects this from 0° 46° 1°, following Kepler’s corrections in letters to Fabricius and Herwart (see the note above). The incorrect number had apparently come from Magini’s ephemerides. For more on the errors in this passage, see Frisch’s note in TIBO III p. 439, note 96.
60 Evidently, Kepler had planned a second part of the Optics, which never appeared. The material which he assembled for this purpose formed the basis for another projected (and uncompleted) work, which he called Hipparchus. The drafts for this work have been published in JKGW XX, i, especially sections VII and IX.
manually intersect, and to these three marks he places the same sign. Afterwards, with the instrument unmove, he writes down the number or sign of the marks, and adds what the pointer showed, what the transversal, what the shadow on the divided diameeter. After that is done, he again applies himself to looking for another moment. Thus everything is done in the same place by the same hands and without wobbling, which should be considered a gain where experienced assistants are wanting. For inexperienced ones disrupt instead of helping.

Next, on the azimuthal square, the tangents of the azimuths will be found, and on the rule, the secants of the distance of the luminous from the zenith, and on the crospiece, the tangents of the same arc, by the application of a proper measurement of inches; and, for a test, these three are used to elicit the same moment of time. An example will follow at the end of the chapter.61

Prostheorem for the Following Problem

That the path of that ellipse by which the sun, let through a slit, illuminates the floor, is a conic section

For the sun’s path in the sky is a circle, a great circle at the equinomes, and nearly a circle (for the joining, one from another, breaks up the full circularity to some extent), least in size at the solstices of the tropics, in between at intermediate places. But now both our opening and the whole globe of the earth is, within the limits of the senses, on an axis perpendicular to all those circles of the natural days. Therefore, the lines connecting the opening with all the points of the circle which the sun is passing along on any day, that is, the rays of the sun coming in through the same hole over an entire day, all joined together from all locations of the sun, form a right cone, by the Conics I 4 and Definition 8 of Book I. But by the same first definition, the same rays of the sun also form another cone inside the chamber or window, similar to the exterior one, since the angles, being vertically opposite,62 are equal. But the floor is placed parallel to the plane of the horizon. And the sun’s rays at whatever place, coming in through the opening, are extended to the floor. Let there be imagined a multitude of continuous bodies of the sun over the whole diurnal circle; therefore, the rays from all of them will go in simultaneously, that is, a conic surface will go in through the opening. And that is cut by the plane of the floor. Therefore, the common section, that is, the part which the solar ellipse describes over the whole day, is a conic section. When, therefore, the sun does not set, but graces the horizon of the opening, the plane of the horizon is parallel to the lowest line of that cone, with the result that the section here is a parabola, by the Conics I 11. But where the sun does not set at all, and all the rays all around fall on the floor for 24 hours, the section is an ellipse, by 13 of the same book. Except right at the pole, where this section is a circle. By 4 of the same book.

Where the sun sets, however, the plane of the horizon does not receive all the rays of the whole circle (that is, non those that are below the horizon), and it is parallel to some plane that, drawn through the vertex of the cone, cuts its base.

61 See p. 424, below.
62 Mētrē ἐσχηάθερεν, Cr. Euchild, Elements, I 15.
namely, the circle of the natural day. Therefore, by the *Comics* I 12, this section is a hyperbola, and to avoid confusion, I repeat to you that the continuation of the ellipses on the floor for the whole day would create the figure of a hyperbola.

The same is true of the ends or nodes of sundial gnomons, which serve as the opening. And as in our chamber, the two light-cones came together at the vertices, so in the art of sundials, the light-cone is joined to the shadow-cone at the common mark of the gnomon. I do not know whether this is remarked by the writers on sundials.

And since at our latitudes all lines of this kind are hyperbolas (except when the sun makes the equinox, for then the path of the ellipses is a straight line, the most obscure of all hyperbolas), the relation of the ellipses of the rays to their hyperbola should be noted. It is known from the preceding that if a perpendicular be dropped from the center of the opening, the axes (or, in common language, the longer diameters) of all the ellipses direct themselves to the point on the floor at which the perpendicular from the small opening falls. For the perpendicular and the ray and the axis of the ellipse are in the same plane of the vertical circle. But to the same point there also tends the axis of the hyperbola, which the ellipses of that day create. Therefore, when the shape and the axis of a hyperbola are given, the point on it is given, from which all the axes of the ellipses are extended, cutting the hyperbola.

**Problem 23**

To elicit the time of the phases from the extension of the ellipses

With a piece of paper laid motionless on the floor, capture or note the hyperbolic path of either or both vertices of the ellipse, and at the same time the drawn axis of a number of ellipses. For at the meridian hour, the axis of the ellipse falls directly on the hyperbolic path, in morning and evening most obliquely. And so, a tangent to the hyperbola being drawn, measure the angle it makes with the elliptical axis. Next, either with a compass or by computation, establish the cone for that day, and its hyperbolic section, and the point lying perpendicularly beneath the vertex of the cone, which will be on the axis of the hyperbola; likewise, the point of the hyperbola at which the observed angle’s set up; and finally, the point on the cone’s base at which falls the straight line this is drawn from the vertex to the point found on the hyperbola. For the arc between this point and the highest, or the one through which the vertical drawn through the center of the base passes, measures the interval of time between the meridian and the phase.

Moreover, the duration of time, and the minutes, even without a clock, you will measure in this way. If you enclose the right side edge of the ellipse just marked with a stylus, and hold it for as long as the luminous ellipse is traversing the stylus, and becomes tangent to the stylus on its left side; at that moment, you will remember to note the axis of the ellipse, and with the stylus switch over to the right side, and so on. From the number of contiguous ellipses you will have the time. For the body of the sun at apogee takes up exactly half a degree.

---

33 Keptr has I 13 here; however, the proposition constructing and defining the hyperbola is I 12.
which measures nearly two minutes. But the figure of the ellipse on the plane is a little wider (because of the width of the opening) than that figure which would be created by an opening with a magnitude of a point.

For example, on 25 February or 7 March 1598, when I lacked suitable instruments, and the cloudy sky made me almost lose hope of observing, I was nevertheless beneath a roof, and I had opened up a crack of uncertain shape and size, directing my efforts to whatever occasions might arise. When it was now noticeably eclipsed, the shadow was standing exactly at the right; it was one or two digits; the ray on the paper was of a silver gray color in magnitude, not such as I would have been able to perceive accurately; for the sun was again hidden in the blink of an eye. On the town clock, it struck one quarter after ten. After an hour, another sneak view. Horn fazed down, at the ray turning a little to the west, and very fast; in the sky, therefore, facing up, and the eclipse on the north. The end was clear. At a little before 12 by the town's time, there were 6 digits in the shadow of the ray. The shadow on the lower left, if you were looking at the ray, on the northern part; when your face is turned to the sun. At a quarter past twelve in the town, the digits were about 4; the inclination about 23. When the digits were about 3 3/4, the inclination appeared to be about 20. When the digits were 3 1/2, with yet another clock striking one quarter, the inclination was about 21. A little after, 3 1/2 digits; inclination about 22. At half before one o'clock on the town clock, 3 1/2 digits; inclination about 23. A little after, about 4 1/2 digits, inclination 21, which seemed very accurate. The cause of the discrepancy was this, that not well remembering the precepts of Marsilius, I neglected it, bring to the observer the circle; described on paper, their diameters divided into twelfths; therefore, when I mark the two circumferences with points, the sun meanwhile was going off, and I was not judging whether the ray was striking the paper perpendicularly. And so also, the moon's diameter appeared most of the time less, but sometimes greater or equal to the solar diameter. In fact, there was also this deficiency, that the indicator of the vertical was nothing more than the length of the paper. And so I acknowledged these obstacles, and therefore, running to reprimand plans, I began to note the end of the eclipse with ellipses, having no doubt that by the computation that I am now presenting, both inclination and time could be obtained from them. Therefore, from this moment up to the end of the eclipse I captured nine ellipses, six of these at the very least touching each other. And thus there were more than twelve minutes left until the end. In fact, when it ended, a little later it struck the third quarter past twelve, I had a wooden quadrant, not greater than two thirds of a foot. So with this, I took the sun's distance from the zenith, after it had struck the third quarter: 54° minus.

On noon of that day the sun's position from lyco was 16° 40'. Pisces, declination 5° 14' south. The altitude of the equator at Gare in Styrca, from concurrent observations, is 42° 58'. Therefore, the sun's meridian altitude was 37° 44'. The distance from the zenith 52° 16', and with parallel, 52° 18'. Let A be the opening, the plane surface BC, the perpendicular AB, the cone on that day DAC with angle DAC 169° 32', the cone's axis AE pointed towards the pole of the world, the hyperbolic section FGC, C the vertex, BC the axis, when in fact the sun is on the meridian, so that ABC is the plane of the triangle DAC through the cone's axis AE. Let side DA be extended until it meets BC at H.
Therefore, $HC$ is the transverse side of the figure, by *Conics* I.12. In numbers, where $AB$ is 100,000, $BC$ will be the tangent of the sun’s distance from the zenith, 129.385; and, because the declination is 5° 14’, $HC$ will be 10° 26’. Therefore $BAH$ is 41° 50’, whose tangent is 89.515. Therefore the remainder $HC$ is 39.870. For the upright side [latus rectum] of the figure, $AI$ must be drawn parallel to $BC$, and $CM$ perpendicular to $BC$ from $C$; and it must be done in such a way that as the square on $AI$ is to the rectangle $DI \cdot IC$, so is $HC$ to $CM$.\(^{64}\) In numbers: where $AB$ is 100,000, $AC$ is the secant of the angle 52° 18’, that is, 163,525. But because $L$ is the center, the base $DC$, and the cone is right, therefore $A\ell C$ is right, and $ACL$ is 5° 14’. But $AIL$ is the altitude of the equator, 42° 58’. Therefore, where $AI$ is 100,000, $AC$ is the secant of the complement of the declination, that is, of 84° 46’, 1,096,346, and $CL$ the tangent of the same, 1,081,778. But $AI$ is the secant of the altitude of the pole, 146,719, and $IL$ is the tangent of the same, 107,362, with the result that $DI$ is 1,199,140, and $IC$ is 984,416. For that reason, the square on $AI$ is to the

\(^{64}\) This is the way the length of the upright side $CM$ is established in *Apollonius* I.12.
rectangle $DI, IC$ approximately as 215 to 11,804. And $HC$ should also be to $CM$ in this ratio. Therefore, in the units of which $HC$ was 39,870, $CM$ will be 2,189,000, approximately. Therefore, the hyperbola is given since the sides of the figure are given. Therefore, by Conics II 1, the angle of the asymptotes is given. For when $HC$ is divided in two at $N$, $N$ will be the center, and $NO$ the asymptote. Therefore, $ONC$ is 82° 19’.55 Thus the hyperbola of that day is obtained.

Now let $PQ$ be the ellipse of illumination. I wish to know how much of an arc of its axis $PQ$, that is, the line $BQ$, makes with the section or line that touches the section at $P$, at any hour of the day. If the sun were traversing a great circle, so that the path of the ellipses would be a straight line such as $CM$, it is obvious that this very line would take the place of the tangent of the angle $CAM$ as well as $CM$, provided that in the former case $CA$, in the

55 In the plane of the paper, draw $CO$ from $C$ perpendicular to $HC$, intersecting $NO$ at $O$. By Apollonius II 1, $\frac{1}{2}HC, CM = CO^2$. But $\tan ONC = \frac{CM}{HC}$. Thus $\tan ONC \approx \sqrt{CM/HC}$. 
latter $CR$ would be the whole sine. But because the path itself is a curved line and is hyperbolic, while the sun’s path is a smaller circle, therefore $PR$, applied ordinately, is the tangent of the angle $PBR$, with $BR$ the radius, and of the angle $PAR$ with $AR$ the radius or the whole sine. But the arc of the great circle between the sun and the meridian, perpendicular to the meridian, is the measure of this angle $PAB$. Let it be proposed for us to continue the computation at these moments, $\beta P 30^\circ, \beta P 45^\circ, \beta P 60^\circ$, after noon. Let the meridian $\beta y$ be set out, on it $\beta$ the south pole. Now let the sun be at $\delta$, a part of the great circle $\gamma \delta$. And because the time measures the angles at the pole, the angles $\beta \delta \delta$ will be $\delta 45^\circ, 7 36^\prime, 11 15^\prime, 15 0^\prime$. But if $\beta \delta \delta$ is right, and $\delta \delta$ is the complement of the sun’s southern declination, that is, $84 46^\prime$. From these knowns there is also sought the side $\beta y$ from the base $\beta$ and adjacent angle, which at the first moment becomes $84 45^\prime 1^\prime$; at the second, $84 43^\prime 1^\prime$; at the third, $84 40^\prime 1^\prime$; at the fourth, $84 35^\prime$. Let $\beta e$ be the depth of the south pole, $47 2^\prime$; the remainders $\beta y$ or $BBA$ will be $37 43^\prime 1^\prime$, $37 41^\prime 1^\prime$, $37 38^\prime$, $37 35^\prime$. The complements of these, $B$, $A$, increased by parallax, are $52 18^\prime 1^\prime$, $52 20^\prime 1^\prime$, $52 24^\prime 1^\prime$, $52 29^\prime 1^\prime$. And as $BB$ is $129 437^\prime$, $129 592^\prime$, $129 853^\prime$, $130 244^\prime$. But previously, $BC$ was $129 885^\prime$. Therefore, $CR$ is $52 207^\prime, 468^\prime, 859^\prime$. So, since $HC$ is $39 870^\prime$, and its half, $NC$ $19 935^\prime$, $NR$ will be $19 997^\prime$, $20 403^\prime$, $20 974^\prime$. Therefore, by Apollonius I 37, let the square of $NC$ be divided by $VR$; the result is $N S$, the line between the $\omega$ sun and the time-touching the section at point $P$, at which the line extended out at right angles from $R$ falls, namely, $19 883^\prime$, $19 710^\prime$, $19 438^\prime$, $19 111^\prime$. Therefore, if you subtract $N S$ from $NR$, the remainder is $\delta \varepsilon$, $104^\prime$, $434^\prime$, $925^\prime$, $1682^\prime$. But by Apollonius I 37, as $HC$ is to $CM$, so will rectangle $NR$, $RS$ be to the square of $RP$. Consequently, $RP$ will be $16 676^\prime$, $21 900^\prime$, $32 170^\prime$, $43 304^\prime$. Previously, however, $BR$ was $129 437^\prime$, $129 592^\prime$, $129 853^\prime$, $130 244^\prime$. Hence, the angles $BBR$ are $4 43^\prime 9^\prime$, $36^\prime 13^\prime 45^\prime$, $18^\prime 35^\prime$. And their complements $BBP$, $85 17^\prime$, $80 22^\prime$, $76 5^\prime$, $71 25^\prime$. But in the triangles $SBR$, sides $BR$ remain, and sides $SR$ were given previously, hence the angles $SPR$ are $0 34^\prime 1^\prime 8^\prime 1^\prime 39^\prime 2^\prime 12^\prime$, which, subtracted from $BBR$, leaves the angles $BPS$ sought, $84^\circ 35^\prime$, $79^\circ 16^\prime$, $74^\circ 26^\prime$, $69^\circ 15^\prime$. You can also, for a test, use other means to seek out the lines $RP$, by which this whole arrangement is governed, by seeking the arcs $\gamma \delta$ of a great circle. For as the sine of the angle $\gamma$ or the whole sine is to the sine of $\beta \delta$, so are the sines of the angles $\beta$ to the sine of the arcs $\gamma \delta$, which come out to be $54 44^\prime 2^\prime$, $7 28^\prime 6^\prime$, $11 12^\prime 7^\prime$, $14 56^\prime 7^\prime$. And these are in fact the angles $PBR$, and the sines $AR$ of the angles $BAR$ are given, namely, $163 365^\prime$, $163 689^\prime$, $163 895^\prime$, $164 206^\prime$. But these, multiplied by the tangents of the arcs $\gamma \delta$ or the angles $PAR$ give the lines $PR$, namely, $10 673^\prime$, $21 459^\prime$, $32 457^\prime$, $43 800^\prime$, not indeed much different from the preceding.66

In our eclipse, therefore, I would like these instructions to be used more familiarly. For the rest, when I took the eclipses, I had not yet thought about the eclipses’ hyperbolic path. From this it happened that I did not keep the paper laid

---

66 In the computations in this paragraph, Franz Hammer’s corrections, which are many, have been adopted without comment.
on the floor carefully enough in the same place. Nevertheless, at the end of the eclipse I noted the two final ellipses more steadily. For I was watching for the end.

The drawing itself, examined with a compass, gives the angle E P S near the end of the eclipse as 70 degrees, with the diameters A B, C D of the two ellipses being connected on the two sides by the line B D, and, on consideration that the line B D cuts the hyperbola at B, D, the angle of the hyperbola itself with A B would have been a little greater; with C D a little less, but equal with some intermediate diameter. Therefore, at three minutes before 1 o'clock, the angle could have been 70°. Therefore, the end of the ellipse was slightly after 3° 50′. But, as I think, could be 13° 2′, 30° 44′, 56′ 53′ 50′. Angle a B D comes out to be 14° 34′.

This is most certain, that the time was after noon. For the ellipses were now moving away from the opening. II shall trust the constancy of the clock over such a short space of time, which, I think, can be done safely, the duration will now be observed. For at the time when I saw a defect of a digit or somewhat less, it struck precisely quarter past ten; therefore, since two minutes after the end it struck three quarters past twelve, the duration was therefore greater than 2 1/2 hours. Now in the thirteen minutes of time the eclipse decreased by 1/3 digits. Therefore, in eight minutes it increases or decreases a digit. And so at the first view it was one digit eclipsed; the duration is 20′ 36″. If, however, the former was less, the latter will also be less.

And since, by two indications, the end of the eclipse falls at hour 12° 55′, when 20′ 36″ are subtracted, the beginning will fall at 10° 19′ or more. Will we have confirmation from observation here too? Let us see. When on my quadrant the sun’s distance from the zenith was shown as 59 degrees, there was not yet any trace of a defect. Computation shows it was eight minutes after ten. Then the sun went under the clouds, and stayed hidden for rather a long time; about 1 1/2 an hour, until, on coming out, it appeared to be one digit eclipsed.

At Hven in Denmark, by a student of Tycho’s who was on the island at that time for the purpose of this observation, the beginning was given as 10° 3′; the end 12° 32′. The duration was 20° 29′. The digits 9 1/2. And in fact the time ought to have been longer here than in Graz. For as the moon was to the north, it is in accord with this that at that elevation of the pole, which surpassed mine by 9 degrees, it appeared 5 minutes more immersed in the sun’s body, and made its course through the sun’s body that much longer. See, therefore, the fallacies of

467 In the Toubier Rudolphine (JGW X p. 227), Kepler again gives the 1598 eclipse as an example, but notes that, since Tycho had left the island for good by that time, the observation was not made. Based on this, the author concludes that these observations must have been made at Frankfurt am Oder, farther to the east by David Opganis.
vision above, from Chapter Six, Tycho himself, as he wrote to Maestlin, observed
the beginning at Wurzburg in damburg with ariete, at 09 52', which at
when would have been 10h 41m 20s, near by Hoving, the difference of meridians
is 3° 8'. Therefore, the middle at Hven was 11h 13m 31s; at Graz, 11h 37m or more.
Consequently, the difference of meridians is about 20 minutes, or 5 degrees,
provided that everything that was used had been correctly established. Honian's
plate makes it not much different, i.e., the difference of meridians is more than 4.
I shall now reveal Problem 17 with this example, and shew the particular
inclinations that come out from my ellipses. First, I will find out the ratios of
the diameters to each other, at the sun's altitudes of 37° and 36°, such as they
were a little before and after the end of the eclipse at the time when I marked the
eclipses. With arc of 52° 45', 53° 0', 53° 15', the differences of the tangents
come out to be 1197, 1212. Therefore, the longer diameter is 2409, while the
shorter, drawn through the axis of the cone (multiplying the secant of 53°
by the tangent of 15°), is 1450. Again, with arcs 53° 45', 54° 0', 54° 15',
the differences of the tangents come out to be 1255, 1271. Therefore, the
longer diameter is 2526, the shorter through the axis of the cone is 1486,
a ratio of 17 to 10. The previous is a little less. The ratios in my sketches,
which were only rough and very quick, comes out less, and therefore flawed, for
the reasons stated in Problem 17; and this is the reason why I would suspect it
must be corrected in this way, from the instructions in Prob. 17. So the longer
diameter in the second of my ellipses was exactly this, expressed by the letters
AB, one horn at the bottom near D, the other on a line GI, perpendicular to
AB. Therefore, with BA divided at the midpoint at E, from E let the circle BG
described, and let LG cut this at G. And let the arc DG be bisected between
the horns in H. DHE will be the inclination, which now comes out to be 22° 13'.

Suppose that you would like to know what the place of the other horn was. About
center E with radius EJ, the size of the shorter diameter in relation to the
other, draw a small circle IK, and connect EG, which will cut the circle at
I; consequently, through I, parallel to BA, you should draw the line CI, which
will cut line GI at C. Therefore, C was one horn, while D was the other. In my
third ellipse, the horn was still about at the bottom, at the apex or vertex of the
eclipse, the inclination by mechanical measurement being 20°; in the fourth, it was
16° 31'; in the fifth, still less, while I judge that the other horn is always at the
bottom vertex, although from the first it was gradually moving away from it. But
in the sixth, where the lowest of the bottom vertex was now capable of being
distinguished from the lowest of the lower horn, the inclination again comes
out to be 22° 51'. In the seventh, 26°, without a doubt erroneously. In the eighth,
20°. In the ninth and last, 22° 51', measured very carefully, right at the end, which
by comparison with all of them is also the most true. For these inclinations vary
slowly at the beginning and end, very quickly in the middle.

Please be indulgent, reader, for my setting before you observation that are
not altogether consistent with themselves. For the eclipse was an important one.
which was awaited with longing by certain astronomers for many years, but was only seen in a few places, the sky being cloudy. For that reason, I think it preferable that these reports of it, of whatever quality, be on record, rather than it be held to be completely unobserved. And the uncertainty is nonetheless not so great that they ought to be repudiated. For it is the nature of my candor that I lay out all the doubts. And the warning was not necessary everywhere; for at the end of the eclipse, two things that are unreliable to the observing, and about which, individually and separately, I can doubt whether I was sufficiently attentive, conspire almost for a single moment, perfectly sincerely and with no prejudice whatsoever, from the observation we set forth, before they were examined: the sun’s altitude, and the angle between the eclipse and the hyperbola. And this consenus itself is not fortuitous, but they are consistent with observations in Saxony and Denmark, and with the actual revised computation of Toche. And I do not know what can be accepted against four agreeing witnesses. Besides, I shall ask this same thing about the inclination in the following problems, whether they are in agreement with these times.

Problem 24

Given the time, the quantity of defect, the diameters of the luminaries, and the inclination to the vertical of the circle through the centers, to dig up the visible altitude of the moon from the sun, as well as the longitude.

The problem is Maestlin’s, but is made easier by our parallactic table. Let the beginning of the observation be given as 10° 27′, when the defect was about one digit; the sun’s diameter, from the above, is 30° 35′. Let the moon’s diameter at this distance from perigee of 49° 24′ and eccentricity of 43° 36′ be assumed to be 32° 44′. Nor is there much uncertainty about this, as was argued above in Prob. 5 and 13. But the inclination of the circles is equal to a right angle, as was evident from observation. Let the meridian circle VP be set out, pole P; zenith V; let the sun be at S; the vertical be VS; the circle of declination PS. And through the center of the visible99 sun 5 let the arc of a great circle go across to the center of the moon, and let this be SL. Its quantity is obtained thus. The sum of the radii is 31° 40′; a digit is 2° 35′; which, subtracted, leaves the distance of the centers. Therefore, SL is 29° 5′. And let VSL be right. It is requisite to use this to examine in detail the moon’s visible altitude from the visible ecliptic SL, that is, the arc LE, and the visible longitude E5. Therefore, by the theory of the primus mobile, let the angle ESV, and before this, the altitude of the nonagesimal degree from raising, which is also useful elsewhere, be sought. So, since the sun is at 16° 45′ Pices, its right ascension is 347° 45′. When 23° 15′.

99 Here, as well as in the statement of the problem, Kepler uses the word “visibilis,” which has about the same meaning as its English cognate. It would perhaps be more idiomatic to substitute “apparent” for “visible,” but this would obscure the context. Kepler evidently wished to bring out between vision and the position of the heavenly body. If he had meant the reader to understand this as “apparent,” he could have used “apparent,” which also has a meaning close to its English cognate.
the times of the sun’s distance from the meridian, are subtracted from this, the
reminders, 324° 32', are the right ascension of the meridian, with which it
divides the heaven at 22° 10' Aquarius, and its declination 14° 11', which is $M$.A.
But $AV$ is equal to the altitude of the pole, 47° 2', therefore $MV$ is 61° 13'.
Now at this moment, 22° 31' Gemini is rising. Therefore, 22° 31' Pisces is at
the onagesimal degree, that is, at $N$; therefore, the arc $MN$ is 30° 01',
and $MNV$ is always right. Therefore, in the right triangle $MVN$, the base
$MV$ and the side $MN$ are given. If you will therefore divide the secant of the
former by the secant of the latter, there comes out the secant of $NV$, the arc
sought, which is here 56° 4', the distance of the onagesimal from the zenith.
The complement of this, 33° 56', is the altitude of the onagesimal, measuring the angle $O$
between the ecliptic and the horizon, by which arcs the latitudinal parallels will later be taken, in accord with Chapter 9.
For Copernicus’s table is very concise, and does not show these arcs adequately.

And so, once $NV$ is found, in the right triangle $SNV$, with the right angle at $N$,
the sides are given. For $S$ is 16° 45' Pisces, $N$ is 22° 31' Pisces therefore $SN$
is 5° 48'. Consequently, divide the tangent of $NV$, increased by the zeros in
the radius, by the sine of $SN$, and out will come the tangent of the angle $NSV$
sought. 86° 7', and $YSM$, 93° 53', towards the moon, which, at the beginning,
is normally on the western side, now closer to the meridian than is the sun. But
from the observation, $LSV$ is 90°. Therefore, the remainder $LSN$ is 3° 53'.

When a perpendicular is dropped from $L$ to $ME$ (let this be $LE$), a third triangle is given us, nearly plane, which is $L.E.S$, in which the base $LS$ and the
angles are given, as a result of which the sides $L.E$, the visible latitude of 1° 57' N.
and $E.S$, the visible longitudinal distance of the moon ahead of the sun, 29° 3',
will not be unknown. But, so as to escape the tenet of multiplying the sines
by the distance of the centers, find the distance of the centers at the head of the
parallactic table, and the angle between the ecliptic and the circle through the
centers at the side of the table: the sines in the table will give the visible latitude.
If you find the complement of this angle at the side, the entry will show the visible
longitude. 88

Problem 25

Given the visible latitude at a certain moment, to find quickly the
visible latitude at another moment at a certain distance from the
former one. It is further required that the distance of the moon from
the center of the earth be known approximately, and the hourly
motion of the moon, and angle and motion of latitude

88 The parallactic table (in the pocket at the back of this book) gives the angle whose sine
is the product of the sines of two other angles. It can thus be used to apply the sine rule
for right spherical triangles, as Kepler does here.
In the example (after 10° 29′), when the latitude is 2° North, let the hour 12° 55′, when the eclipse ended, be proposed for us. And let the hourly motion of the moon from the sun be 3′ 30″. From Tycho. Therefore, to 29° 28′ is credited 1° 22′ 58″ of true motion of the moon from the sun. The moon also departed by about the same amount from the node. And hence it is standing about at degree 10 from the node, its true latitude over an arc of this size increased 7′ 4″ to the north. But parallax too increases its visible latitude, which is clear thus. At 12° 55′, the right ascension of the meridian is 2° 8′. Therefore, 24° 48′ Cancer is rising. The point on the meridian is 2° 17′ Aries, with declination 0° 51′ north. Consequently, the arc between the culminating point and the zenith (previously it was 51°) is now 46° 11′. But MN is 22′ 7″, the amount between the culminating point and the nonagesimal. Hence, V.N comes out as 41° 27′, which previously was 56° 4′. With these two arcs, by the instructions in Ch. 9, I take from the parallaxic table, under the heading of 55 radians (the amount now adopted for the moon's distance from earth), the parallaxes of latitude decreased 10° 27′, or from the sun 10° 21′, under the heading of 56 radians the decrease would be only 10 seconds less, while there is the same amount of increase for the visible latitude. Therefore, 1° 57′; 7° 4′, and 10° 22′, together make the sum of 19° 23′. This, then, is the visible latitude deduced in this particular way at the end of the eclipse.

Now, by the converse of Problem 24, with the latitude, such as ought to appear, adopts among the Given, we shall establish the inclination, such as ought to be observed, so that we may compare it with our observed inclination. Let S now be the meridian, with I.S.N. now closer to the eastern horizon. The end of the eclipse is when the circums of the luminaries touch each other. Therefore, S.I is the sum of the radii, 31° 40′, while E.L is 19° 27′. Hence, E.S.F. comes out to be 37° 30′. So S.N is sought in the following manner. First, V.N has been found at this last moment to be 41° 27′. And S.N between the sun and the nonagesimal is 57° 56′. Therefore, the angle between the eclipse and the vertical comes out to be 55° 8′. I subtrah S.N.; these remains 17° 29′ between the circle passing through the centers and the vertical. It was observed to be 22° 1′. The difference is slight. For conversely, if you will assume 22° 1′, the visible latitude will come out to be 17° 6′, only 2′ minutes less, and these minutes may be missed for various reasons. For example, because the shadow of the first moment did not come exactly at the right, for this image was "seen for barely a moment. Or because we are uncertain about the quantity of the defect, for he who would observe these things precisely in such a small ray without an instrument? Or because, at the end, of the inexpérience of a new art, and the inclusions being not everywhere perfect. Finally, because

35° This should have been 41° 8′.
I did not take the precautions of the second chapter here, on account of the smallness and the unknown quantity of the opening. For, as is evident from the last diagram, when the elliptical ray is diminished at the edges, the inclination will also be diminished. However that may be, I have taken by the example how much the observed inclinations contribute to finding the observed latitude for the visible longitude at the end of the eclipse, from the angle V.S.N. 55° 8', let V.S.L. observed to be 22° 1', be subtracted, and the remainder, E.S.E. is 32° 38'. But it was computed to be 37° 39'; consequently, S.E. in the former case is 57° 22', in the latter, 52° 21'. The sines of these, multiplied by the sum of the radii, make, in the former case, 26° 40'; in the latter, 25° 4'; the moon's visible longitude beyond the sun, previously, it was 29° 3' before the sun. Therefore, in 26° 40', the moon's visible motion from the sun (the two longitudes taken together), is either 55° 14' or 54° 7', the former from the observed inclination, the latter from the computed one.

Problem 26

From a given visible longitude and latitude at certain moments, and with distances of the moon and sun from the center of the earth adopted from elsewhere, to establish the true longitude and latitude; likewise, the hourly motion as well, and the instant of true conjunction; or, on the contrary, assuming the hourly motion, to find the moon's distance from earth, approximately.

Parallax, mixed with the true motion, establishes the visible motion; when separated, it leaves the true motion. Parallax, in turn, is brought about by the nearness of a celestial body to the earth's center. And so the problem is obvious enough from the conversion of the instructions for computing eclipses only here, for the use of the instructions introduced in their place in Chapter 9 above. I repeat what should also be stated at the same time, in order that the use of the observed inclinations in establishing the moon's true position may be evident, and in order that the sun's eclipses may be freed from suspicion, by a demonstration that those eclipses contribute more to the investigation of the moon's motions than the lunar ones do. Let the distance of 1150 earth radii between the sun and the earth be adopted, as Tycho wished it, on which more elsewhere. This total, found at the head of our parallactic table, presents a horizontal refraction of about 3 minutes. The moon's distance from the earth, in turn, is taken as 55 radii, as above. Its horizontal parallax, taken proportionally from the head of our table, will be 62° 30'. The solar parallax of 2', subtracted from this leaves a horizontal parallax of the moon from the sun of 59° 30'\footnote{The solar parallax is, of course, much smaller than Kepler's figure, but the lunar distance corresponding to an apparent diameter of 32° 4' is 57°. The decrease in the moon's parallax would thus be about equal to that of the sun's parallax, leaving the difference about the same as Kepler's.}.

Let it now be the first moment, when the distance between the transagesimal and the zenith was 56° 4', with which, under the heading of 59°, is taken exactly
48 minutes 5"., while under the heading of 30" is taken 24° 53". The sun, 49° 22'., is the latitudinal parallax of the moon from the sun.

But the apparent latitude of the moon from the sun was 1° 57' north. Therefore, when added to the parallax of latitude, this gives the true latitude, 5° 19'.

For the longitudinal parallax, with the altitude of the nonagesimal 33° 56', I extract under the same columns of 59° and 30' the maximum longitudinal parallax of 33° 13'. Therefore, under the columns 33° and 13°, through the moon's visible distance from the nonagesimal of 6° 17' the moon being at 16° 14' Piscis to the sight, the nonagesimal at 22° 31' Piscis, I extract, with a double entry, the correct longitudinal parallax of the moon from the sun, 3° 37' in this place; and this is westward, because the moon is east of the nonagesimal. Therefore, when 3° 37' of parallax are subtracted from 29° 3', the moon's visicle distance from the sun, the remainder is 25° 26', the moon's true distance from the sun, to the west. At the other moment, when the distance between the earth and the nonagesimal is 41° 27', with this entered under the same columns of 59° and 30', as before, I extract the latitudinal parallax of the moon from the sun, 39° 23'. Suppose that the visible latitude of 17° 6' north was found correctly in Prob. 25. Therefore, when this is added to the moon's parallax from the sun, the true latitude will be 56° 29'. For the longitude at this moment, as before, I do as follows. Because N V is 41° 27' the altitude of the nonagesimal is consequently 48° 33'. With this, entered under the headings of 59° and 30', I extract the new headings of 44° 34'. Under these new columns, through the visible distance from the nonagesimal of 37° 31' (for the moon is at 17° 17' Piscis, the nonagesimal at 24° 48' Aries) I extract the moon's longitudinal parallax from the sun, in this place 27° 9' to the west. Add this to the visible amount of superceding deduced from the observation, which was 26° 40', and the result is the moon's true longitude from the sun, 53° 49'. Before, it was 25° 26' before the sun. Therefore, in its true motion, the moon over 26° 26' will have moved 1° 19' 15' from the sun; even less, if we use the computed inclination of the end. Above, from Tycho, we took this hourly motion as 1° 22' 30'. If this be true, an error of estimation in the beginning of the eclipse would be indicated, and the moon would have very slightly (but not at a digit) encroached upon the sun; perhaps indeed a slightly larger diameter of the moon would be required, or the sun clock might have been adjusted in mind the interval. Finally, and in turn, all the observations being held correct, either Tycho's hourly motion would be too great, or the parallax too small. Meanwhile, while the arc of the moon's true motion is diminished, the interval of latitude computed from Tycho is also diminished; and thus the computed visible latitude of 19° 22' will approach closer to the observed value of 17° 6', nor would the duration be augmented excessively over that which was fixed in Denmark.

From this variety, the diligent, talented, and circumspect astronomer easily sees which things fit together with which and what things out of all of them may be sought most securely by the observations and may be adopted for the establishment of canons, and which, on the other hand, being rough and not fully known without a large error, may nonetheless be added. Finally, he sees how important it is for matters astronomical and geographical, to observe and note the inclinations of the phases exactly, and indeed optically, by an opening.

For the instant of true conjunction, it is requisite that ye be certain of
the moon's hourly motion. We shall therefore grant, with many conjectures in agreement, that in the beginning not a whole digit was observed, but nonetheless we shall subtract something from Tycho's hourly motion. For besides, I have a number of causes of this matter, which I am going to set forth in the second part, Conjectures. 72 Let the truth, then, be in the middle, and let an hourly motion of 32'.58' be adopted. For even if it had not been eclipsed at all at the first instant of time, there would nonetheless not result as great a true motion for 2 hours, 24" as results from Tycho's hourly motion, with the parallaxes; for which the evidence is more reliable, staying the same. So, at the end, at a hour 12° 55', the moon had passed 55° 49' beyond the sun's true position. If 32'.58' make one hour, what do 53° 49' make? The rule will produce: 1° 38'.29'. Therefore, the instant of true conjunction at Gratz will have been 11° 17'. Provided that he has the moment of the end correctly, we are plainly certain of the middle of the true conjunction within one minute.

If you should wish to work backwards from the adopted hourly motion to seek out the parallaxes, the route is not so flat, but is circuitous or with a sidelong glance. For you must call upon Fortune, that you adopt that very parallax immediately at the beginning which you choose; that is, which would lead you back to the adopted hourly motion at the closest place, by the method just explained. Should you err from the true parallax, the work has to be done again, and by comparison of the errors with the differences of the parallaxes, the truth has to be palpated by a blind procedure, as it's regular falsity.

Problem 17

At a given elevation of the pole, the beginning and end, or moments, being visible, and the sun's place known, with hourly motion and diameters of the luminaries also chosen, and the qualitative motion of latitude (taken roughly), and finally the distances of the luminaries from the center of the earth, to investigate the instant of true conjunction and the true latitude; and from it the difference of meridians as well.

Let's play. For there is no reward for this work more certain than this pleasure. And if it is allowed, we make it our practice even to work with song. I want to know the instant at which the true conjunction occurred in Denmark, so that a more reliable difference of meridians may be had. At the same time, I also desire to know whether an observer hindered by deceptions of vision will note the beginning earlier, the end later. For this will easily become evident if we obtain a greater latitude by our vision. We shall also not be forgetful, that in our latitude, at the end of the eclipse there is an uncertainty of two minutes. The altitude of the pole at Gratz is 55° 54'.45'. Let the moon's hourly motion be taken as 32°.58', as just now. The sun of the midday is 31°.40'. We know that the moon is ready to be

72. See the footnote on p. 381, above.
73. Conjectures. I have not found this word in the dictionaries, but desire it conjecturally from "cancer," used, by way of mineral theory. Cf. Gauss's Dictionary of Music and Musicians, s.v. Conjectures.
north, and is running away from the ascending node. Let the moon's horizontal parallax from the sun be 59° 30', as before. The beginning is 10° 36'. The end, 12° 32'. The sun's position at the beginning is 16° 43' 27" Pisces at the end, 16° 49' 42".

Before everything, let the parallaxes be established. So, at the beginning, the sun's right ascension is 330° 48'. Subtracting from this 29° 15', the time of the sun's distance from the meridian, following the line of the observation at Hven reviewed in Prob. 23, leaves the right ascension of the meridian, 318° 33', with which it divides the sky at 16° 5' Aquarius. And the declination of this point is 16° 5' south. Hence, in the diagram of Prob. 24, since AM is 16° 5', AV 55° 54' 45", therefore MV is 72°. Further, at this moment 18° 24' Gemini is rising, and N is 18° 24' Pisces. Therefore, MV is 32° 21', and MV is 68° 33', its complement 21° 28". By these, I take the moon's parallaxes from the sun, 56° 22' of latitude, a titular 21° 45' of longitude. And since the sun is at 16° 43' Pisces, and the nonagesimal is at 18° 24', therefore, MV is 1° 41'.

I increase it by about 30 minutes, by which the moon precedes the sun to the sight, so that it becomes 2° 11', with which, from the columns of 21° and 45°, I take the true parallax of the longitude as 0° 48' to the west. Let these be kept ready.

At the end of the eclipse, the sun's right ascension (at 16° 50' Pisces) is 347° 54'. Add 8°, the time of the sun's distance from the meridian, the right ascension of the meridian will become 355° 54', with which 25° 32' Pisces is on the meridian. And the declination of this is 1° 47' south, which is MA. Therefore, MV is 57° 45'. Further, 27° 43' Cancer is rising at this moment, and N is at 27° 43' Aries. Therefore, MV is 32° 11', and MV is 50° 44'. And its complement is 39° 16', by which I take the parallaxes of the moon from the
sun, 46° 3′ of latitude, a trinity 37° 39′ of longitude. And since the sun is at 16° 50′, and the meridian at 27° 43′, the moon is therefore 40° 53′. I decrease it by about 25 minutes, by which the moon follows the sun to the sight, so that it becomes 40° 28′, with which, under the head 37° and 39′ just found, I take the true parallels of longitude of the moon from the sun to be 24° 26′ to the west. Let these again be kept ready.

Now, because the intervening time is 25° 29′, and the moon’s hourly motion from the sun is 32° 50′, therefore, the moon’s motion is 1° 21′ 32′. By an arc of this quantity about the node, the latitude varies by 6° 57′ at an approximately distance from the node of ten degrees (it being posited that the angle of latitude is 4° 58′ 13″). And because the moon is ascending to the north, therefore, at the end, the true northern latitude will by greater by 6° 57′. But the visible latitude is also increased because of parallax. For at the beginning, the moon’s parallax of latitude from the sun is 56° 22′, at the end, 46° 3′, the difference is 10° 19′, which adds to the difference of the visible latitudes, I therefore add 6° 57′; the result is 17° 46′, the excess of visible latitude at the end over the initial amount. In the same way, since both the parallaxes of longitude retard the moon, I subtract the smaller, 0° 48′, from the greater, 24° 26′; there remains the retardation, 23° 38′; this, in turn, subtracted from the moon’s true motion, which was 1° 21′ 32′, leaves 57° 54′, the resulting visible motion of the moon from the sun within the time of duration. Let each straight line A'B be set out, representing the visible motion of the moon of 57° 54′, and at eight angles to it let BC be drawn, with length 17° 46′, the difference of the visible latitudes; therefore, CA being joined, will be the moon’s visible path. Upon AC let the isosceles triangle ACF be constructed, so that each leg may have 31° 40′, the sum of the semidiameters, and from A, C, let circles of the moon’s diameter be described, from F a circle of the sun’s diameter, touching the diameters of the moon, and through F let straight line DE be drawn, parallel to B A, and let CB be extended to D, and AE be made equal to it. DE will be the visible ecliptic, and AE, CD the visible latitudes, from which afterwards the true ones are easily obtained by the mediation of the parallaxes.

To C B there needs to be applied with line BD or DE, unacted from those on F C or F A, which remain two sines whose roots together are equal to E D. If one prefers to extract what is sought by algebra, he will come to that equation where the cube and a constant

---

76 Although the results of these computations are figured because of errors in the data, these errors are irrelevant to Kepler’s purpose, which is to show the method of carrying out such computations. Hammer (JKGW II, pp. 458-59) gives corrected values from Kepler in the Tatrikischen Reduktions II pp. 110-11, JKGW X, pp. 225-27.
number are equated to the squares and the positions (i.e., the first power of the variable).\textsuperscript{77} For us, the geometric way is open. For as $AB$ is to $AC$, so is the whole sine to the tangent of angle $BAC$, which becomes $16° 36' \frac{1}{2}''$. In turn, as the whole sine is to $AE$, so is the secant of angle $EAC$ to $AC$, which becomes $1° 25' \frac{1}{2}''$, whose half, $AG$, is $30° 12' \frac{1}{2}''$. Therefore, as $AF$ is to $AG$, so is the whole sine to the sine of $\frac{1}{2}FG$, whose complement $FAG$ is $17° 27'$. When $BAG$, $16° 36' \frac{1}{2}''$, is subtracted from this, the remainder, $FAB$ or $AEF$, is $0° 50' \frac{1}{2}''$. Finally, as the whole sine is to the sine of angle $AEE$, so is $AF$ to $AE$ or $BD$, which is what we sought, which becomes $0° 28'$. Therefore, the visible latitude at the beginning of the eclipse is $0° 28' \text{north}$, at the end $17° 44'$. When the transits of parallaxes are added, the former comes out to be $56° 50' \text{north}$, the latter $1° 47' \text{north}$. See how much greater it is than the one observed in Graz, $51° 30'$ at the beginning, $56° 30'$ at the end, or the sum $58° 20'$. And since this computation of ours is more rightly consistent with the computation of Tycho, who shows a true latitude at the middle of the eclipse of $34° 11'$, we evidently conclude that the usual error crept up on the observer, so that he did not take note of the beginning and end, but moments close to these, because of weakening vision in the sun's height light. It should not trouble you, either, that I attribute to him an error of $51'\frac{1}{2}$ minutes at the true latitude. For the principles by which I obtained this are set up thus, with the moon, especially, passing almost through the middle of the sun's body, and touching it from the slanting sides, so that if you will add even the slightest amount to the time, this sum just stated would be removed from the latitude, with the two small moons of our diagram settling deeper into the sun's body, as if their chains had been loosed.

This also is evidence that the latitude was really smaller, that the observer came up with $51'\frac{1}{2}$ digits. And in fact it is certain from Ch. 3, that whether it was contemplated through an opening or with eyes directed at the sun, the eclipse in either case was greater than $51'\frac{1}{2}$ digits. If you compute the length $FG$ from this diagram (for at $G$ the eclipse is greatest) it will come out $90' 30'$, the distance of the centers under the maximum obscuration, and when the axis of the moon's diameter over the distance of the sun is subtracted from this, the remainder is

\textsuperscript{77} This is an equation of the form $y^2 + ay = bx^2 + c$. Kepler is using Cardano's terminology; c.f. Cardano, The Great Art, trs. Wimse. However, the problem sketched out by Kepler reduces to the equation $2AB/AF - x^2 = AB^2 + CB^2 - 2CBx$ (where $BD = MF$, $x$), which is not a cubic equation. Hammer (1998 BII p. 459) explains that it is a cubic equation because the fourth powers of $x$ are canceled. This last statement is true, but the explanation ignores the fact that there are no third powers of $x$. Witsch (2002, 2 p. 441) works out the solution without addressing the discrepancy between Kepler's statement about the cubic equation and his own analysis, which has so values. In any case, as the next sentence shows, Kepler chose another way to the solution.
If the ratio of the lunar diameter to the solar is observed through an opening, the proportion of the opening being unknown, but if the true ratio of the diameters is known from elsewhere, so reckon how much the observation of digits shall have erred from the truth, and the rest.

On 21 July 1590, my teacher Maestlin observed an eclipse of the sun at Tübingen beneath a broad and dark roof, the sun’s ray being admitted through the roof tiles. I shall communicate the description of the observation as the author supplied it plentifully to me, so that I may use this example to display the application of a number of the problems above. We are missing the beginning. Therefore, when the sun was eclipsed by a semidiameter, its altitude above the eastern horizon was 26°; the inclination, observed as in Prob. 14, was 88°. The moon was higher than the sun on the ray, lower in the heavens.

About the middle of the eclipse he measured the ratio of diameters in the way that is in Prob. 7, and found that where the sun was 24, the moon occupied 23, while the ratio of the distance of the centers to the sun’s radius was as 59 to 88.

After the maximum obscuration, when it was again seen to be eclipsed by a semidiameter, the sun’s altitude was found to be 33°. The inclination of the circle through the centers was 2° to the vertical, by which arc the moon was west of the sun, and south, as before.

When it was one fourth eclipsed, the sun’s altitude was 33°. The inclination of the circle through the centers was 19° to the vertical. Further, the moon was then made to be east of the vertical.

At the end of the entire eclipse, the sun’s altitude was 41°; the circle through
the center-comprehended angle of 30° with the vertical. The moon was to the east.

In the ray CBDi or SHD end set out on centers B, A, and, because AB is the moon’s radius, BF the sun’s, the ratio of BF to BA from observation, will be as 24 to 23; therefore the whole AF will be 47. Further, since it was demonstrated above in Ch. 2 that the edge of a ray admitted through its opening is enlarged, accordingly, about the same centers B and A let the interior arc be described, with radius BH for the former, AF for the latter, so that FB and HH are equal. Therefore, AF will be the radius of the sun, and BH the radius of the sun, which are obtained thus. Since the sun is one sign from apogee, as the whole sign is to 30 seconds, by which it is increased to the sight from apogee to its mean distance, so is the versed sine of its departure from apogee to the increase of this place, approximately. So the diameter is 30° 47, the half 15° 23. Or by our parallactic table, find the difference of the mean and least diameters there 30° 7 at the head, the degree of the complement of the diameter from apogee (or this example) at the side. What the entry shows, subtract from the head; add the remainder to the minimum diameter. If the distance from apogee exceeds a quadrant, the excess must be found at the side, and what the entry shows must be added to the head, and the sum added to the diameter at apogee. We will proceed in the same way with the moon. The moon was 47 degrees 50’ from apogee; by the complement of 27, and the eccentricity of 43°, as in the above example, the diameter of the moon is obtained as 30° 47, the half 15° 23. Therefore, where AF is 15° 23”, BH is 15° 2”. And because BF and HH are equal, therefore, BH, FF are also equal. Therefore, the sum of AF and BH is equal to the sum of AF, FF, that is, AB, BF. Therefore, AF, 47 of Marsden’s units, is equal to the true measure of 30° 27’. Therefore, if 47 gives 30° 27’, what will 25 make in this measure? The rule produces 15° 54”, which is AF, and AF is 15° 23”, the obscured part is 15° 32”, when it was considered by Marsden to be eclipsed by the half diameter. And because £ is 15° 32’, and AI is 30° 27’, therefore the remainder AB, the correct visible distance of the centers by Prob. 12, is 14° 55’.

Problem 29

From the altitude of the sun or of a star with known declination, and a known altitude of the pole, to find east and distinguish the time or elongation of the sun from the meridian.

1st. The “half-sine” is, as usual, the number of units in the radius opposite which the trigonometric functions are compared. The “versed sine” here is 1 - cos 2°, where A is the sun’s true anomaly, by this proportion, Kepler makes the change in the sun’s apparent diameter proportional to the versed sines. This is a form of proportionality of which Kepler who found see, for example, his planetary speed rule in Ch. 29 of the Mysterium cosmographicum (1596, p. 201, and the notes on p. 249, and his refraction rule (Ch. 4 above).
We enter the fray in one form of triangle, with Wittich’s procedure of adding and subtracting. For although we do not diminish the labor, we nonetheless go up to the summit and have a view of our path before we eyes. The valleys, however, conceal Wittich. Because his demonstration is general, carried over from the circle to the sphere, this one of mine is especially adapted to the sphere.

Imagine that a plane parallel to the horizon passes through the center of that circle which at that moment the moon occupies in carrying out its diurnal motion. Then as the whole sines is to the sine of the altitude of the pole, so is the sine of the declination of the sun or of the star to the sine of the altitude or depth of this plane above [or below] the horizon, according to whether the sun was to the north or south. Next, consider that, as the ecliptic is inclined above the equator, the equator above the horizon, so is the parallel [upon which the sun travels] inclined above this plane, and the sines of the degrees beginning from the plane are proportional to the sines of the altitudes above the plane; those, that is, the greatest of which is the difference of the sines of the altitude of the plane and of the meridian altitude of the star; or, contrariwise [i.e., if the declination is south], the aggregate of the depth of the plane and of the meridian altitude of the star. And so, subtract the sine of the altitude of the plane, to the sine of the

79 For Paul Wittich (d. 1556), see Gingerich and Westman, “The Wittich Connection.” Wittich never published any of his work; Kepler knew of it from Joost Bürgi, who had known Wittich when both were working at the court of William of Hesse.

80 In the adjacent diagram, $P$ is the pole. $N$, the plane of the horizon projected on the meridian plane (the plane of the paper). $T$, the projection of the equator, and $Z$ the zenith. The sun over the course of a day moves on a path that is very close to a circle parallel to the equator. Let the projection of that circle be $SH$. $H$ being its center. Kepler’s imaginary plane therefore passes through $H$, let its projection be $BC$, and let $K$ be the intersection of $BC$ and $EZ$. Then the sine of arc $AP$, the polar elevation, is $KT / HT$, and the sine of arc $AS$, the unit’s declination, is $HT / ST$, and the sine of arc $BS$, the altitude of Kepler’s plane, is $KT \times TE$. The relation stated follows directly.

The “parallel” is the circle about $H$, whose projection is $SH$; it is parallel to the equator $AT$. The “sines of the altitudes above the plane” are the lines $SE$ and $DF$, which are in the same ratio as $SH$ and $DH$. $D$ being the projection of sun’s position. Now imagine the sun’s circle about $H$, perpendicular to the plane of the paper, passing through $S$, then through the sun’s position (call this $D$), and intersecting Kepler’s “plane” $BC$ parallel to the horizon at a point behind $H$ (call it $H$). The line drawn from $D$, intersecting $HH'$ at right angles is the sine of $H / HD$ (with $HH'$ or $SH$ as radius), and is equal to $DH$, and this angle is the complement of the sun’s distance from the meridian. But $DH$ is equal to the perpendicular from $D$, that is, the sine of the complement of arc $DS$ to $DS$, $DH$, or $DH$, $ST$. And $ST = \sin SYM = \sin C \sin M$, while $DL$ is the observed altitude of the sun diminished by the sine of $CTM$. 

791
altitude add the sine of the depth, and of the meridian, and of the temporal or observed arc. Then, as the first of the two remainders or sums is to the second, so is the whole sine to the sine of the complement (if the star is higher than the plane) or of the excess over a quadrant (if it is lower) of the stars distance from the meridian. An example is in the eclipse under consideration, and at the first moment. Let the altitude of the pole at Tübingen be 48° 24'. For all tables make the latitude of Tübingen and Augsburg the same. But at the position of Augsburg, or Gegglingen, a little farther south at half a mile or 2', the altitude of the pole was found very accurately to be 48° 22', as you see in vol. I p. 361 and following of Tycho's _Persoonssamle._ In fact Maestlin too records this altitude at Tübingen in 1586 at the end of his book of spheres. Let the sun's position be roughly 7° 30' Leo, because the time is not known precisely; the declination of this is 18° 28'. Hence, the sine of the altitude of the plane is 23,687. And because the altitude of the equator is 41° 36', I add the declination of 18° 28', hence, the sun's meridian altitude is 60° 4°. The sun's observed altitude is 26'. The sines of these are 86,661 and 43,837, diminished by the altitude of the plane, the remainders are 62,974 and 20,150. And the latter divided by the former (increased by the zeroes in the radius), gives 31,097, the sine of the arc 18° 40', whose complement, 71° 20', measures the time of the distance from the meridian. Therefore, 15° 31' were eclipsed at 26° 14' 20'' am.

I shall also follow the same method at the other moment, when the sun's altitude was 33°, and in the two remaining ones, when the sun's altitude was 37°, and 41° 5'. Except that here I shall mix in an addition/subtraction in the first part, to avoid multiplication. The sun's position at 7° 15° is shown by Tycho's computation to be 25° 2° Leo. Therefore, at the end of the eclipse, it is really at 7° 30' Leo. The declination from 7° 25° Leo to 7° 30' Leo decreases 2 minutes. And thus can safely use this declination unchanged.

<table>
<thead>
<tr>
<th>Declination</th>
<th>18</th>
<th>28</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abs. of equit.</td>
<td>41</td>
<td>36</td>
</tr>
<tr>
<td>Sums</td>
<td>66</td>
<td>41</td>
</tr>
<tr>
<td>Difference</td>
<td>25</td>
<td>8</td>
</tr>
<tr>
<td>Remainder</td>
<td>3927</td>
<td>3927</td>
</tr>
<tr>
<td>Half</td>
<td>23687</td>
<td></td>
</tr>
<tr>
<td>Latitude of the plane sought.</td>
<td>23687</td>
<td></td>
</tr>
<tr>
<td>Right ascension</td>
<td>62974</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Alitudes of the sun</th>
<th>35° 30'</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sines</td>
<td>54,446</td>
</tr>
<tr>
<td>Alitude of the plane</td>
<td>23687</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Alitude of the plane</th>
<th>23687</th>
</tr>
</thead>
</table>

---

82 _TBGHI_ p. 348.

83 The "librimum sphaericum" would most likely be Book III of Maestlin's _Comment._ cf. fol. ii, p. 10, of the 1610 edition. "This [third] book is entirely and exclusively Spherical..." However, Book III contains no computation of the polar altitude. At the end of Book IV (cf. 1410 p. 541), he gives the latitude of Tübingen as 48° 30', but does not show the means of determining it; it is only included as part of an example.

84 _Prosthaphaeresis._ This is the method, described above, of using a trigonometric identity to substitute addition or subtraction for multiplication; see note 49, above.
These are sufficient to set out this problem clearly. For the rest, we shall continue the example by Problems 24 and 26, first taking the visible latitude, longitude, and so on, then the true ones, at these four moments.

<table>
<thead>
<tr>
<th>Sun's position</th>
<th>Lat.</th>
<th>Lon.</th>
<th>Lat. 7° 27' 30&quot;</th>
<th>Lon. 7° 28' 12&quot;</th>
<th>Lon. 7° 29' 14&quot;</th>
<th>Lon. 7° 29' 14&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right ascension</td>
<td>128° 51'</td>
<td>139° 53'</td>
<td>129° 54'</td>
<td>129° 55'</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance from meridian</td>
<td>31° 20'</td>
<td>65° 45'</td>
<td>54° 12'</td>
<td>47° 52'</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RA, Dec. of Mer.</td>
<td>26° 30'</td>
<td>69° 6'</td>
<td>75° 42'</td>
<td>82° 3'</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cumulating</td>
<td>0° 41'</td>
<td>10° 44'</td>
<td>16° 3', Gen. 22° 13'</td>
<td>Gen. 22° 13'</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decl. of Sun</td>
<td>20° 17'</td>
<td>22° 8'</td>
<td>22° 32'</td>
<td>23° 16'</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alt. of pole</td>
<td>25° 24'</td>
<td>48° 24'</td>
<td>48° 24'</td>
<td>48° 24'</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Side: 400
- deg of Prob. 24: 28° 2°
- 6th Dec. of House: 146° 31' (192° 8')
- Resulting rising deg.: 7° 5' (Vir. 14° 3', 10° 34', Vir. 24° 32', Vir.)
- On the Nautograph: 7° 5' (Gen. 14° 5', Gen. 19° 34', Gen. 24° 12', Gen.)
- Therefore, 6° RM: 8° 24'
- Correction shows 9° V: 27° 20'
- Complements: 67° 40'
- In triangle NYS
- N = 60° 20'
- by SV: 53° 24'
- or by SV, NYS is found 53° 24'

a. The "horoscope" is the same as the Ascendant. That is, the degree of the ecliptic that is on the eastern horizon at the given moment. Its "obliquity equinox" is the degree of the celestial equator that is on the eastern horizon at the same moment. r is therefore 6° east of the Right Ascension of the Meridian (as a comparison of the numbers in the table shows). It is a useful number because it is equivalent to the 6° between the equinox point and the horizon, and this, together with the location of the pole and the obliquity of the ecliptic, allow one to calculate the ascendant (the Keplerian in the next line of the table).

These things so far are from the motion of the planet itself. Now, in the diagram of Problem 24, 6 in this particular instance is not between N, M, but N is closest to M—note this to avoid confusion. And thus, if, facing the sun, you number the angle, N SV from the upper part of the verticle to the ecliptic; towards the right and west, 30° 43' at the first moment, and the angle from the lower part of the verticle to the circle through the centers of the luminaries, 88° from observation, the remainder between the circle through the centers and the ecliptic will be 61° 17'. The moon is ahead of the sun to the south. At the second moment, the upper angle was 31° 36', the lower was observed to be 2° 30'. Therefore, the remainder in the semicircle is 145° 54' between the circle through the centers and the western part of the ecliptic; therefore, between this same circle and the eastern part (the moon being to the south, as before) is 34° 6'. Here, therefore, the moon is nearer to the east, and beyond the sun.
In the other two moments, because the inclinations are rather large, it is a good idea to find out whether the precaution of Problem 14 is necessary. The sun’s altitude is 37° 15’. The inclination, 19°, is in fact in the plane of the instrument, which was set perpendicularly, not to the sun, but to the azimuthal point of the horizon. Now when two planes are tangent to a sphere on the same great circle, or are equidistant from the tangent planes, their mutual intersection is a straight line, perpendicular to the plane of that great circle; and a line from the center of the sphere extended on through the plane of the great circle so as to meet that mutual intersection, will fall upon it at right angles. For by Euclid XI. 4., a line perpendicular to a plane is perpendicular to all the lines of the plane. In the present undertaking, we have three such planes. For the great circle here is the vertical circle drawn through the center of the sun. Imagine that it is touched by one plane; the plane of the horizon is therefore equidistant from it. Imagine that it is touched by another plane at its intersection with the horizon; we have said that the plane of the instrument was equidistant from that plane. Imagine thirdly that it is touched at the center of the sun by a third plane. Therefore, the intersection of this plane and the plane of the horizon falls outside the sphere. But the intersection of the plane of the instrument with the plane of the horizon falls within the sphere.

Let $AXY$ be the plane of the horizon, $STV$ that of the instrument, $SXY$ the third plane. A the center of the sphere, $S$ the sun, $TV, XY$ the intersectants. In the plane $TSV$, let another line $SV$ be set up at $TS$ making the angle $TSV 19°$, and let $AV$ be extended to $XY$ and let $ST$ be joined. And because $ATV, ATS, AXY$ are right, $TS$ will be the sine of the sun’s altitude, 60,529. But $TSV$ was found to be 19°. Therefore, $TV$ is 20,842. Further, $AT$ is the sine of the complement of the altitude, 79,600. But the angle $TSX$ is equal to the sun’s altitude, 37° 15°. Therefore, $TX$ is 60,627. Therefore, the whole $AX$ is 125,627. But as $AT$ is to $TV$, so is $AX$ to $XY$, which accordingly becomes 32,893. And because $STX$ is right, and $FGX$ is 37° 15°, therefore, as the whole is to $ST$, so is the secant of $TSX$ to $SX$, which becomes 76,651. Therefore, because $SCY$ is right (for $SX$ is in the plane of the vertical, $XY$ in the plane of the horizon), as $XS$ is to $XY$, so is the whole to the tangent $SXY$ sought, which becomes 23° 23’ greater than before.

---

85. This is the tablet upon which the inclination was measured, not set perpendicularly to the ray coming in from the sun, but in a vertical plane set perpendicularly to a line coming in from the sun’s terminus. This would have the effect of distorting the inclination (just as it distorts the sun’s image), making the measured inclination smaller than the true inclination.

89. This is the plane perpendicular to the ray coming from the sun, it is required to find the angle $XY$ which is the true inclination.
Thus at the end of the eclipse, when the sun's altitude was 41° 15', the inclination on the instrument being 30°, the angle X SY was found right on the circumference of the sphere, by a similar process, to be 37° 31'. And this being so, the inclination is correct in both cases.

Since, therefore, at the third moment, the angle on the upper right between the ecliptic and the vertical is 32° 35', as was proved above from the time, and the lower left angle is equal to it, while the angle of the vertical and the circle through the centers, likewise on the lower left, is 23° 23', therefore, when the latter is subtracted from the former, there remains between the circle through the centers and the ecliptic 9° 12', the moon being to the south.

At the fourth moment, the prior angle above was 34° 15', the latter of the same conditions was 37° 31', and greater. Therefore, when the former is subtracted from the latter, there remains between the ecliptic and the one through the centers 3° 15', the moon now being to the north.

For the rest, in Problem 144 I advised that this method is risky, because of the wobbling of the hands. How much it will be this time will not be concealed in the outcome.

Now, with those angles found, we shall establish the visible longitude and latitude at all 4 moments. For in the small triangles LSE of Prob. 24, the bases LS and the angles are given, and consequently the sides L E of latitude, S E of longitude, as well. Moreover, the bases at the first and second moment are equal, because in body cases it appeared to be eclipsed by a semidiameter. Therefore the distance or L S was found above to be 14° 35'.

At the third moment, because one quarter of the enlarged ray appeared to be eclipsed; therefore, let this be subtracted from the sum of the radius, as because the sun's radius is 24°, the moon's 23°, the sum 47°, a quarter of the sun's diameter 12°, therefore the remainder is 35° in the enlarged ray. But the centers remain in their places, by Problem 12. And the lunar radius is diminished by the same amount that the solar is increased; and so the sum of the radii also remains the same, as also in Problem 28.

Therefore, the ratio of the sum to the distance of the centers is the same. But because the sum of the radius is really 30° 27', therefore, as 47° is to 35°, so is the sum to the distance of centers, 22° 40'. But at the end, the distance of the centers is equal to the sum of the radii, 30° 27'. I nevertheless am employing this degree of precision only for the sake of the example. For as far as concerns the issue itself, I recall this in the midst of describing the openings were changed, and so the quantity will very likely have been different in the later times; and not everything corresponds to this exactly. But I carry on with the example.

Now, with the bases known, since they are hardly different from straight lines, we shall multiply them by the sizes of the angles, and, the last five digits
being dropped, the latitudes will come out, with the arcs of the complements being the visible longitudes of the moon from the sun.

\[\text{Section} \quad \text{before the sun} \quad \text{Second} \quad \text{after the sun} \quad \text{Third} \quad \text{Fourth} \]

<table>
<thead>
<tr>
<th>Latitude</th>
<th>7</th>
<th>10°</th>
<th>12° 21'</th>
<th>22° 25'</th>
<th>30° 24'</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15°23'</td>
<td>A</td>
<td>8° 22'</td>
<td>X</td>
<td>33° South</td>
</tr>
</tbody>
</table>

The same thing will result if you will follow the short cut appended to Problem 24.

Up to this point, we have treated the example by Problem 24. Now, to give full and sufficient attempts to Problem 26 also, let us also consider parallaxes. And because Tycho Brahe gave the moon a distance of 56\text{,}1 semi-diameters at its mean distance, therefore, using an eccentricity of 43\text{,}6 which I consider close to the truth, at the anomaly of this eclipse the moon will be 58\text{,}1 semi-diameters distant from earth. Consequently, the horizontal parallax is 58\text{,}4', while the sun's is 2° 58'\text{.} Therefore, the moon's parallax in excess of the sun's is about 56 at the bottom.

So, by the arcs IV found above, under the heading of 56 I take the parallaxes of latitude 25° 42'; 24° 16'; 24° 1'; 23° 42'. The southern visible latitudes subtracted from these, and the northern ones added, make the true latitudes 12° 39'; 16° 14'; 20° 24'; 25° 26'.

Then, using the complements of IV I take new headings from the parallactic table, 49° 50'; 50° 20'; 50° 36'; 50° 45'. And beneath these heads, using the visible distances of the moon from the montes alpes (or the approximate distances), that is, 60° 13'; 53° 36'; 48° 15'; 43° 47'. I take the parallaxes of longitude of the moon from the sun, 43° 15'; 41° 31'; 37° 45'; 37° 7'. I increase these by the moon's observed precessions (abound of the sun), and diminish them by the separations, because the eclipse is to the east: the results are the true longitudes of the moon before the sun, 50° 25'; 29° 10'; 15° 22'; 4° 43'. And thus the true conjunction followed the end of the eclipse. And because the hourly motion of the moon from the sun resulting from this is 29° 46' (Tycho gives 27° 56'), therefore, dividing the remaining 4° 43' by this shows about 10 minutes by which the true conjunction falls after the end of the eclipse, that is, at 8\text{th} 58\text{,}10' am.

But about this true longitude one must have a little doubt, and about the latitude not a little. For within half an hour, that true latitude can vary by barely 4 minutes; here, there are almost 13. And even if we use the last uncorrected inclinations of 59 and 30', there will still be 9 minutes in the variation of latitudes. And so I attribute this to the shaking of the hands and to the error-prone method of observing. Tycho's computation at the meridian of Hven, puts the moment of true conjunction back at an apparent time of 9° 29'. Therefore, the difference of meridians would be 1 degree, excessively small. Let us therefore consult the
observation at Urania, so that we may also at the same time have something more certain about the moon's latitude.\[\text{34a}\] At 6\textdegree\ 53\textprimeminutes, the eclipse was just noticed; at 6\textdegree\ 56\textprimeminutes, the whole sun was shining. The observers judged that the eclipse had begun at 6\textdegree\ 59\textprimeminutes, and ended at 8\textdegree\ 58\textprimeminutes, so that the duration would have been 28 minutes. Pictures were also added, the first of which represents the moon a little higher than the sun. And at 76\textdegree\ 15\textprimeminutes, the luminaries were in equal distance. At 8\textdegree\ 2\textprimeminutes, it was added, that the sun was one fifth eclipsed, and that the horns, head foremost, were turned equally downward. There is no more trustworthy notation. We shall consider the last first. One fifth of the diameter is 6 minutes. Therefore, the distance of the center is 25° 27', and that is right on the vertical. But the time of 8\textdegree\ 21\textprimeminutes (so that we might take this short cut too) shows the altitude of the sun, and the altitude shows the combined parallax, or the parallax of longitude-latitude,\[\text{34b}\] which, added to the moon's altitude, immediately establishes the true altitude. From this, afterwards, by the mediation of the angle of the vertical with the ecliptic, the true latitude and longitude of the moon is given immediately.\[\text{34c}\]

<table>
<thead>
<tr>
<th>Declination of the sun</th>
<th>Alt. of equat.</th>
<th>48</th>
<th>28</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>52</td>
<td>53</td>
<td>2055</td>
</tr>
<tr>
<td></td>
<td>2688</td>
<td>22254</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3354</td>
<td>elevation of the vertical was above the plane.</td>
<td></td>
</tr>
</tbody>
</table>

The distance from noon of 8\textdegree\ 21\textprimeminutes is 3° 39\textprimeminutes; the degrees, 54° 45'.

Therefore, as the whole sine is to 53,154, so is the sine of the complement, 57,715, to the sun's elevation above the plane at that time.

<table>
<thead>
<tr>
<th>53154</th>
<th>32</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>52</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>58</td>
<td>9985</td>
</tr>
<tr>
<td>63368</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Dropped from the sun to the plane

| 3984 |
| 29234 |

On the horizon: 58° 48'. Sun's latitude: 54° 42'. Complement: 16°.

To this distance of the sun from the earth, the visible distance of the centers is added (because the moon is lower), making 55° 43° 27'. This, under the heading of 56° in the parallactic table shows a parallax of 46° 16'. The visible distance subtracted from this leaves 20° 46', the true distance of centers.

But as the sine of the complement of the sun's altitude is to the sine of the angle 54° 45', so is the sine of the altitude of the equator to the sine of the

\[\text{34c}\] The observations are in TBOO XII pp. 8-15.

\[\text{34b}\] See Ch. 8 p. 325.

\[\text{34a}\] The computations below follow the same procedure as the one used earlier in this same problem. Kepler therefore thought it unnecessary to label each step. As before, multiplications are avoided by using the addition/subtraction methods.
angle between the ecliptic and the meridian, 33° 49', the complement of which is 56° 11'. Hence, under the headings of 20° and 40°, which is the distance of the centers, shows the moon's true latitude, 11° 33' north, the longitude 17° 14' before the moon.

I was expecting that the Tychoic observations would help Maestlin on the latitude. But I go away more uncertain than before. For Tycho's computation, founded upon lunar eclipses, shows a latitude of 16° 47' at the middle of the eclipse. At this moment, therefore, it ought to have become greater, because it is after the middle. Give me a greater eclipse at this moment, and I shall construct a greater latitude. And so you see how much the sense of vision deceives. Perhaps, too, for they wrote 1/2. For how could it happen that for Maestlin the eclipse, in restoring a quarter of the diameter, took no more than 25 minutes of time, for the Tychoic observers, in restoring a fifth, fully 35°? And so a third is 10°; previously it was 6°; from this we have constructed a latitude of about 15° 30' before it. I, for my part, drive these things home, though with some uncertainty, for no empty reason. For I wish to lay it out before astronomers, how frequent are the opportunities for being deluded, and, in turn, how great and how greatly desired is the usefulness, if diligence be applied in observations of this kind.

But because at Tübingen the time of the end has no evident error, let us also examine the observed end at Uraniborg. Let the moon's true latitude at the end be 17° north, and the end be exactly at 4° 10'. The degree of distance from the meridian is 4°. The right ascension of the meridian, 44° 55'. The culmination degree 25° 20' Gemini, with declination 23° 26'. This, subtracted from the altitude of the pole, 55° 54° 10', leaves 3° 28', the side M1, in the diagram of Prob. 24. And because 17° 55' is rising, the coexisting point on the ecliptic is therefore 26° 37' Virgo, and the mean ascendant is 26° 37' Gemini. Therefore, the side NAM is 1° 17', and NA by computation is 3° 26', showing a latitudinal parallel (under the heading of 56° of 30° 3'). Its complement, 57° 34', shows a heading of 45° 17'. Let the moon be exactly at 8° Leo, to the sight. It is therefore 41° 23' from the mean ascendant, which, from the headings of 27° and 17° shows a longitudinal parallel of 1° 15'. The latitude, in turn, is placed at 17° north, the latitudinal parallel 30° 3'. Therefore, the visible latitude is 13° 3' south. But the visible distance of centers is 30° 27'. Therefore, from the base and the side, the remaining side of visible longitude is 27° 31'. And the parallel of longitude is 31° 15'. When the former is subtracted from the latter, the remainder is 3° 44', the interval between the true positions of the sun and the moon at that time in Delawitt. If we use the same latitude at Tübingen, and subtract 23° 42' from the parallel of latitude, the visible latitude will be 6° 42' south. And the base of the small
triangle is 30° 27'. Therefore, the side of longitude is 29° 42'. But the parallax of longitude is 35° 7'. And so the difference of the moon's positions is 1° 31', which make 3 minutes of time. Therefore, at Tbingen, at 8° 51', 39' (with 3 minutes added at the end), the moon was at the place where it was at Uraniborg at 9° 0'. The difference of meridians is 8° 19 minutes, in degrees 2° 8'. Less, too, if at Hven the eclipse had ended at 8° 58'. And we do not differ much using this way: at Tbingen, the defect was seen equally [i.e., equal in size] at 7° 14′ and 7° 57′. Therefore, the middle of the eclipse is close to the middle time, that is, at 7° 30′. But at Hven it began at 6° 50′ ended at 6° 0′. Half the duration is 1° 5′. Therefore, the middle is at 7° 55′. But the parallax of longitude at Tbingen exceeds the Danish by 4°. Therefore, the eclipse at Tbingen really happens earlier by 8 minutes of time. And thus, when the difference of parallaxes is removed, the middle at Tbingen would be 7° 44′, the difference of meridians, 11° of time or 2° 45′ of the equator. And because at Hven the moon has 3° 44′ left to reach the sun, and thus almost 8° of time, the true conjunction is at an apparent time of 9° 8′. Tycho places it at 9° 2′ apparent, a difference of 6′ of time. 3 minutes of the moon's motion. From this, the certainty of the Tychonic computation at this place is evident. Below, in Prob. 52, there is more on this eclipse.

Problem 30
From the time and inclinations of the principal phases, correctly observed, to give the angle of visual latitude, or the angle which the moon's visual path makes with the ecliptic. Where this angle is portentous: and a warning about Problems, Reinhold, and their disciples.

This is the very thing we have just set out in both of the preceding examples, and we have set out several times. In the diagram of Prob. 27, given the moments of time and the inclinations, and the distance of centers $FA, FC$, that is, once the phase is designated, the moon's visual longitude from the sun, $FE, FD$, will not be unknown; likewise, the visual latitude $EA, DC$, by Problem 24.

Therefore, when the smaller latitude $EA$ is subtracted from the greater $EC$, if they were in the same direction; added, if they were in different directions; the side $BC$ will be obtained. But $BA$ and $DE$ are equal, and $DE$ is composed of the moon's amount of preceding [precessio] $EF$ and its excess [superatio] $FD$. Thus, with the sides with the right angle being given, the angle $BAC$ will not be unknown. So, in the eclipse of 1598, at the beginning the moon's latitude was 1° 57′ south. The moon was 29° 3′ in front of the sun. At the end, the latitude was either 17° 6′ or 19° 21′, and the moon...
was beyond the sun by either $26^\circ 40'$ or $25^\circ 4'$. Consequently, $AB$ is $54'$ and $BC$ is approximately $20'$, and $BAC$ is about $21'$.

In the other example, we shall proceed gradually, following, not the mistrusted visual latitude of the end, such as is taken from the example, but the one that followed from the presupposed right latitude. Therefore, when the moon was $7^\circ 10'$ before the sun, the visual latitude was $13^\circ 3'$ south. At the end, we have calculated that it ought to have been at a distance of $29^\circ 42'$ beyond the sun, with a visual latitude of $6^\circ 42'$. Subtracting the latter from the former gives $6^\circ 21'$, which will now be side $BC$. When in turn the longitudes are added, they make $BA$ $36^\circ 52'$. Therefore, angle $BAC$ is about $10'$. See another, mechanical way below, in Prob. 31.

Further, since the angle of true latitude is not greater than $5'$, it cannot be said in how great a confusion I was stuck for how long, while Podlerny and Reinhold presented me an absurd undertaking. For although Maestlin warned me in letters about the paralaxes which bring this about, still turning before my eyes were the method of Podlerny, Precepts 63, 64, and 65 of the Protonics, and the examples of Magini, all of whom treat paralaxes first, then at length proceed to this angle, or to the initial and final visual latitude; and this they seldom vary by more than 6 minutes from the beginning to the end of the greatest paralaxes. Arguments also entered in if the authorities were to neglect parallax, how could the beginnings and ends of eclipses come out right? Indeed, how could Podlerny use these correctly to construct the inclinations, to which he attributes a great deal in the foretelling of events, so much so that he asserts that the regions indicated are those towards which the inclinations are directed?

And so I was of the erroneous opinion for a very long time, that some miraculous inequality lay hidden in the moon's motion, not noticed by the authorities, and that this shows itself chiefly around eclipses, when the moon is passing across the ecliptic.

And thus I have thought it worth while to warn others as well, if any perchance will be sailing this sea, to beware of this ledge.

Such a person will therefore note first, that Podlerny openly asserts, in the last chapters of Book 6, that he was not going to follow the highest precision in this undertaking; and consequently judged that these inclinations in the directions of the winds should not be very precisely noted. But Reinhold, whom

---

92) Previously (at the end of Prob. 25, on p. 396 above), Kepler had given the latitude, correctly, as north. In Problem 32 below, he recomputes the angle $BAC$ using the correct latitude and finds it to be $15^\circ 12' 40''$. Again, evidence of haste.

Magini followed, does not give visual latitudes of the moon at the beginning and end despite his having called them "visual": note this well. For the examples of eclipses, and the more method of computing proposed in Prob. 25: and the discussion of Ptolemy himself on the variation of parallaxes, cry out in objection. Reinhold enters the table of the moon's latitude through the moon's motion, corresponding in minutes and seconds to the incidence and emergence, and from this asserts that he is referring to the visual latitudes. But that table of latitude was constructed from a constant angle of 5 degrees. 69 It would therefore be required that the parallax of latitude not vary from beginning to end, if the visual latitude could truly be taken by Reinhold's method. But Ptolemy himself asserts, and the tables bear witness, that the parallaxes of latitude change substantially at every moment. It is therefore not exactly the visual latitude, but quasi-visual, that is taken by Precept 64. For the parallax of latitude is indeed applied, but it is not the proper one, but one carried over from the mean to the ends of the eclipses: and it is not the true diagram of the eclipse that is constructed following Precept 65 of the Protretics, but one nearly true. For most of the time, the visible path of the moon is twice, thrice, or four times more oblique to the ecliptic; and consequently, the maximum darkening differs considerably from the instant of visual conjunction in longitude, and the times of incidence and emergence vary.

I would also add here an example from Walther's observations, which finally led me back to the path after straying for a very long time. In this, I learned that the same thing that happened, to my wonder, at conjunctions and at the nodes, is sometimes seen at the time between the months, when the moon is halved, thirty-seven degrees removed from the node.

Thus, Bernard Walther relates that on 12 January 1482, in the morning, two and a half hours before sunrise, the moon, now nearly bisected, was seen by him to move around towards Saturn, which it would also cover later, when it was around the meridian. The description is as follows. When the moon was at its last quadrature, or thereabouts, it was in any case darkened from the direction of the west. And when it was first noticed, which was about 2 1/2 hours before sunrise, Saturn was to the east, and, as it seemed, to the south of the moon, distant by two moons. Afterwards clouds had come in the way, so that I was unable to see the beginning [of the occultation], I judged it for certain, however, that the moon was going to overtake Saturn with its southern horn. However, after I saw Saturn again, it was distant on the diameter by two digits or thereabouts from the northern horn; and then, in turn, it began to reappear. Then it had now passed across the meridian. But at the time that I judged to be the middle of this eclipse, I took the moon's altitude, when it was almost on the meridian line, and found 32 degrees.

What I set down above, that the moon was at first going to overtake Saturn with the southern horn, DOES NOT APPEAR POSSIBLE, CONSIDERING THE...

69 This is the inclination of the moon's true path to the ecliptic. However, parallax intervenes to make the apparent angle different.
MOON'S PATH. But this was in fact very obviously apparent, that Saturn was at a distance of two degrees or thereabouts on the moon's diameter from the northern horn. He is worthy, and a well-known author, and the observation careful, and especially there is that, on account of which we have introduced this, that we should apply no less diligence in weighing it.

Therefore, let the moon's diameter or the section dividing the shining part from the dark, be \( AL \), and let \( E \) be the darkened part at the western side of the ecliptic (or the great circle between the sun and the moon), \( D \) at the eastern side towards the sun. Let a line \( AC \) be drawn at right angles to \( LA \) tangent to the moon at \( A \), and let it be \( AC \). Since, then, the moon is a little more than a quadrant away from the sun, a great circle through \( A \) extended will pass through the poles both of the ecliptic and of the great circle drawn through the centers of the luminaries, and will cut both at right angles. And the arcs of those circles about this place of a quadrant will be nearly parallel, but \( AL \) also cuts line \( AC \) at right angles. Therefore, \( AC \) is in a way parallel to both the ecliptic and to the circle through the luminaries. Let \( AC \) be made twice \( AL \), as account with Saturn's having been seen at the beginning at two moons' distance from the moon, and let the star Saturn be placed at \( C \), which, in this place, will really be seen as south of the moon at \( AL \) (because \( AC \) is parallel to the ecliptic) and in any event in such a position that if the moon's center were to approach the sun directly towards \( D \), it would appear that it would catch Saturn at \( C \) exactly with its horn \( A \). Now, when Saturn was uncovered, because it was then seen to be two digits from the northern horn in the moon's diameter, let the diameter \( AB \) be divided into 6 parts, and let \( J F \) be a sixth, or two digits, and through \( F \) let a straight line be drawn at right angles to \( LA \), cutting the circumference at \( G \) and \( H \), and let a line equal to \( GH \) be further extended on \( AC \) and let this be \( C \), and through points \( C \) and \( F \) let the diagram or circle of the moon be described. For since the moon, as before, is situated at quadrature, the diameters or boundaries of the luminous part in the

---

49

---

49 Emphasis added by Kepler. He also made some minor changes, such as altering "vel citta" (or beyond) to "vel circa" (or thereabouts). The quotation is from Jonas de Reginmontano, "Scriptor de trocutio" (Nürnberg 1544) 29-50 (Observationes facie per dactylarum visum Bernardum Walthersiam).

50 The "circles" are the ecliptic and a great circle through the moon and the sun, which is within a few degrees of the ecliptic. These two circles intersect at the sun, and since the moon is about a quadrant from the sun, the parts of these circles in that region are at their greatest distance from each other, and are nearly parallel.
two diagrams will remain approximately parallel, and will be at right angles to the same CA. And thus will this observation of Wajtter have been delineated geometrically.

Now let $CI$ be bisected at $K$, and let $K, F$ be joined. And let the diameter $LA$ in some way be given, a measure, such as $200,000$. If $F$ will be the sixth part, that is, $33,333$. Consequently, $FA$ is $186,667$, and $FH$ or $KC$, $74,528$, while $CA$ is $400,000$. And the whole $KA$ is therefore $474,528$. But as $KA$ is to the whole sine, so is $AF$ to the tangent of angle $FKA$. Therefore, angle $FKA$ is $19'21'$. Namely, the angle that the visual path of point $F$, which is $FK$, makes with the ecliptic, to which $KA$ is nearly parallel here. Here, then, although the moon is not carried by such a precipitous motion in latitude as where right at the nodes, still, it is simultaneously tending towards the descending node while Sagittarius and Capricorn and Aquarius are rising through which signs the angle between the ecliptic and the horizon is decreasing, and in turn, the lunate sensible parallelism is increasing. As a result of these things, by both its true motion and its visual semblance it is simultaneously carried to the south. Therefore, something happened even to the experience astronomer' rather, of such a sort that not only seemed to him a marvel, but even led him to doubt the usefulness of his own eyes.

Problem 31

Whether it is possible for the beginning of some solar eclipse to decline towards the east, and for the end of another to decline to the west; but for the beginning of an eclipse the moon to be in the west, and at another time for the end to be in the east.

This is among the paradigms proposed by Ptolemy. For it befits the sun to be eclipsed by the ordinary way from the west in the beginning, and to be made full from the east; and it befits the moon, on the contrary, to begin from the east, and to end from the west. Let us begin from the moon. About center A, let the circle of the earth’s shadow $BCDE$ be described, through A let the straight line $FG$ be drawn representing the vertex. Through the same let the arc of the ecliptic $HI$ closed to the west also be drawn, so that $FAH$ may be the angle between the ecliptic and the vertical. And let $KL$ be drawn at right angles to $HJ$. Should the center of the moon be at $K$ or $L$, it will be the moment of true conjunction. Then, accordingly, let the sum of the radii of the moon and the shadow equal the moon’s true latitude, decreasing southern at $K$, so that the moon’s path may be $KM$ towards the ecliptic; increasing northern at $M$.\[\text{\textsuperscript{49}}\]
L, so that the moon's path may be LN away from the ecliptic. In both cases, then, some small part of the moon will be eclipsed (or in its particular matter as well, the authorities require this slight correction), for the moon, a little past K and a little before L, becomes closer to the center A because of the obliquity of the moon's path with respect to the eclipse; and in that point of its path upon which a perpendicular from A falls, the eclipse is greatest. And, for the sake of a measure as the scint, of 5 degrees (the maximum latitude), 10°32' is to the radius, so the sum of the radii, which we shall take as 60 minutes, to the distance of the centers at a maximum eclipse, 59°46'. Thus 14° fall into eclipse, a small part, certainly, not as much as a hundredth. Nevertheless, we are following the highest points here. Nothing prevents the occurrence of the same thing we are demonstrating here, even when the moon at K has taken a little of the shadow. Now the beginning of the eclipse will be at K, the end at L, and the point of contact of the circle of C, or the index of the eclipse, will verge to the west at the right, as well as the whole duration; while the end E will verge towards the east. Also, both can take place on an eastern arc. For let OP be an arc close to rising, and FAP be the angle, while QR is the perpendicular, and the moon at Q is northern descending, its path being SQ, while at R it is southern, likewise descending, or moving away from the node, by the path RT. Then, in the former case, the eclipse B at the beginning verges to the west at the right, but the final defect D will verge to the east.

Saca an eclipse of the moon was seen most recently on 8/18 November 1603. Since this began about 65 degrees from the zenith, counting to the left, it ended with the shadow not quite reaching the zenith, but still verging to the left, towards the east. For the eclipse was in the southeastern quadrant, with the moon descending on the meridian.

It began ten minutes after the right shoulder of Aquarius culminated, while already half an hour before, the light was observed to be becoming palmed at this very part, or somewhat to the lower left. And when the sun was at 25°35' Virgo. The time is thus demonstrated to be the 5th 21'0.

It ended three minutes after the head of Andromeda culminated. Therefore, the time was 8°11'0'; the duration, 15°50'3. The computation of Tycho gives 25°39', in fact considered that about the middle less than a quarter was missing, but others argued that more than a quarter was hidden. Of the circumference, certainly less than 1, more than 1, was considered to be gone. Those things are nevertheless in agreement; that the defect was smaller, and the duration shorter.

57 a Aquarii.
58 Albetera, a Andromedae.
than in the computation, and the shadow did not reach to the zenith. The middle is 7° 16′, which very minute the computation of Tycho shows at the meridian of Hven for the true conjunction in longitude, which differs somewhat from the middle of the eclipse.

The darkened circumference was seen, while the very bright Pleiades, distant by a few degrees, were hardly seen; so brightly was the moon illuminated, even in the shadow—which you should ascribe to the above.

As for the computation, it is fully consistent with this phenomenon. For at 85° 17′, the angle between the vertical and the ecliptic is 52° 48′, whose sine, multiplied by the sum of the radii, 60° 16′ (for Tycho makes the radius of the shadow 44° 6′, of which in its place), shows the latitude at this moment, 53° 40′; the amount it would have been had the shadow ended exactly at the zenith. Now, since it declined slightly to the left, the angle between the ecliptic and the line passing through the centers is also slightly larger. Let it be larger by 5 degrees, which is the amount of the declining to the left. Therefore, the latitude at this moment is 55° 48′. At the middle, then, an hour earlier, it is about 55° 20′, and subtracting 44° 6′ from this leaves 9° 14′ of the moon’s body in the shadow, a little more than 3 digits. Thus, therefore, the computation also requires this phenomenon. Tycho Brahe also saw something closely similar.

For on 19 Oct. 1594, at 55° 56′ in the morning, the moon began to be darkened at its top edge, or, as the picture was captured in the other observatory, a little to the right, although this eclipse appeared quite large to those farther to the west. The moon therefore began to be darkened from the west, and was again made full from the same side (although beneath the earth, at Hven).

But the account and the amount are more obvious in solar eclipses, because of the parallaxes. For if you will adopt an angle of 20 degrees for the moon’s visual path, as it had more than once come out to be, the secant, 186.418, divided into the sum of the radii—let this be taken as only 30—makes it 2.84, the remaining 1·1 units is hardly different from a digit in quantity. And so, when at the very instant of conjunction, the visual sum of the radii is equal to the visual latitude of the moon, it can still be eclipsed by a digit.

It is, moreover, sufficient for the furthering of the demonstration to recount again the extremes of the occasion upon which this happens to occur. So again, as in the lunar eclipses, and as much as for the moon’s true motion, one needs the moon’s latitude to be either northern decreasing, and the eclipse in the east, or southern decreasing, and the eclipse in the west, if the eclipse of the sun is to begin from the east. Again, if it is to end from the west, the moon’s latitude should be either northern increasing, the eclipse being in the west, or southern increasing, the eclipse being in the east.

89 See JSXO XIV pp. 317–8.
90 Here Kepler refers to the following exercise.

To p. 413. Therefore, note that the true conjunction in longitude is one thing, the apparent conjunction in longitude is another, and the apparent conjunction according to nearness is yet another. For the moon, at the same visual longitude in the sun, is nevertheless not at the greatest visual nearness, unless the eclipse is exactly central.
But as far as concerns parallaxes, at whose charge is the governance of the case, those things demonstrated in Chapter Nine must be considered, and Copernicus's table of the the angles of the horizon must be inspected. Thus when Arres is rising, the angles begin to increase, up to Libra, and when Cancer is rising they increase maximally. At that time, as a consequence, the parallels of latitude is decreasing maximally, and the moon, in whatever sign it is above the horizon, is carried visibly to the north more than it is by its true motion to the south, and much more if it is ascending in its true motion. Then, as a consequence, when the sun stands to the west in a small southern eclipse, what was proposed can possibly happen, that the sun begins to be eclipsed from the east. On the contrary, a small northern eclipse starting to the east will be able to be lifted from the west.

On the contrary, when Capricorn is rising, from Libra to Arres, the angle of the horizon is decreased, the parallax is increased, and the moon in its visual motion is carried to the south. Therefore, northern eclipses that start in the eastern quadrant will begin to be eclipsed from the east; southern eclipses in the eastern quadrant will end from the west. Even so, this cause is so obvious that it is valid almost at the meridian itself, with the assistance of the others. For at the meridian the moon is driven swiftly backwards by its visual illusion. And so it appears to ascend to the north by almost as much, at least in suitable signs.

On 12 or 22 July 1599, in the morning, right at sunrise, a slight eclipse was seen by Tycho at Prague, almost at the vertex of the solar body. Now the moon, though with its true motion is descended towards the downward-leading node, was on the contrary carried by a visual illusion mostly to the north, with the parallaxes of latitude decreasing (for Leo was rising). Therefore, the inclination was a minimum, and the end looked towards the east. So, in southern latitudes, where the eclipse is small, the eclipse must have stood from the west from beginning to end. On 20 May 1593, at Zerbst, the sun was observed to be eclipsed by two digits from below, at the beginning, the eclipse verged slightly to the left, at the end more so. It therefore began from the east, when the moon in its motion of latitude was struggling up towards the north. At Hren it was not seen to be eclipsed at all, as the computation of time shows at 20' 51'.

On 16 February 1588 at Hren, at 1 h 32 after moon, the sun began to be eclipsed, it ended at 2h 51'. At the beginning, the eclipse is depicted to decline from the vertical at about 96° to the right, which, while it was ascending, still did not come to the vertex of the solar body. For at the end, the inclination to the right and west is depicted as still 12° of 55 degrees. Leo was rising. The moon's true northern latitude was decreasing, but to the sight it was increased more, with the parallaxes of latitude diminished. On the other hand,
through the great increments of longitudinal parallax for both the heading in the table and the fractional part of the heading were increasing, for the former as the angle of the horizon was increasing, and the latter because the moon was close to the nonagesimal, the moon was greatly curtailed in its motion from west to east.

But that no doubt may remain, I shall here compute the inclinations, such as ought to have been seen at the beginning and the end. The sun was at 7° 17' Poles | 7° 20' Joices. The right ascension was 339° 0' | 339° 3'. The distance in time from the meridian was 23° 0' | 42° 45'. Therefore, the right ascension of the meridian was 2° 0' | 21° 48'. The degrees on the meridian are 2° 11' Arietis | 25° 34' Arietis, the northern declination of which is 0° 52' | 9° 12'. The altitude of the pole is 35° 55'. Therefore, the altitude of the culminating points is 34° 57' | 43° 17'. But 1° 56' Leo | 15° 6' Leo are rising. And the quadrant,115 of these is on the nonagesimal. Therefore, between the nonagesimal and the culminating point is 29° 35' | 21° 32'. Hence, the distance of the nonagesimal from the zenith is 48° 45' | 42° 31'. Under the headings of 37° 20', the moon's parallax from the sun, they select the parallaxes of latitude, 43° 5' | 38° 44'. But their complements, 41° 15' | 42° 29', elicit the headings of 37° 49' | 42° 16'. And because the sun or moon is distant from the nonagesimal by about 54° 39' | 67° 86', by these, order the headings found, the parallaxes of longitude are elicited, 51° 11' | 59° 6'. But through the same distances of the sun from the nonagesimal, the angles between the ecliptic and the vertical come out to be 54° 24' | 44° 44'. Now, let the sun's radius be taken as 15° 20'; the moon's, 135° 58'; the sum, 31° 18', the base of the small triangle. From Tycho's computation, let the latitude also be taken as 1° 8' | 1° 5'. When the parallaxes of latitude are subtracted from these, the residuals are 24° 35' | 26° 16', one side of the small triangle about the right angle. From the base and the side given the angles opposite the latitude, 52° 45' | 57° 3'. And, so as to make the agreement evident, from the same procedure come the sides of the longitude, 19° 0' | 17° 21', the former before, the latter after the sun. The former, subtracted from the parallax of longitude, the latter added, each to its own, give the true longitudes beyond the sun, 12° 1' | 8° 27'. Therefore, in 1° 14' the moon's true motion is 14° 26', so the hourly motion is nearly 54', slightly greater than is correct, because, just as the beginning is always observed later than is correct, the end is observed earlier than the true. But to the angles, let AB be the vertical, CB the ecliptic. Angle CBA is 54° 24' | 44° 44'; let BE be the arc through the centers; and EBD is 52° 45' | 57° 3'. Therefore, ABD at the beginning of the eclipse is 125° 35', as a result of which ABE is 72° 51'. But at the end, because ABC is 44° 44' and the greater EBC is

115 This is astrological terminology, meaning that the nonagesimal is 0' removed from the rising degree.
57° 3', therefore $EBA$ is the excess, 12° 19', $E$ still standing to the right of the vertical, or towards the west, which was what was to be sought out through the computation. But that the first angle comes out twice as great as mine, which was taken from Tycho's diagram, I think happened because of an erroneous picture, or because in measuring I had by mistake said the simple angle instead of the double.

A completely similar eclipse was seen on 23 Sept. or 3 Oct. 1595. For Maestlin, in the dissertation on eclipses published in 1596, in thesis 53, describes it thus: "In the beginning, a little past noon, the edge of the eclipsed sun declined from the vertical, not to the west, but 9° degrees to the east; it was, moreover, eclipsed by 2 digits and a half, no more, by the testimony of careful observation." This was at Tübingen.

At Strassburg, as I have found in Tycho's observations, it was noted by a certain observer to have begun before eleven o'clock, finished at one o'clock precisely. All the diagrams show that the shadow always declined to the left from the vertex.

I also observed the same eclipse at Graz: it began for me exactly at the vertex, when the sun's distance from the zenith was 51° 2', by a small wooden quadrant. When there appeared to me to be three digits missing on the diameter, the sun's distance was 55° from the zenith. A little later, the eclipse appeared to decline by 15 degrees. The moon's diameter appeared to be less than the solar on the sky. At Ulm, they observed its end at 26° go. Four digits were eclipsed.

In this eclipse, the true and the visual motions of the moon were in the same direction. For the moon was revolting towards the downward-bearing node. And with Sagittarius and Capricorn rising, the angle of the horizon was decreasing very rapidly, and the parallax of latitude was increasing.

And because it was nowhere observed under adequate circumstances, it will be in vain to call it back to computation. Only from these given, we shall write the occasion of also solving Problem 30 differently. For the angle is great.

About center A let the circle $OC$ of the sun's body be described, on which let $OA$ represent the vertical, $AC$ the horizon, $AB$ the ecliptic, and let $OAB$ be 57 degrees. For if $OA$ were the meridian, it would cut $AB$ at A, the 10th degree of Libra (which is the sun's position) at an angle of 66° 48'. But now $OA$ is beyond the meridian towards the

\[108 \text{ Cf. TBOO XII pp. 366-72.}\]
nonagesimal, and is therefore a little brighter. Because for Maxelin the beginning inclining 9 degrees to the east, therefore, let OP be 9°, and let a straight line be sent out from A through P, while from P let the moon's diameter PM be extended, and about center M let the circle of the moon's diameter PG be drawn. And because the maximum obscuration was $\frac{2}{3}$ digits, let the radius AQ be divided so that $QI = \frac{2}{3}, IA = \frac{1}{2},$ and about center A with radius AI let a small circle be delineated, to which a straight line is tangent that also is tangent to the circle PG, and let the point of tangency be I. Then let AI be extended, and from I let the moon's radius FN be extended, let MN be joined and extended to the common intersection with the ecliptic B3: let this be D. MDA will be the visual angle of the moon's path about 27 degrees, as mechanics shows. For because in the beginning the moon's center is certainly at M, it will therefore go either on a higher path MN, and will therefore not conceal $\frac{2}{3}$ digits, or a lower one, and will therefore conceal more. Therefore, it will seem to go only on path MND.

From my observation, the angle comes out a little smaller, for as it is only the visible angle, it is thus different in different places. And for me, the altitude of the pole was smaller, and also the east was closer. For in Prob. 27 above, there was 4° 30' of longitude between Hven and Græ. Either, in Prob. 29, between Tübingen and Hven was 2° 45'. Therefore, between Tübingen and Græ is about 7° 15', approximately.

Since for me, then, it began at the vertex, let AO be extended, and let OK be the moon's radius, OH its circle, RE three digits, EH tangent to the moon's body at E, EL the moon's radius, and let the points KL be joined, and let these and the ecliptic be extended until they meet at F. They will make an angle of about 20°. However, I am not sure whether the defect might not have come out greater. For clouds covered half of the time, and they concealed both ends, and perhaps also the maximum defect.

**Problem 32**

Whether the visual path of the moon's center is a straight line; that is, whether from observation of the beginning and end the quantity of the maximum defect may be obtained in the usual way without error. Here is treated the correction to be appended to Problem 30, and the usual precepts about forming a plat of an eclipse.

Then, as is sufficiently clear, that a space so small as the little amount that the moon makes up in three hours is equivalent to a straight line, even though a curve is drawn around the center of the earth. In fact, that is not in question now, but rather, whether the moon's center is seen on the same great circle for the whole time of the duration of the eclipse? I say that it is not necessary that this always happen. For the path can become curved. For since the true latitude and the parallax in longitude and latitude combine for the moon's path—though as regards the true latitude, about the nodes it is nearly proportional to the spaces traversed, and so this does not affect the straight line at all. For in equal times it is very nearly uniformly borne off to the sides, and also uniformly carried forward to the ear. But because, by a legitimate cause, the parallaxes of longitude vary nonuniformly, and more rapidly in proportion to their nearness
to the novaseminal, it can as a result happen that the true latitude, increasing uniformly is applied to the nonuniformly increasing visual longitudes, and a bending o’ the path results from this. This happens much more so as a result of the parallaxes of latitude. For the signs of the zodiac rise nonuniformly, and over three hours it can happen that the angle of the horizon increases slowly at the beginning, quickly at the end; and therefore (following what was shown in ch. 9) the parallaxes of latitude will vary slowly at the beginning, quickly at the end (and likewise the observed latitude), whence again it happens that the moon’s composite path becomes curviform. However that may be, the slight difference that exists as a result does not appear perceptible, except around the meridian and the nonaseminal. Consequently, one must proceed using examples.

In the eclipse of 1598, I was convinced that about the middle of the darkening, as if at a single moment of time, when the sun was penetrating the clouds, I saw a rather large defect, i.e., the remaining horn being exceedingly thinned. It is in fact impossible that I saw a defect of this quantity, if we should state that the moon’s visual path is straight, and should make a diagram based on this: much less so, if those things be taken into account which Tycho had annotated for us above in this observed eclipse. For it ought to have been greater at Hamburg and in Denmark than in Styria, because it was to the north. Further, the horns were in front on the ray upturned in the sky, but for a short time inclined by an estimated 10 degrees. And Lessenow had almost persuaded me, in stating that he had seen it very close to central at Torgau, When I wrote to Maestlin in 1598 on this subject, he, while not asserting anything, ascribed everything that I had truly observed to a doubtful observation of the beginning and to the parallaxes, giving support rather vividly to this very problem. However that may be, this too is pertinent, that, since this image was seen fleetingy, I might have set the paper askew on the ray, whence the elliptical section of the cone produced the image of a longer, and thus thinner, horn. We shall nevertheless consult the computation. In Problem 25, the moon’s visual path was 55° 43’; the latitude at the beginning, 1° 57’ north; at the end, 17° 6’; the difference is 15° 9’; In the diagram of Prob. 27, duplicated here, BC is 15° 9’, BA is 55° 43’; therefore, BAC is 15° 12’ 40’’, and AC 27° 44’ approximately, while AG is 28° 52’ and AF 31° 40’’. Hence, FAG is 24° 36’ 35’’, and the distance of the centers FG at maximum darkening is 13° 1’. But the radius of the moon is 16° 22’’. Therefore, it goes past the sun’s center by 3° 21’’.

---

157 This is the eclipse whose observation Kepler describes in Problem 23.

158 Kepler to Maestlin, 15 March 1598, no. 89 in JKGW XIII p. 179; Maestlin to Kepler, 2/12 May 1598, no. 97 in JKGW XIII p. 208.
which, added to the sun’s radius of 15° 18’, makes 18° 39’, of which 30° 35° are 12 digits. Therefore no more than 7 ½ digits would have been eclipsed. Even less by the computed latitude of the end, and this in the case where we assume that some imperceptible amount was eclipsed at the beginning. But if we should say that about one digit was eclipsed, the centers will be brought a little closer at the middle, by about 12’.

Let me then make a test through parallaxes at maximum darkening, the point of which is in a somewhat different place than at the point of visual conjunction. A line perpendicular to DE, set up from F, cuts AC at H. And H is the visual place of conjunction, while G is that of maximum darkening, because FH is longer than FG, since the angle H is acute, G right. Nor should it trouble you that the authorities measure the quantity of greatest darkening in H, for they do it inaccurately because it matters little. G, however, is the midpoint between C, A, inasmuch as FC, FA are taken as equal. On the supposition that the visual motion is also proportional to the time, the time of greatest darkening would have been 11½ 41¼’, or a little before. In 1 hour 14’, the moon’s motion is 40° 29’, which corresponds a change in latitude of 5° 24’. But at the beginning, the true latitude was 51° 19’, as in Prob. 26. Therefore, it is now 54° 43’. The sun’s right ascension is 347° 51’. Subtract 4° 45’, which make 19 minutes’ distance from the meridian, and there remains 134° 6’, the right ascension of the meridian, with which it divides the sky at 11° 40’ Piscos. Its declination is 7° 12’ 45”; I add this to the elevation of the pole, 47° 2’, MV will be 54° 14’ 45”. The rising sign is 9° 33’ Cancer. Hence, MN is 27° 53’. And VN is 48° 38’, showing, under the [parallactic table] headings of 59° 30’, a latitudinal parallax of 44° 40’, which, subtracted from the true latitude, 54° 43’, leaves the visual latitude, 10° 3’. The complement of VN, 41° 22’, shows a heading of 39° 20’. And because the moon is visually closest to the sun at 16° 46’ Piscos, NS will be 22° 47’, which from the headings found shows a longitudinal parallax of 15° 14’.

But the moon’s true motion in 20° 28’ was found to be 1° 19° 15’; at the half, therefore, it was 39° 37’. At the beginning, though, in its true motion it was 25° 26’ before the sun. Therefore, it is now 14° 11’ after the sun, truly, and by parallax visibly 1° 14’ before the sun. And in the diagram repeated here, FGH and CRB are similar triangles, with the result that HFG is also 15° 12’.

108 Although Franz Hammer (DKGW II p. 461) notes an error in the computation of the rising point, stating that it should have been 18° 39’ Cancer, my recomputation, using Kepler’s right ascension and solar elevation, shows a rising degree of 9° 36’ Cancer, not much different from Kepler’s. Hammer cites an article on this subject by Lalinde, titled “Enterre de Kepler. Sur la courbure de l’orbite apparente de la Lune,” in Connaissance des Temps, VF, Année (1798) pp. 239–243. Lalinde claims that the supposed “curvature” of the path is the result of Kepler’s errors.
thus, where $FG$ is 10$^\circ$ 3$'$ (if you suppose that the moon is now a point $G$ of maximum darkening, with this visible latitude, the moon's longitudinal distance $GH$ from the visible point of conjunction would be 2 minutes 48$''$. But because the moon is only 1$'$ 14$''$ before $H$, it has therefore now gone past the point $G$ of maximum darkening by 3 minutes of time. And since the visible latitude is increasing, 3 minutes of time earlier it was less than 10$^\circ$ 3$. And so the difference between the visual latitudes obtained from the computation of parallaxes, and that obtained from the drawing of the beginning and end and the middle proportional [to them], is more than 3 minutes, or 1$''$. This eclips was therefore more than 8$''$, digits. And thus it has been proven that this is sufficiently evident in eclipses occurring at midday.

As a result, in Problem 27 above, if you leave this to the Tychoan observers, that the error of the eyes is equal, not only at the beginning and end, but also right at the middle, there will nonetheless be a cause for 9$''$ digits having appeared, more, that is, than is derived from the proportion of the beginnity and the end.

In the eclipse of 1590,\textsuperscript{130} when it was near maximum darkening, the distance of the centers was to the radius of the sun (including the "fringe") as 59 to 88, and where the sun is 24 the moon is 23. Therefore, where the distance of centers is 59 the moon's diameter is 84$. And so in these units, the sum of the radius is 172$. Therefore following the example of Prob. 28 and 29, as 172$^2$ is to 59 so is 30$^\circ$ 27$''$ to 10$^\circ$ 25$, the required distance of the centers is the usual units. And because the moon's radius is 15$^\circ$ 25$, the excess of 5$''$ reaches beyond the half of the sun. There were accordingly eight digits eclipsed. We shall see what follows if we proceed proportionally, as if the moon's path were a straight line.

Since, then, a semidiameter was seen to be eclipsed at two moments, one before, the other after the middle, and since the distance of the centers was 14$^\circ$ 5$, and the longitude accordingly was 7$^\circ$ 10$''$ before the sun, and 12$^\circ$ 21$''$ after the sun, therefore, the sum or visual path was 19$^\circ$ 31$. The latitude for the former was 17$^\circ$ 3$, for the latter 8$^\circ$ 22$, south, so that the difference was 4$^\circ$ 43$. Consequently, $AC$ is 18$^\circ$ 7$, but $AG$ is 4$^\circ$ 43$, and the distance $GF$ of the centers is 11$^\circ$ 52$. The difference from the observations is 1$^\circ$ 27$. As a result, the digits would have been only 7$. And so, if these things hold true that have been established by the observations, in this eclipse as well the amount eclipsed was perceptibly greater than that which was derived from the diagram and the moments of equal eclipse at the middle.

Further, so that this may be established in doubtful instances, and so that

\textsuperscript{130} 1600 XII pp 8-10, and other observations mentioned in Problems 28-9 above.
the reader may at once see almost all the things I have hitherto presented in a 
crippled way for various reasons—these things are not so because of the difficulty 
of observation, or because they relate to Plato’s *Republic*, but, once care is taken 
they can be brought to complete perfection—I now add two very clear instances 
of the two most recent solar eclipses, by which I shall display an example of 
almost the entire art presented in these 32 Problems.

**First Example**

In 1600, on June 30 or July 1, at Graz in Styria, I was outdoors with the 
wooden instrument whose complements description appears in Prob. 1, with a sur-
rounding curtain. And since the instrument had not yet been divided into its 
scales, I carved in 15 marks with a knife, and identified them in sequence with 
numbers; and whatever was seen at the individual marks on the panel, I recorded 
separately on paper; but I shall show you most concisely the series of observa-
tions, described from the paper slip. For I labor the "additions-subtractions," lack-
ing artfulness and unsatisfactory, that I wish had not been Ptolemy’s principal ob-
servations, 110 in contrast, write these things as if I considered myself per-
suaded that they were going to fall into the hands of posterity. Consequently, so 
that a free and open judgment may be left to it either way, nothing will either be 
contrived or concealed.

<table>
<thead>
<tr>
<th>Districts on the</th>
<th>Angles of the circle within the</th>
<th>Digits of the</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shadow.</td>
<td>circle through the center, numbered</td>
<td>circle through the center up to the index</td>
</tr>
<tr>
<td></td>
<td>(circle should oppose the shadow on the ray</td>
<td></td>
</tr>
<tr>
<td></td>
<td>therefore, on the same side as the moon in the sky)</td>
<td></td>
</tr>
<tr>
<td>1. 62(^\circ) or 72(^\circ)</td>
<td>3 ( \frac{1}{2} )</td>
<td>8</td>
</tr>
<tr>
<td>2. 58</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>3. 52(^\circ)</td>
<td>46</td>
<td>5</td>
</tr>
<tr>
<td>4. 34</td>
<td>more than 4</td>
<td>6</td>
</tr>
<tr>
<td>5. 13 approximately</td>
<td>Towards the east.</td>
<td>approximately,</td>
</tr>
<tr>
<td>7. 4</td>
<td>8. 22 approximately</td>
<td>Towards the west.</td>
</tr>
<tr>
<td>9. 32</td>
<td>Very pale and blurry.</td>
<td>approximately,</td>
</tr>
<tr>
<td>10. 47</td>
<td>11. 59</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>less than 5</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>approximately</td>
<td></td>
</tr>
</tbody>
</table>

110 Kepler here uses the Greek phrase τὰ προσθέσεις καὶ υποστάσεις to describe 
Ptolemy’s “corrections.” This refers to Ptolemy’s practice of *suppressing* the actual 
observations and presenting only adjusted positions based on them.
Notes

Angles of the vertical with the
circle through the centres

13. 64. 15’, precisely
14. 80° more than 2

by estimate.
not by measurement.

Since now (on the ray) I could no longer perceive whether any part was
exposed or not (thought it was still eclipsed by the moon), and the puller alone
was responsible, the vertical angle was not yet 90°.
15. A little beyond the end, it was not yet 90° from the vertex.

Note on these notes: indicators of the azimuths and altitudes: at the end,
the crosspiece or horizontal rule was hanging down, and was giving too great an
altitude.

Then all the configurations through the cracks in the roof had this form on
the plane of the horizon. 107

On Azimuths and Altitudes at these 15 Moments

When the observation was finished, my azimuthal quadrangle 108 was di-
vided, on one side, set up on the meridian, into 2000 units, and on the other,
which looked east and west, into 1,200 of the same units in each direction. In
the preceding days I had now, by estimation pointed this rectangle at the meri-
dian, and once it was recorded thus, I sought in the usual way, by altitudes of
the sun before and after noon, equal on both sides, the one meridian of the azimuthal
plane, and found that the meridian of the instrument inclined to the west, by I
degree 4 minutes. And when that arc was subtracted from the arcs found by the
numbering of divisions, the remaining were the true azimuths, or the directions

---

107 In the 1604 edition, the diagram accompanying these words had, at the top, the letter
“z,” followed by the sun symbol, and at the bottom, the word “ad,” followed by the sun
symbol. However, since the woodcut was not very clear, both later editions misunder-
stood Kepler’s meaning and, reading the “z” in “ad,” as spurious, put “A [sun sign]”
at the top and “[sun sign]” at the bottom, as if the letters were merely labels for the
ends of the ellipse.

108 This is the base of the instrument, upon which the crosspiece pivots, carrying the rest
of the instrument.
on the horizon of the verticals passing through the sun. And so that I might have no doubt about the projection of the meridian found, I sought the altitude of the pole from the distance of the verticals that showed the same altitude of the sun, and from both the sun's declination and the altitude being known. In the triangle between the pole O, the zenith V, the sun S, SY was given from the sun's altitude, NP from the sun's declination, and SYP from half the sum of the two azimuths. As a result, the complement VP of the pole's altitude was also given, which came out to 47° 16', which of course is for the setting up of this operation and of the wooden instrument, very closely approaching the true altitude of 47° 2'. And so the meridian was certain. The following is the sequence of numerical measures.

<table>
<thead>
<tr>
<th>Moment of time</th>
<th>Reckoning ordered on the line towards the west</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>794</td>
</tr>
<tr>
<td>2</td>
<td>1071</td>
</tr>
</tbody>
</table>

However, domestic affairs hindered me from dividing the crosspiece, the post, and the rule at the same time, and counting the divisions intercepted at the moments of observation. For only a few days before, I had returned from Bohemia, and I had meanwhile gotten the instrument ready, and now I was immediately preparing myself for a new journey to Prague with my family. And so when I returned to Syria in 1601, I carefully and at leisure examined the instrument, which I had left behind, and which was found undamaged. And so, while the post or perpendicular had 3,640 units, other parts had the following:

- Image
<table>
<thead>
<tr>
<th>Hyperb.</th>
<th>Beta or</th>
<th>This Yields</th>
<th>Corresponding</th>
<th>On the</th>
<th>These Altitudes</th>
<th>The above Azimuths T</th>
<th>Over by</th>
</tr>
</thead>
<tbody>
<tr>
<td>degree</td>
<td>Longitude</td>
<td>a Propen-</td>
<td>Distance of Sun</td>
<td>Bear</td>
<td>Give at</td>
<td>Turn 90° or 270°</td>
<td></td>
</tr>
<tr>
<td>or Rad.</td>
<td></td>
<td>dicator</td>
<td>from the Zenith</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>at the Hyper.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>504</td>
<td>2450</td>
<td>5040</td>
<td>25.56</td>
<td>25.56</td>
<td>20.18</td>
<td>17.° +</td>
</tr>
<tr>
<td>2</td>
<td>679</td>
<td>2615</td>
<td>5041</td>
<td>27.27</td>
<td>27.27</td>
<td>25.91</td>
<td>9°</td>
</tr>
<tr>
<td>3</td>
<td>653</td>
<td>2675</td>
<td>5039</td>
<td>27.30</td>
<td>27.30</td>
<td>25.33</td>
<td>7°</td>
</tr>
<tr>
<td>4</td>
<td>570</td>
<td>2750</td>
<td>5056</td>
<td>28.36</td>
<td>28.36</td>
<td>26.35</td>
<td>37° +</td>
</tr>
<tr>
<td>5</td>
<td>581</td>
<td>2896</td>
<td>5056</td>
<td>29.33</td>
<td>29.33</td>
<td>41.8</td>
<td>6°</td>
</tr>
<tr>
<td>6</td>
<td>589</td>
<td>3173</td>
<td>5040</td>
<td>32.32</td>
<td>32.32</td>
<td>46.18</td>
<td>2°</td>
</tr>
<tr>
<td>7</td>
<td>601</td>
<td>3265</td>
<td>5040</td>
<td>33.2</td>
<td>33.2</td>
<td>50.40</td>
<td>33° -</td>
</tr>
<tr>
<td>8</td>
<td>611</td>
<td>3468</td>
<td>5038</td>
<td>34.1</td>
<td>34.1</td>
<td>54.30</td>
<td>34°</td>
</tr>
<tr>
<td>9</td>
<td>616</td>
<td>3537</td>
<td>5038</td>
<td>35.6</td>
<td>35.6</td>
<td>55.48</td>
<td>24°</td>
</tr>
<tr>
<td>10</td>
<td>626</td>
<td>3723</td>
<td>5040</td>
<td>36.27</td>
<td>36.27</td>
<td>58.46</td>
<td>36°</td>
</tr>
<tr>
<td>11</td>
<td>646</td>
<td>3995</td>
<td>5040</td>
<td>36.42</td>
<td>36.42</td>
<td>62.15</td>
<td>17° +</td>
</tr>
<tr>
<td>12</td>
<td>644</td>
<td>4014</td>
<td>5042</td>
<td>36.33</td>
<td>36.33</td>
<td>62.52</td>
<td>50° -</td>
</tr>
<tr>
<td>13</td>
<td>662</td>
<td>4396</td>
<td>5042</td>
<td>41.4</td>
<td>41.4</td>
<td>67.19</td>
<td>54°</td>
</tr>
<tr>
<td>14</td>
<td>685</td>
<td>4635</td>
<td>5044</td>
<td>42.38</td>
<td>42.38</td>
<td>68.24</td>
<td>3° +</td>
</tr>
<tr>
<td>15</td>
<td>686</td>
<td>4797</td>
<td>5040</td>
<td>43.78</td>
<td>43.78</td>
<td>71.55</td>
<td>68°</td>
</tr>
</tbody>
</table>

It is accordingly apparent, that when the crossepiece was moved along, with the pivot of the post adhering to it quite firmly, the instrument followed, pulled from its mounting by force, the bindings slacking, and that this happened after moment 5. The error in time, however, was very slight. And on the other hand, in the last moments, the role from which I had hung the curvain hindered the waist somewhat, which had recoasted it as moving across, preventing the rule from following the sun's descent in elevation. So if at the end of the time you establish the time from the altitude, and again from the azimuth, you will find a difference of 4 minutes. For the altitude gives a tone of 5° 59' 36", the azimuth 2° 59' 23". And since it is certain that both the altitude and the azimuth are slightly in error, the mean of 5° 59' 36", taken as true, will be no more than one minute from the truth itself. The others, I stand by the altitudes, with the azimuths giving their testimony from a distance. This I did with all the more diligence, both because the foundations of lunar demonstrations will be able to be laid upon this eclipse in upon a concession, and because I had no little agreement with the late Tycho Brahe about the time of the beginning, who, having grasped some slight opportunity, called the whole record of times into doubt. But I have no doubt that if his life had continued to a time when I could have showed him this consensus, he would have submitted. The times, then, along with the necessary data from the first motion,14 are as according to the previous problems. The sun's declination was 22° 17', and 16° because of parallaxes. The pole's altitude was 47° 2'.

14 The daily rotation of the earth.
Therefore, from the beginning and end, the middle at Graz was estimated by me to be at \(10^\circ 47'11''\), the duration 28 \(20'\). For the Tychoan observers, in the case of Bezark, which is 5 German miles to the northeast of Prag, the middle was \(10^\circ 46'11''\), the duration 28 \(30'\). The limits were judged by them to be about 5, while to me they appeared 6, while still in the ris and thus after the removal of the shell, 7.

And Tycho’s birth could not have been treated, lest his observation ne undertaken by mine, nor ought he to have considered mine by his, nor to have considered a about the beginning was seen by me, the farthest to the east, at 11 \(3^0 11''\) and by him, farther west, at 12 \(40'\). For I was also 3 degrees farther south, and saw the sun more greatly eclipsed by about the same number of shades, and thus saw the beginning earlier and the end later, and its darkness beneath the same more, and the darkness beneath the same more. He observed, however, there was no eclipse in the moon’s path which was in the east and -

242

On the estimation of the moon’s diameter in the eclipse, see Part 12 above. Now, as you see both from the table and from the digits, the maximum
darkening was between 20 0 and 30 3[10], closer to the latter. Therefore, the
visual conjunction with respect to longitude was about 20 30, nearer by half an
hour to the end than to the beginning. For the increments of longitudinal parallax
decrease after moon. And because the sun is at 18° Carces; its radius will be
15° 1'. But the moon's simple anomaly, 8 signs 14° 30', through an eccentricity
of 4336, indicates a radius of 16° 10', where it is 15° 15' at apogee. Therefore,
the sum of the radii is 31° 11'. This is the distance of centers at the beginning
and end of the eclipse.

But in order to obtain this same thing at the other moments, note that in
these problems it has been more than once repeated from Ch. 2 that the digits
eclipsed on the enlarged ray number the true approach of the centers. But the dig-
its eclipsed on the ray are twelfths of the enlarged ray, and the enlarged ray of
the sun, as pointed out in Prob. 2 above, had 105° 3, more correctly 106°. Therefore,
the centers, and the others in which the digits were noted, these
units are given. When, therefore, we subtracted from the half-day, 53, along
with the radius of the opening, 8° 3, the remainder is the units returning to the
center of the sun, and the number of minutes in which this amount will be ob-
tained from the unshelled ray, which is 89°. For if 89° are 3° 1', how much,
then, will the remainder to the center be? Whatever comes out here, added to
the moon's radius, establishes the true distance of centers, and if the sum of the
units from the radius of the opening and the digits together exceeds the units of
the radius of the ray, the excess, reduced to minutes of 53, will have to be sub-
tracted from the moon's radius, so that the true distance of centers may again be
obtained.

With these things set up thus, and with the angle adopted which we had
found most recently, the visual longitude and latitude at all moments is estab-
lished, which I present in the following table.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Units Lost</th>
<th>Remainder in the Center</th>
<th>Value</th>
<th>Distance of the Center</th>
<th>Apparent Latitude</th>
<th>Apparent Longitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2. approx.</td>
<td>42° 14' approx.</td>
<td>14.16</td>
<td>14.26 approx.</td>
<td>5° 49'</td>
<td>12° 17'</td>
</tr>
<tr>
<td>2</td>
<td>20°</td>
<td>24° 8'</td>
<td>24.18</td>
<td>24.18</td>
<td>3° 52'</td>
<td>23° 10'</td>
</tr>
<tr>
<td>3</td>
<td>36°</td>
<td>18° 6.7</td>
<td>18.7</td>
<td>18.17</td>
<td>5° 22</td>
<td>20.45</td>
</tr>
<tr>
<td>5</td>
<td>35° beyond</td>
<td>9° minutes</td>
<td>3.12</td>
<td>22.22 minutes</td>
<td>11.1</td>
<td>15.55</td>
</tr>
<tr>
<td>6</td>
<td>33° approx.</td>
<td>2° 14' approx.</td>
<td>2.42</td>
<td>19.136 approx.</td>
<td>12.24</td>
<td>5.16</td>
</tr>
<tr>
<td>8</td>
<td>53° plus</td>
<td>8° plus</td>
<td>8.23</td>
<td>32.28 minutes</td>
<td>13.25</td>
<td>1.40</td>
</tr>
<tr>
<td>10</td>
<td>44° approx.</td>
<td>0° 11' approx.</td>
<td>16.21</td>
<td>16.21 approx.</td>
<td>14.45</td>
<td>5.54</td>
</tr>
<tr>
<td>55</td>
<td>41° below</td>
<td>0° plus</td>
<td>0.11</td>
<td>16.21 plus</td>
<td>14.45</td>
<td>5.54</td>
</tr>
</tbody>
</table>

115 These are the seconds of a minute, which we know in the note above.
First, you see what use it is to have many moments rather than the beginning and the end. For unless another moment, and the rest after it, had immediately supported the first with their consensus, I would have been left within 5 minutes of initial latitude in flat doubt in which I had fallen from the gap in the numbering of the inclination, in passing over an entire decade of degrees.

Second, you also see, in the eleventh moment, how easily error is produced. For it is in the notes that there were less than 5 digits in the defects. And so here, from the supposition of 5 digits precisely, the visual latitude would become decidedly retrograde and decreasing. As a result, it is good to seek this latitude more from the measurement of the observation.

Third, it is clearly evident that the maximum darkening was a moment 7 and 8, when the moon was not yet visibly in conjunction with the sun with respect to longitude. The digits on the ray were 6; in the sky, after removal of the shell, 7.

Fourth, the angle of visual latitude is notable, as is evident from the last and thirteenth moments, which are reliable. For the difference in latitudes is 12° 6', while the moon visibly traverses 47° 38' of longitude. The angle of visual latitude is therefore 14° 16'. For the parallax of latitude was increasing, and at the same time the moon was descending southwards in its true motion. The reason why this angle was not larger is the altitude of the sign, in which the parallels are small, and the increments themselves are slight.

Fifth, the moon's visual path was curved, as is evident from moments 1, 8, and 13. For if 1 and 8 are compared, once 28° 16' of longitude the moon was carried up 5° 49' in altitude. Therefore, the angle of latitude, 15° 27', is greater than before, from moments 1 and 13.

The direction in which this eclipse is useful to us in testing the moon's motions, for the sake of which we have put in so much work in examining it, will follow in the second part [of the Optics], to which place the parallaxes of this eclipse are also deferred.

Another example

On 14/24 December 1691, at Prague in Bohemia, we saw the sun eclipsed on the north in the afternoon, in the following manner.

I had brought my panels with me from Styria along with a foot measure, with which I prepared a rule of the same length as the one in Styria. The room was very dark. The rule with the panel rested on a crescent, and was set upright
upon it in the plane of the vertical, as was described in Prob. 15, since the thickness of the wall did not bear the whole apparatus of the instrument, described in the first problem. Further at the end the opening also had to be changed, since the sun struck the south wall too obliquely.

The time: were noted, not from the sun’s altitude, but more conveniently from the Tychoic clock, with a hand for minutes and seconds. We had made a test of this clock the night before, as to whether it was of the right speed, by observing transits of the fixed stars, and on the preceding moon, when the hand was made fast at the point of 12th, we held up its motion until the sun’s center, by the Tychoic quadrants, had come to the meridian. Further, by a small brass quadrant of a foot and a half, we took note rather crudely of the azimuths of the sun between the initial observations.

There follows a true and reliable transit of the paper on which the observations were recorded.

<table>
<thead>
<tr>
<th>Time</th>
<th>Azimuth on the Small Quadrant</th>
<th>Vertical Distance Scale from South to the West on the Instrument that, with the Houses</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>1 Towards the east, approximate</td>
<td>70. more read 72, as in the next.</td>
</tr>
<tr>
<td>12:50: 0</td>
<td>12. 30 to the west</td>
<td>72. curve down from proceeding.</td>
</tr>
<tr>
<td>1.10</td>
<td>1.15. 20.</td>
<td>73. uncertain.</td>
</tr>
<tr>
<td>1.47:30</td>
<td>Beginning.</td>
<td>74.</td>
</tr>
<tr>
<td>1.49:45</td>
<td>1 digit</td>
<td>75.</td>
</tr>
<tr>
<td>1.25:15</td>
<td>118. 30.</td>
<td>76.</td>
</tr>
<tr>
<td>1.24:30</td>
<td>1.30. 0.</td>
<td>77.</td>
</tr>
<tr>
<td>1.29:15</td>
<td>21. 40.</td>
<td>78.</td>
</tr>
<tr>
<td>1.35:30</td>
<td>2 digits</td>
<td>79.</td>
</tr>
<tr>
<td>1.42:0</td>
<td>72. 15. more certain</td>
<td>80.</td>
</tr>
<tr>
<td>1.44:30</td>
<td>76.</td>
<td>81.</td>
</tr>
<tr>
<td>2.20:30</td>
<td>3 digits</td>
<td>82.</td>
</tr>
<tr>
<td>2.25:30</td>
<td>86.</td>
<td>83.</td>
</tr>
<tr>
<td>2.30:0</td>
<td>71. digits</td>
<td>84.</td>
</tr>
<tr>
<td>2.45:30</td>
<td>71. digits</td>
<td>85.</td>
</tr>
<tr>
<td>3.05:20</td>
<td>7 digits.</td>
<td>86.</td>
</tr>
<tr>
<td>8 digits.</td>
<td>87.</td>
<td>88.</td>
</tr>
<tr>
<td>3.5:0</td>
<td>8 digits.</td>
<td>89.</td>
</tr>
<tr>
<td>3.90</td>
<td>Not yet zero.</td>
<td>90.</td>
</tr>
<tr>
<td>9.15:0</td>
<td></td>
<td>91.</td>
</tr>
<tr>
<td>9.21:0</td>
<td>The sun skinned the visual horizon, with a few degrees of altitude.</td>
<td>92.</td>
</tr>
</tbody>
</table>

(Continued)
My small moon subtended 57 where the sun was 31', but the shadow perceptibly surpassed it. See Prob. 13.

The five azimuths, increased by one degree, by which the instrument declined from the meridian at the east, show the times.

<table>
<thead>
<tr>
<th>Less than the truth</th>
<th>12 56 50</th>
<th>2 39</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 8 44</td>
<td>2 56'</td>
<td></td>
</tr>
<tr>
<td>1 22 36</td>
<td>2 39'</td>
<td></td>
</tr>
<tr>
<td>1 55 0</td>
<td>0 39'</td>
<td></td>
</tr>
<tr>
<td>1 43 25</td>
<td>7 2'</td>
<td></td>
</tr>
</tbody>
</table>

This gives evidence that the declination of the instrument was a little greater, about 6, to the east. And it was impossible to distinguish half a degree's difference on such a small instrument and in this procedure, so that the sun may equally illuminate the instrument on both sides.

There follow the requisites from the first motion.

<table>
<thead>
<tr>
<th>Times</th>
<th>Calibrating Point</th>
<th>On the Nonrevolution</th>
<th>Distance of the Sun from the Nonrevolution</th>
<th>Angle between the Sun and the Eclipse</th>
<th>Angle between the Eclipse and the Circle through the Centers</th>
</tr>
</thead>
<tbody>
<tr>
<td>South</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 37'</td>
<td>20 45 Cap.</td>
<td>15 44 Aquarius</td>
<td>70 4</td>
<td>42 51'</td>
<td>76 9</td>
</tr>
<tr>
<td>1 44'</td>
<td>21 17 Cap.</td>
<td>16 41 Aquarius</td>
<td>69 50</td>
<td>43 48</td>
<td>74 44</td>
</tr>
<tr>
<td>2 23'</td>
<td>22 6 Cap.</td>
<td>18 3 Aquarius</td>
<td>49 36</td>
<td>45 35</td>
<td>74 15</td>
</tr>
<tr>
<td>2 24'</td>
<td>22 26 Cap.</td>
<td>18 10 Aquarius</td>
<td>69 21</td>
<td>46 56</td>
<td>74 52</td>
</tr>
<tr>
<td>2 35'</td>
<td>25 2 Cap.</td>
<td>23 6 Aquarius</td>
<td>68 30</td>
<td>50 6</td>
<td>73 17</td>
</tr>
<tr>
<td>1 44'</td>
<td>27 10 Can.</td>
<td>26 22 Aquarius</td>
<td>67 58</td>
<td>53 27</td>
<td>71 59</td>
</tr>
<tr>
<td>2 26'</td>
<td>5 51 Aquarius</td>
<td>8 24 Pisces</td>
<td>64 46</td>
<td>65 29</td>
<td>68 47</td>
</tr>
<tr>
<td>2 30'</td>
<td>8 11 Aquarius</td>
<td>11 17 Pisces</td>
<td>65 53</td>
<td>68 21</td>
<td>65 30</td>
</tr>
<tr>
<td>2 43'</td>
<td>11 32 Aquarius</td>
<td>15 8 Pisces</td>
<td>62 36</td>
<td>72 22</td>
<td>60 44</td>
</tr>
<tr>
<td>3 23'</td>
<td>13 54 Aquarius</td>
<td>17 44 Pisces</td>
<td>61 40</td>
<td>74 47</td>
<td>62 31</td>
</tr>
<tr>
<td>3 25'</td>
<td>15 35 Aquarius</td>
<td>19 35 Pisces</td>
<td>60 51</td>
<td>76 38</td>
<td>61 39</td>
</tr>
<tr>
<td>5 92'</td>
<td>18 5 Aquarius</td>
<td>22 3 Pisces</td>
<td>60 3</td>
<td>70 5</td>
<td>60 30</td>
</tr>
<tr>
<td>3 35'</td>
<td>19 38 Aquarius</td>
<td>23 25 Pisces</td>
<td>59 31</td>
<td>80 22</td>
<td>59 12</td>
</tr>
<tr>
<td>1 7'  21 1 Aquarius</td>
<td>24 26 Pisces</td>
<td>58 54</td>
<td>81 57</td>
<td>50 8</td>
<td></td>
</tr>
</tbody>
</table>

The diameter of the sun with its border reached 110 units. That of the moon or shadow reached 75, on which see Prob. 13. Had the sum is 52', So whatever the amount of the eclipse will have been, will be subtracted from this, and the true distance of the centers in our units will remain. However, this should be compared, not to the quantity 110 of the diameter with the border, but to the unshelled diameter, 93', following this line of reasoning: if 93' make 31 minutes, what is our remainder? Now these distances of the centers, according to their size angles with the eclipse; and their complements, will be deduced by our parallactic table, in longitude and latitude, in the following table.
Here, because the eclipse is very large, and at the middle the distance of centers is small, the moments of greatest darkness and of visual conjunction therefore hardly differ in longitude. The latter was at 1h 5m, the former a little before. The eclipse, accordingly, was slightly greater than 8 digits on the ray. Subtract the distance of centers, 6° 22', and less than that, from the moon’s radius, 15° 15’, and there remains 8° 57’, and more, and the sun’s radius, added to this, makes 24° 23’ and more, eclipsed, which, in the sky, are about 92 digits or more than that. And the image of the eclipse was precisely this on the ray, where the inner lines show the true image of the eclipse after the inscribing of the ray.

The moon’s path is again curved, for if you add 30° 35’ and 0° 56’, the former from 1° 17’ 30” and the latter from 2° 5, 3°, the longitudinal path is 31° 31’. When the latitudes are added, the latitudinal path is 8° 33’. The angle is therefore 15° 11’. If you do the same thing with 1° 17’ 30” and 2° 21”, the angle will become 13° 26’. So, up to this point, the eclipse is always greater in the middle than is gathered from the beginning and end.

[In the 1604 edition, Kepler’s endnote followed here. They have been included as footnotes, or appendixes, when they are lengthy, at the appropriate places in the text.]

Conclusion

And so up to this point we have dealt with the deceptions of vision, and, treating the subject through the procedures and operations of visual perception, we have brought it down pretty much within the limits of books 4, 5, and 6 of Plato’s Alcibiades. Time, then, to sound the retreat, so as not to attempt all at once.
Besides, if God should prolong my life and vigor, I shall set forth the use of these observations in another book, which you might consider either as the second part of this work, or as an appendix. I shall everywhere interlace the three books of Ptolemy just mentioned with new problems, highly ingenious and delightful, on the second movable,110 and I shall more briefly, and more completely show how to use observations, fewer in number and easily comparable, to investigate the same things that Ptolemy investigated. This seems all the more necessary, since, in my book, the linear theory was published without the proofs of demonstrations, in the form of the book of Prognostications.111 It was moved in its appendix, which form allowed the entire subject onto a few pages.111 And because the chief aim of this book will be to investigate the sizes and distances of the three bodies of the Sun, Moon, and Earth, while indeed, as is clear from Theon, Hipparchus worked through the same material in a separate book, to which he gave his very title, so therefore, that it may be happy and fortunate, the writer of the book shall be Hipparchus.112 Farewell, reader, and to follow through my efforts with your good wishes, remember that he who has begun well has half of the achievement.120

110 Seconds movebiles. These included the sun, moon, and planets, as distinguished from the prior movebiles, the outermost body that performed the daily rotation of the heavens. Kepler is of course using conventional terminology here, and does not mean to endorse the cosmology from which these terms came.

111 That is, preliminary exercises.

112 The appendix, which appears on pp. 320–23 of volume III of TBOO (not volume II, as stated in the note in MPGW II, p. 462), was written by Kepler. In (p. 321) Kepler explains that, although Brahe had not originally planned to include the linear theory, he expanded the work in the course of printing so as to include a brief account of the moon. The “few pages” were actually more than fifty.

113 Hipparchus’s book On the Sizes of the Sun and Moon is lost. Its title is known from Autolycus’s commentary on the Timaeus, 200B, which Theon of Smyrna quoted extensively in his own commentary on the Timaeus. Kepler’s Hipparchus was never completed, though some of the ideas and theories of that work appeared in other books that Kepler published. For an account of the Hipparchus, and a transcript of the surviving manuscripts, see JEGW XX, pp. 181–268 and 501–36.

120 Heraclitus, Eikodemos 1.2, 40.
[Kepler's] Index of Memorable Things and Questions, and of Passages Added from the Authorities

[Page numbers are those of the 1604 edition (in the inner margins). Kepler originally included line numbers; these have been omitted. Entries have been sorted by English alphabetical order.]

A
Air, altitude of, 129, 134.
summer brightness of, during the day, where from, 255, 259.
its force, 303.
color of air, 258.
in respect to density, 127, 140.
matter of, what kind, 117, 128-9.
motion of, 144.
opacitv and shadow, 248, 270.
proportion to water, 128.
brightness in the sun, 22, 299, 301.
difference between air and aether, 77, 79, 130.
Aether, essence of, 259, 265, 301.
Albategnius on the sun's eventuality, 147, 330.
on eclipse of the sun, 269, 367.
on the sun's diameter, 343.
on determining the shape of the moon's phase, 237, 243.
on the heavenly bodies' light, 260.
on the moon's diameter, 349.
Alexandria, elevation of the pole at, 149.
siting and air and refraction, 149.
Alhazen the Arab on the place of the image, 57.
refuted, 58.
on refractions, 77.
on the heavenly bodies, 130, 150.
refuted regarding the perpendicularity of surfaces, 67.
opinion on the cause of refractions refuted, 84.
Alhazen's error about the means of vision, 158.
Alps, healthful sky of, 135.
Ammianus Marcellinus, 293.
Amm. the Arab, son of, on the moon's light, 227, 229.
'Avétszôr, definition, 5.
Amnagaros on the moon's light, 226.
Anaximander's opinion of the sun, 222.
Anatolius Constantimopolitani, 294.
Anthonius, 294.
Anthonius, definition, 229.
Antonius Maria, Copernicus's teacher, on the change in the elevation of the pole, 148.
Apollonius, 75, 92, 95, 97ff, 198.
243-4, 363-4.
Aquaependentius, 159.
Archelaus, 289.
Archimedes, 75, 343.
on the visual angle, 212.
examined, 214.
Aristarchus, probable opinion, 256.
on the sun's diameter, 343.
Aristotle, 222.
on the air's altitude, 134.
nature, 128, 359.
on light, the pelletid, the opaque, color, refuted, 10, 13, 25.
29ff.
on the moon's disc, 233.
on the lines of the moon's phases, 243, 245.
opinion on the sun's round ray examined, 38.
occasion of erring, 57.
passage from the Mechanica on recrudescence, 331.
on the balance, 17.
on winds, 129, 132.
Aristotle (cont.)

opinion and questions on vision, 162, 185, 202, 204, 205, 207, 209, 212, 219, 311, 326, 330, refuted, 210.

unfair to the ancients, 205, 229, observations of, 304, 306.

Arteries, measure of the pulse of, 327.

Artists, a trick of, 258.

Astrological theses, 128, 261.

Astronomy, parts of, 1, physical part, 223, 349, 358, precision of astronomy, how great, 271, distribution of the subject, 3.

Astronomer, need of circumspection in the, 391, 408.

Astronomers PUT ON NOTICE, 252, 351, 358, 360.

Atlas, 259.

Augsburg, elevation of the pole at, 398.

Austrian plain, altitude above the Ocean, 135.

Aventine on Mercury in the sun, 306.

B

Balance, working of, demonstrated, 17.

Baltic sea, 136.

Bede, 294.

Bending, what is, 5, 18.

Berossus, 226.

Bodin, 134.

his opinion on islands, 139.

Bohemia in winter, colder than Denmark, 136.

Book, second part or appendix of this, 360, 367, 375, 391, 430, 449.

Brow use of, 162.

Bunting, 290-2.

C

Camera obscura representing external objects, 51.

Candle, how to make it appear to burn upon a paper, 195.

Canopus, 148-9.

Cardano on the nature of air, 128.

on the redness of the eclipsed moon, 273, on a comet, 266.

on the balance, 17, refuted, 19.

Carinthia, mountains of, suffused with aroma, 34.

Catostratics, foundations of, demonstrated, 56ff.

circumstances of revising Catostratics, 90.

Cats, shining eyes of, 166.

Cebrenus, 294.

Cerebrum, structure of, 163.

Chaldeans, observations of, 307.

[Charlemagne, see Life.]

Chasmata, properties and causes of, 280.

Circle of illumination, 229, 233.

Clavius, 285, 296-9, 302.

Cleomedes on a remarkable apparition of an eclipse of the moon, 145.

on an eclipse of the sun, 289, 292.

on Canopus, 148.

on Palliciftum [Aldebaran], 149.

on the moon’s light, 226-9, 253, 272, 298.

on communicated light, 261.

on the viewable motion of the sun, 326.

passage from Cleomedes corrected, 149.

passage explained, 132, 141-2.

Clouds, why they may hang in the air, 133.

Coin seen in water, example of, 132.

Cold, an inactive quality, 12.

Color, definition, 11, 32.

whether colors shine by night, 12.

1 Entries in square brackets are cross references provided by the translator.
how they shine, 33.
bleached by light, 28.
origin of, 11.
two kinds of, 12 marg.
Comet, how to represent on a wall, 267.
whether comets born, 267.
why they spread illness, 267.
tails of, 260.
beards or tails, origin of, 264.
kinds of, 264.
why curved, 232.
their shapes, 265.
Horn-shaped, origin of, 267.
why they may change, 267.
their bodies are pellicid, 265.
wintry, 266.
examples, 266, 267, 335.
light, origin of, 264.
motion of, not circular, 335.
a suspicion about the cometary matter, 259, 304.
Commandino, 214.
Conjunction, great, of Saturn and Jupiter, what it may mean, 305.
Cone, sections of, 92.
description of, 95.
calculation of conic sections in various ways, 97ff, 380ff.
Cone, visual, 176.
Copernican earth motion, of what sort, 227, 326.
hypotheses of Copernicus true, 260.
justified, 256, 321.
reconciled with scripture, 127.
explained, 321.
defended, 322, 325ff.
presented in Euclid’s Optics, 331ff.
Copernicus’s observation of a conjunction of Venus and the moon, 306.
Crystalline humor, 167.
use of, 176.
why it may protrude, 198.
experiment concerning the crystalline, 208.
opitical performance of the cap of the crystalline, 177.
of a crystalline lens, 181.
Cydias, 289.
Cyrtopus-Leovius, 295.
D
Danube, river, 135.
Dark things, easily ignited by light, 28, 220.
whether they drive rays together, 28.
Darkest, becomes an active quality, 12.
witnesses of daytime darkness, 288–9.
instances, 285, 290ff.
whence they may arise, 203.
Democritus, opinion on vision, 205.
reconciled, 205.
Dionysius of Halicarnassus, 290.
Dioptras, account of, variety and flaws, 216.
Distance, by what aids it may be seen, 60–3, 311.
contributness for measuring the distance of an object from one standpoint, 165.
Dresden, Theater of Wonders at, 18., 224.
Dutch voyage to the north, 138.
E
Eagles, why they seek high places, 159.
Ears, your own, to what extent you can see them, 175.
Earth, how much higher than sea, 135.
reflects light, 229.
less than water, 251.
it’s globe endowed with an animate faculty, 27.
it’s workings, 224.
illumination of, 233.
diurnal motion of, 312.
Kepler’s Index

Earth (cont.)

for Copernicus, of what kind, 227.
defense of, 325ff.
purpose of, in nature, 322.
Penumbra, 239, 241.
shadow of (umbra), 267ff.
whether it is deeply dark, 278.
its anatomy, 279.
length, 270, 304.
a new way of measuring the earth’s orb, 251.
Eccentric, 328.
and explanation of, 333.

Eclipses

magnitudes of, procedure for measuring, 349ff.
procedures for computing are faulty, 407, 411, 413, 420.
inclinations of, 360ff.
use of them, 360, 391.
absurdity, 360, 411ff.
ways of noting the times of, 367ff.
purpose of, in nature, 3.

Eclipse of the moon,
is not by the earth’s shadow, 268, 270, 271.
remarkable, 136.
necessity of partial eclipses, 350.
why the beginnings and ends are uncertain, 238, 240.
various and ancient, 349, 367–8, 371–2, 412.

Eclipses of the sun
of 1560, 265, 296.
of 1567, 297ff.
of 1590, 303, 395–400ff.
of 1600, 357, 422–430. 7

7 The eclipses of 1600 and 1601 are listed twice in Kepler’s index, with the page numbers shown here.

of 1601, 359, 430–end.
of 1600, 4, 40.
of 1601, 285.
of 1605, 252, 358.
observation of, 359, 378, 414, 416.
image on a wall, 54.
whether the whole sun can be covered, 297, 296ff, 345.
comparison and rule of eclipses of the sun, 303.
color of, 303.
faulty diagram of, from Protenicus, 408.
main eclipses of the sun, what peculiarities they have, 295, 303.
total, do not always bring on night, 287.
which ones bring on night, 287, 296, 303.
usefulness of, 389.
Elbe, river, 135.
Empedocles on light, 30, 32.
defense of, 210.
on an eclipse of the sun, 288.
Epicycle, what is, 330.
Ecliptic circle, causes of, 330.
Equinoxes, a pair of on the same day, and why, 146.
The ancients’ procedure for observing them, 146, 150.
Eteesian winds, origin of, 144.
Euclid, 67.
was a Pythagorean, 331.
proposed the opinion of Copernicus in his Optics, 331.
principles of Euclid’s Catoptrics refuted, 56.
Optics, 7, 233.
Empy, wind, properties of, 133.
Eyes, that it may be nourished, 204.
anatomy of, 158.
causes of color of, 164.
causes of structure of, 63–4, 161, 197–9.
why in some places we use a single, 212.
why two, and account of their position, 62. 159-61, 310. 312.
etymology of "ouclus", 159.
parts of, 163f.
diagrams of parts of the eye, 177.
sparklings of, where from, 165.
 likeness in an onion, 163.
objects touching the eye, why they are not seen, 35.
much seeing harms the eyes, 35.
Eyebrows, uses of, 162-3.
Eyelashes, uses of, 162-3.
Eyelids, causes of, 161.

F
[Fabricius, Paul, see Paul Fabricius.]
Faces of humans, why erect, 160.
Fenel, on the humors of the eye, 204.
Figures, natural geometrical, in fossils, 224.
Fire, to ignite by cold water or ice, 194.
Fireflies, 25.
Fishes, eyes of, why they are so, 64.
Fixed stars,
forward motion of, from equinoxes, how it is to be investigated, 152-3.
by eclipses of the moon, 370f.
refraction of, why smaller than the solar, 112, 137.
whether they rotate, 262.
Flame,
in what materials, 224.
how it may be preserved in a closed vessel, 26.
Fossils, miracles of, 224.
Frosteathes, 274.
Funck, 296.

G
Galen, 160.
Gemma, Cornelius, on eclipse of the moon, 274.
on a remarkable phase of the moon, 256.
on a prodigious darkening of the sun, 259.
on a total eclipse of the sun, 295-6.
on the new star, 166.
on vision, 208.
observations of, 343, 349.
observation of eclipse, 297.
Gemma Frisius,
on the radius of the eclipsed sun, his way of observing, and its flaws, 350.
Gilbert, William, magnetics, 148, 222.
Ginger, 12, 22.
Giraffes' neck, why long, 158.
Gnomons, useful remark concerning, 377.
Graz,
longitude of, 368, 384, 395.
latitude, 380, 424.
Greenland, clouds of, 140.
Guidabaldo on the working of the balance, refuted, 17, 19.

H
Haloes, 23.
what they are, 133.
properties of, 157.
Haly on the comet, 266.
Hartmann, 38.
Hearing, account of, 34.
Heart, flame in, 26, 224.
whether it is also a spark of light, 26.
Heat, all comes from the soul and from light, 25-7.
Heavenly bodies,
why greater near the horizon, 133.
altitude of, from the earth's center, 310.
motion of, whether rotated by the eyes, 307, 324.
rising and setting of, a tint of vision, 327.
see stars.
Periplus, 290–1.

Hesse, refractions in, 135.


Huygens, shop of, 216, 298, 347.

Huygens's forms, 347, on refraction, 146–7.

Huygens's on diameters, 34, 93, 343.

Huygens's of the sun, 292.

On the Sizes and Distances, 292.

Huygens, 161.

Huygens, mountain in Schwabia, 137.

Huygens, 138.

Huygens, 298.

Huygens, what may make the, 309.

Hyperbola, 367.

Hyperbola, in the eye, 137.

Hyperbolic mirror measures refractive, 105.

Hyperbolic mirror measures refractive, 198.

Hypothetical, 377.

Hypothesis, 260.

Hypothesis, 260.

Hypothesis, 328.

I

Isidore, see Image [image] and Picture Image [species].

Isidore, of light in closed eyes, what it is, 28.

Isidore, of being acted upon, 29.

Isidore, essential characteristic of optical images, 29.

Isidore, images, 29.

Isidore, hanging in air, 180–1.

Isidore, in globes of a denser medium, 178.

Isidore, in eye, what it is, 205.

Isidore, what it is, 60.

by what things it may be made, 181.

place of an image in a mirror, 56ff, 69, 70, 72ff.

cause of, 59f.

whether its place is always on the perpendicular, 69, 70, 72.

properties of an image in a mirror, 85–9.

distinction between an image and a picture, see picture.

Images [imagines], 295.

in water, whether they measure refractive, 88–9.

of refraction in a globe, 178f.

Instrument, ellipse, 335, 354, 358.

construction of, 335, 354, 358.

figure, 339.

and of parts, 352, 354, 355, 362.

Instruments of Tycho, comparison with the Longgrave's, 217.

iris of the eye, origin of its name, 165.

Isaiah, 269.

Italy has frictions, 146.

J


on eclipse of the sun, 299, 419.

opinion of, on vision, examined, 299.

Jordanius, on the balance, 17.

Jourdain, 294.

Jostel, Melchior, his protosaphetic computation, 367.

Jullius Capitolineus, 295.

Julius Obsequens, 283.

Jusius on the comet, 266.

L

Leonard, account of, 200.

property of, 173.

Leovitius, see Cyprium.

Levi, Rabbi, 214.

Levi, on the visual angle, 214.

taken to ivc, 215.

Liechberg, 294.

Life of Charlemagne, 306.
Life of Louis the Pious, 294-5.
Light, whether it is a flowing down, 31, 33.
communicated, 13, 22, 27, 31, 228.
sits essential property, 261.
analogical being, 36.
is a certain quantity, 7-10, 23.
archetype of, 7.
delusion of, 31.
praise of, 7.
shape of, 37.
local motion of, 7-9, 11, 13, 15, 21-2.
nature of, 5, 31.
origin of, 6, 15, 224.
properties of, 13, 25, 28.
causes of refraction of, 16, 17.
four kinds of, 13, 22.
whether, if light were a property, it could move by itself, 36.
Astronomy, 239.
Livy, 292.
[Louis the Pious, see Life.]
Ludwig, L. B. von Dietrichstein, 201.
Luminous bodies are poured out into the sense of vision, 217-8.
Lupus, etymology of, 159.
Lycothenes, 296.

M
Machinery, optical, through crystals, 109, 195.
in need of optics, 96.
Macropho-
in error on the cause of refractions, 84.
at vision, 209.
passage of, 228.
Maestlin, Michael, his way of observing, 351, 361, 370.
things observed by, 297-304, 305, 306, 349.
his observation of the sun's diameter, 342.
of an eclipse of the sun, 361, 395-6, 416, 427.
 teachings of, 386ff, 411.
on a total eclipse of the sun, 290.
its computation, 291.
on the eclipsed ray of the sun, 39.
on the moon's body, 248.
on the illumination of the new moon by the earth, 254.
his observation of an eclipse of the moon, 275.
a remarkable one, 130, 145.
his diligence, 157.
his testimony on refraction, 156.
Magnet, nature of, 225.
Marcellinus, 291.
Martianus Capella, on an eclipse of the sun, 289.
an obscure passage of his, 150.
Mars, star of, when it is of prodigious magnitude, 333.
Commentary on Mars, 2, 328, 329, 339.
Menings of the cæsærum, 163.
Mercator, Chronology of, 290, 297.
Meeus, seen in the sun, when, 306.
Meridians, to obtain the difference of, by eclipses, 368, 392, 395.
Minstrels, 289.
Mirror, convex parabolic, 75.
glass mirrors, why they multiply the repercussions, 143.
account of the making of mirrors, 23 note.
Mivius's opinion, 130 note.
Moon, rippling, 218.
 why, 200, 219.
difficult to observe, 218.
lacks its own light, 228, 253-4, 256, 272-3, 277-8.
Moon (cont.)
to know whether it is waxing or waning, 246.
air poured about the moon, 302.
horns of, in which way they are of
use to astronomers, 246.
body of, what kind, 228-9; 248, 250, 252-3; 256.
observations and measure of its
diameter of, 343ff, 349.
disc cf., why plane, 253-4.
face of, 226.
unexpected angle of visible latitude
spots of, 234; 246ff, 251.
habitants of, what sort, 250.
moon's light,
amplified in the sense of vision,
217-8.
source of, 226.
light of, what sort, 228, 242, 243, 252.
illuminated by earth, 252; 254-5.
by the heavenly bodies, 277.
motion of, what sort, 227.
observations of, 217-8; 237; 246, 258; 300; 306; 344; 347-8; 409.
pull of, in eclipse, 242, 277.
phases of, 233ff.
by which lines they are shaped,
243; 245.
first phase, 257.
whether it can be "old and new"
["very new", i.e.], 257.
how long the moon is full or new,
237.
redness in eclipse, source of, 271, 412.
shadow of, 258.
measure of its diameter, 343ff.
what may be put in between moon
and earth, 129.
Purgatory in the moon, by popular
fable, 250.
Moses, 7, 222-3.
Motion of bodies, by what causes may
it be deflected, 16-7; 20-1.
Mountains, shadow of, how great in
eclipse of the moon, 248.
Mountains said to belong to the moon,
what can they do with regard to
refractions? 149.
Mueh, river in Sylva, 251.
N
Neckar, properties of the springs of,
137.
Nights, unequal brightness of, 296.
Nile, source of, 149.
Nose, palisade of, how it is useful, 162
and in notes.
O
Ob, river, 142.
Observing,
precautions, procedure, stems.
certain mean of, 263, 286ff.
Observations-
more crude, defense of, 385, 391, 405.
defense of very minute circumstances
of, 422, 426.
of the ancients, source of errors,
134.
Ocean, why its northern coast is less
cold, 136.
Olympos, altitude of, 134.
Ophius, what it is, 12.
"Olly", what it is, Euclids, 56.
Optics,
by what method it should be treated,
76.
neglect of, 190, 210.
Ovid,
contradiction in, 159.
his optical fable, 327.
P
Pain, what is, 29.
Parallaxes, ways of deducing in longitude and latitude, 156, 316. theory of, 307ff.
what parallax is, 312–15. its purpose in nature is of the highest order, 313, 321. use in astronomy, 315. difficulty in, 315. parallaxes of the orb. what they are, 321. force of parallaxes, 323. parallactic table, and the need for it, 320. its use, 320, 387, 397. examples, 387ff, 433.
Parhelic, 23. and parhelion, what their characteristics are, 157.
Paul Fabricius, 296.
Felliniad, what it is, 10, 31–3, 35, 77.
Pena, J., on the substance of air and ether, 129.
Peruvian mountains, altitude and cold of, 134.
Peebach on the reappearance of the moon, 257. eclipse of the moon, 371.
Picture, through globes of a denser medium, 178, 193ff. of seen objects in the eye, 168. comparison of visual pictures with ordinary ones, 199. on the inversion of the visual picture, 206. difference between picture and image, 178, 180, 193, 208.
Pindar, 289.
Psene, J., on the round ray of the sun, criticized, 38.
Planets, forces of, on weather, 274. various conjunctions, 305. shadows of, 304. why they appear to precede beneath the fixed stars, 308, 313. why they stand still, retrogress, 330–I.
Plutarch, on absurd inclination of eclipses, 41ff. on an eclipse of the sun, 202–90, 293. on a remarkable eclipse of the moon, 144. on Canopus, 148. saying of, 3. passage of, explained, 234. and corrected, 235–6. others, 246, 257.
Plutarch, On the Face in the Moon, 222. 227–9, 234, 247–8, 250. against, 251, 273, 277, 293, 299. on daytime darkness, 288.
Polc, attitude of, whether it may change, 148. a unique way of finding it, 424. Pomus, air at, what sort, 296. Porta, G. B., on lenses, 200–1. on vision, 209. optical device of, 51. another examined, 180, 182, 195.
Optics, 201, 210.
Posidonius, on Canopus, 148. on the moon’s light, 228, 229, 253.
Problem proposed to geometers and algebrists, 109. another for geometers, and opticians and natural philosophers, 113. another for opticians and geometers, 190.
Ptolemaeus, on a cone, 305. on planets, 305.
Proclus Diadochus, 147, 288, 299, conjecture from him about refraction, 149.
Proclus of Lycia, cited in favor of refraction, 148.
Ποιοτική (μεταφορά), what it can mean, 333 and note.
Prostaphaeretic computation of triangles, 367.
Ptolemy, on repercussion, 67.
on the observation of heavenly bodies, 260.
on the observations of the ancient- 134, 304, 306.
on dioptra, 216.
on diameters, 298-301, 343.
on Hipparchus’s eclipse, 292.
calculation of, 149, 230, 231-2.
of latitude, on what foundation, 306.
hyposes of, explained, 328.
on parallaxes, 316.
suspicion about him, 147, 422.
conjecture from him about refraction, 146.
consideration in his Geography, 148.
Purpose, consideration of, 
foreign to catoptrics, 57-4, 67, 69.
and to refraction, 54.
Pythagorean on the matter of the moon, 229.

R
Rain, where from, 128.
Rainbow, in the morning, 263.
cause of, 133.
colors, 11.
its way they exist, 266, 274, 282.
order of colors, 266.
associated with moisture, 266.
properties of, 157.
Rays, what they are, 8, 9.
the word used otherwise, 31, 37, 339.
Refraction
does not vary with the distance of heavenly bodies from earth, 112.
why they are sometimes greater.
short cut for dividing refractions into latitude and longitude, 155.
resulting sought \textit{a priori} from one refraction in a plane, 114.
the same in a spherical medium, 166.
for all times, 144, 156.
when they should be observed, 177-8.
what they alter over places and times, 134, 278.
table of, established by demonstrations, 128.
comparison with Tychonic tables, 126.
notice to the reader, 126.
calculation of, from observations, 123.
way of computing, 126.
measure of, 76ff.
from comets, 96, 105.
nature of, 187, 226.
true cause of their quantity sought, 109ff.
use of, in astronomy, 77, 130ff, 143.
against those who impugn them, 127, 131, 156.
refraction of a globe, 183ff.
various categories of, 196.
Regiomontanus, 150, 153, 367, 371.
Reinhold, on the moon’s light, 228, 233-4, 242.
limited regarding visible motion and quantity of latitude and of the diagram of eclipses of the sun, 407-8, 411, 413.
on (the moon’s) spots, 247.
phase, 237, 252–3, 257.
redness, 272.
greenish, 272, 352.
on the eclipsed ray of the sun, 39.
on parallax, 316.
correct in the circle of illumination,
230–1.
his exceptional commentary on
Peurbach, 230.

Repercussion,
of movesable, causes, 14.
whether it measures refractions, 90.
suede of, 237.
Reflex, true etymology, 166.
another cause of likeness, 174.
Rhoia, Alps of, why permanently
snowy, 135.
winters of, colder than Danish, 136.
Rhodes,
aer of, 144.
altitude of pole of, 149.
longitude of, 149.
Ringing bodies, what they are, 10.
Risner, F., 132, 150.
Rivers, fluctuation of, how great, 135.
Rohrmann,
on the substance of air and aether,
129.
on dioptras, 217.
his debate with Tycho about
refractions, 77.
observation of refractions, 135, 
138.
opinions on refractions, 136–7.
on the causes of refractions, related,
79, 81, 83, 112.

S
Scaliger, J. G., 257.
Schekel, mountain in Styria, altitude of,
251.
Schwabia, altitude of, 135.
Second, of time, how great, 327.
Senses, finite, 24.
analogy of, 30, 33.
account of, 34.
Sea captains’ observational procedure
compared with astronomical, 309.
Serene, 245.
Shadow, body, and sun seen
simultaneously, 175.
Sighting, account of, 211.
Smell, account of sense of, 33.
Signatures on the ratio of diameters of
the sun and moon, 289, 298, 
343.
Sound what it is, 34.
Sparkings, of hair and eyes, 165.
Specacle, a delightful optical, 51.
popular notice of it, 209.
another optical spectacles, 177–8.
190ff, 196, 251, 267, 274.
Spherical, image of the Trinity, 6.
“Spider’s web” of the eye, whence it is
truly named, 167.

Spirits,
keepers of light, 29.
function of, in vision, 204.
whether they are some quantity,
209.
responsibility of for flawed vision,
220.
visual, in what way they contribute
to vision, 166–70.
whether they should be considered
by the optician, 169, 220.
Stars,
all give heat, 25.
why they are sometimes hidden,
259.
and why small stars are next to
large, 219.
seen by day, 259.
why seem more purely at dawn, 221.
fixed, see Fixed stars.
why they seem to run through
clouds, 328.
new star, suspicion of a, 237 and
note.
how they can be seen above the
horizon when located beneath
it, 151.
Stars (cont.)

- why greater near the horizon, 133, 327.
- colors of, 261.
- why their diameters are sometimes greater, 132-3.
- what they may signify, 132.
- visual distances, to measure, 308-9.
- what they may be, 318.
- where and how they may be changed by refraction, 130-1, 133.
- place of, what it may be called, 308.
- lights, what kind, 262-3.
- refracted in air, 130.
- where from, 260-1.
- mutual occultation, 304.
- location, 308ff.
- twinkling of, 133, 262-3.
- [Stella, Tulemann; see Tulemann.]
- Stecichorus, 289.
- Stork, why they raise their necks, 150.

Sun.

- why it and the moon appear equal, 308.
- animate, 223.
- how it may shine, 226.
- being at rest for Copernicus, how it may appear to move, 334-5.
- is not round near the horizon, 131.
- image of the Holy Trinity, 226.
- aquant of, 338.
- degree of solar heat, how to measure, 85.
- matter of its body, what sort, 222.
- like a stone, for the ancients, 222.
- pellucid, 223.
- watery, 223.
- light of eclipsed sun amplified in the sense of vision, 218, 286.
- diameter seen greater than is correct, 302.
- diameter, observation of, 339ff.
- the ancients' procedure of observing, 147.

comparison with the lunar, 344-5, 353, 356, 360.
- variation of, 341, 343.
- eccentricity, whether it was once greater, 146.
- must be halved, 330.
- eclipse of, see eclipse.
- place in the world, 7.
- light of, what sort, 221.
- function of, 222-3.
- parallax of, uncertain, 304, 351.
- prodigious contemnus of, 259.
- ray of, admitted perpendicularly through a narrow opening, why round, 370ff.
- admitted obliquely, is an ellipse, 363.
- the path of this ellipse is a hyperbola, 376.
- return before proper time is prodigious, 138.
- departure from the ecliptic 150.

Συνολον των Τελεμάχων, 239.

T

- Tables, Rudolphian, 2.
- Tacitus, Cornel., 293.
- Tarsus of the eye, 161.
- Tartar, 142.
- Thales Miletus, on the moon's light, 226.
- eclipse of, 290.
- Theon, on the moon's redness, 272.
- on the eclipse of Hipparchus, 292.
- on daytime darkness, 299.
- Thought-bound people, eyes of, 179.
- Thucydides, 291.
- Tulemann Stella, 296.
- Transparent, see pellucid, 3.
- Triangles, spherical, neat solution of, 397-8.
- Tübinger, horizon of, 137.
- refraction of, 137.
- longitude of, 403, 406.
latitude of, 398.
Twilight, 23.
altitude of, against the common opinion, 287.
cause of, 143.
magnitude of, 78.
refractions are not caused by their matter, 117.
Tycho Brahe, why he shrunk the moon at conjunction, 252, 285ff.
on the substance of air and ether, 329.
on the center of vision, 215-6.
on the illumination of a comet, 264.
color of, 266.
tail of, 323.
motion of, 335.
on the element of fire, 129.
on the observations of the ancients, 134, 148.
on Waller, 156.
debate with Rothmann about refractions, 77.
diaptræ of, 216.
hypotheses of, 260.
comparison of them with the Copernican, 321.
explanation of, 322.
instrumens of, 285, 328.
his Mechanica, 1, 309.
what may be desired in his tubes, 339.
digits of an eclipse in his way of observing, 350.
inclinations, 366.
times, 367.
place of the moon, 370.
diameter of the sun through a slit, 332.
his observations defended, 309-10.
thought through, 350, 368, 380, 394, 407, 405, 413, 426-7, 437.
things observed about refractions, 130, 135, 137, 150, 152, 157.
in an eclipse of the moon, 145.
in the equinoxes, 146.
opinion on refractions, 126, 130-7.
tables of refractions of the sun, 83.
86, 112, 1176.
the tables themselves 121ff.
tables of refraction, how careful, 119-20.
opinion on the causes of refractions, 78, 9, 83, 112.
on the whirling of stars, 262.
for Tycho, the moon is illuminated by Venus, 254, 277.
states of Tycho's astronomy, 2.
against the Tychoanæas, 127, 309.

Uraniburg, longitude of, 368, 384, 395, 405, 409.
latitude of, 392.
Uranus vessel filled with clear water, what optical displays it may produce, 178, 193.

V
Valla, L., 149.
Venus,
whether it might illuminate comets, 264.
whether it might illuminate the moon, 254.
whether it might be able to cover a small part of the sun, 305.
why it may be seen by day, 260.
paltof, 260.
brightness of, 260.
setting of, 136.
obervation of, 262-3, 305.
very great observation, 333.
at conjunction with the sun, 258.
Venus (cont.)
at conjunction with Mercury, 305.
Vision,
most distinct, 172.
double vision of one object, where from, 183.
indirect, 172.
comparison of direct and indirect, 173.
perpendicular, 174.
of a point, 172.
how it happens, 60, 156, 211, 211.
hitherto not known, 168.
causes of confused vision, 200.
twofold causes of vision of distant objects, 200.
conflict of opinions on vision, 205.
Vision, sense of,
why it is confused when the head is inverted, 63.
in what manner it is to be considered, 201.
why vision of tender eyes is flawed, 200, 219.
errors of, 61.
various, about the luminous body, 217, 242.
about the location of the stars, 313.
about motion, 324ff.
breadth of, how it may be investigated, 213-4.
objects of, what they are, 61.
confused vision, circumstances of, 202.
how astronomers use vision, 211, 217.
Visual angle, center of, where, 212, 244-6.
Virginius, 135.
Wagges, mountains of, altitude, 135.
Walach, Ber.
passage on refraction lucidly explained, 150.

another, on a conjunction of Saturn and the moon, explained, 300.
on an eclipse of the sun, 295.
his diligence, 159.
Water shines out most brightly than earth, 251, 252.
Weser, river, 135.
Walden of Stesyl, 251.
Wilhelm, Landgrave of Hesse, his observation of refraction, 136, 149.
his dioptrar, 217.
Winds, what they are, 129.
Windel, on the place of the image, corrected, 57, 182.
on the light of the heavenly bodies, refuted, 260.
on the illumination of the moon, 229, 231, 233, 237, 240,
243-5.
on the moon’s spots, 247.
on the redness of the eclipsed moon, 273.
on the moon seen in an eclipse of the sun, 252.
astroonomical propositions examined, 129-30ff.
on refractions, 77, 86, 225.
on their causes, 84.
table of reflectors of water, 83.
described, 115.
judgement of, 116.
on refraction of the heavenly bodies, 150.
taken to task on the perpendicularity of surfaces, 67.
noted on refractions of a globe, 189.
taken to task on the round ay of the sun, 37, 58.
on the parabolic mirror, 92.
on the earth’s shadow, 268.
rect of, restored, 132.
on errors of vision in motion and size, 327.
on vision, 172.
opinion of, examined, 203ff., 209.
error on the center of vision, 215.
errors on the means of vision, 158, 166, 168-9, 174.
what is lacking in Witsen, 4, 23, 243-4.
on a total eclipse of the sun, 289.
Witsenian trigonometric short cuts, 397.
Wood, rotten, 25.
World formed by reason, not by the motions of elements, 223.

X
Xerxes, eclipse of, 290.
most account, 291.

Y
Year’s beginning from Easter, where, 366.
Τῆρολει (Theroleon), what the word may mean, 333.

Z
Zembo, refraction of, 138.
elevation of pole, 138, 140.
location, 142, 274.
Zenith, by what thing it is established, 309, 327.
Bibliography

This bibliography does not cover the substantial secondary literature on Kepler’s *Optics*. Its purpose is to provide bibliographical information about the works and editions cited in the text and footnotes. Additional bibliographical information is provided in the notes, especially where it is mentioned only once in the course of the book. Included herein are also the abbreviations by which frequently cited works are referenced. —Translator.


Alhazen (i.e., ibn al-Haitham), *Alhacenii Arabis libri sexum in Thesaurus opticsorum*.


Aristotle, *De Anima*, see *On the Soul*.


Chevalley, Catherine, tr., see Kepler.


Clavemedes, *De mundo*: *ive circulatis inspectiones metronum libri duo*, Basel, 1547, Latin translation of the preceding.


Crombie, A. C. G., translator, see Kepler.


JKGW: Johannes Kepler Gesammelte Werke, see Kepler.

JKNS: Johannes Kepler, New Astronomy, see Kepler.

JKOS: Johannes Kepleri, _Astronomi Opera Omnia_, see Kepler.


Kepler, _Johannes, Astronomia nova_ (Heidelberg 1609), see Kepler, Johannes, _New Astronomy_.


Kepler, Johannes, Mysterium cosmographicum (Tübingen, 1596), in KGW I.


Livy, Pseudo., see Julius Obsequens.


Maestlin, Michael, Disputation de evisibilibus solis et lunae. (Def. Marcus ab Ho- kooldo). Tübingen 1596.

Maestlin, Michael, Epytum astronomiae. Tübingen: 1610.


Nennequau, Otto, see HAMA.


Plutarch. The Face in the Moon, see Moralia Vol. XII.


Potta, Giovanni Baptista. Magia naturalis (Naples 1589).

Proclus Diodoerus. Hypotyposis astronomicarum positionum, Latin trs. by Georgio Valla, Basel 1541 and 1551, in Prolemy, Opera; Greek text in TLG.


Rosen, Edward. Kepler's Somnium, see Kepler, Somnium.


TBOO: Tichonis Brahei Dani Opera Omnia, see Brahe, Tycho.


Thesaurus, see Opticae thesaurus.

TLG: Thesaurus Linguae Graecae: Canon of Greek Authors and Works, Luci Berkowitz and Maria C. Pantela, eds. (Irvine: University of California, 1999), CD-ROM.
accommodation, 187.
Agrippa of Birby, 317.
Al-Battani, 158, 206, 267, 301, 340, 374.
Alhazen, 159, 206.
Alhazen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
Alhacen, 206, 267, 301.
clocks, 374, 376, 430.
Combra, eclipse at, 308.
motion of, 345–6.
Commandino, Federico, 229.
Commentaries on Mars, see Astronomia nova.
conic sections, 5, 214.
Constantine X, 305.
convergence reaction, 80.
crystalline humor, 214–5, 219.
as a lens, 191–218.
Curtius, Albertus, 13.
Cydonia, 360.
darkness in daytime, historical examples of, 299–309.
de Claro Monte, Joannes, see Liechtenberg.
Democritus, 225–1.
Dennushi, 420.
depth perception, 79–83.
de Veer, Gerrit S, 151.
Drieichstein, Ludwig von, 216.
designs, 248.
Diogenes Laertius, 237, 241.
Diogenes, Calendar of, 315.
Dionysius of Halicarnassus, 301.
dioptra, 231.
Dresden, 193.
earth, circle of illumination of, see circle of illumination.
eccentricity, whether it changes, 158–9.
eclipses, 5, 156.
measurement of, 56–7.
lunar, 279–295.
eclipse, lunar, of −134, 377.
of −145, 377.
of 926, 285.
of 1460, 378.
of 1539, 285.
of 1580, 285.
of 1588, 286–7, 381.
of 1594, 415.
of 1595, 375.
of 1598, 286, 375.
of 1599, 286.
of 1601, 378.
of 1602, 376.
of 1603, 286, 379, 414.
solar, of 1588, 416–8.
of 1590, 410, 427.
of 1593, 416.
of 1594, 418.
of 1598, 384–399, 420.
of 1599, 416.
of 1600, 423–432.
eclipse, 356–8.
Empedocles, 44, 47, 224, 300, 301.
equinoxes, determination of, 157–8.
Eratosthenes, 304.
eyes, 197, 370.
eyeglasses, 216–8.
eye, anatomy of, 171–9, 188–191.
function of, 179–88.
Fabricius, Paul, 308.
Fabricius ab Aquapendente, Hieronymus, 171.
Fernel, Jean, 219.
Finland, 294.
foxiis, 239.
Friesland, 250.
Frisch, Christian, 380.
Frodardus, 245.
Funk, Johann, 308.
Galen, 173.
Geber, 250.
Geggingen, latunde of, 401.
Gilbert, William, 159, 237.
Guadarrama the Younger, Ignacius, 305.
Greece, 261-2, 269, 268, 311, 315-6, 352, 357, 384, 398, 394, 397-8, 416-9, 423.
Guido de la Luna Monte, 29-33.
Guineas, 294.
Hale, 277.
Hamberg, 420.
Hammer, Ernst, 347, 380.
Hartmann, Georg, 55.
heavenly bodies, positions of, 319, 353.
heliocentric astronomy, optical presentation of, 342-5.
Herodotus, 302-3.
Hipparchus, 301-11, 313, 348, 380.
Hodder, 310.
Hofburg, mountain, 149.
Homer, 300.
Hondius, Joannes, 388.
Hofa, 149.
Hveit, 134, 379-6, 381, 388, 391-4, 398, 400, 408, 414-6, 419.
Illumination, full and partial, 252-6.
image, inversion of, 181.
place of, 83-91, 192.
ingegaments, signs of, 230-2.
instrument, ecliptic, 347-50.
Istiti, John, 374.
iris, 86.
Issaik, 300.
Jahir, see Geber.
Jesuit, Johannes, 711, 170, 219, 221, 240.
Jordanus, 306.
Jovan III, 305.
Julius Capitellus, 365.
Justinus, Marcus Junius, 277.
Kepler, Johannes, Astronomia nova, 13, 339, 353.

De fundamentis astrolabeae perspectivae, 314.

lens, of the eye, see crystalline humor, lenses, 76.
Leibnitz, Cyprias, 367.
Levi ben Gershom, 229.
Liechtenberger, Johann, 306.
light and colors, theory of, 5, 17-54.
Ligastri, John of, 283.
Linné, see Carl Linnaeus.
Livv, 303.
Lothair, 307.
Louis, Sr., 107.
Lucatla, 24.
Ludwig, see Louis the Pious.
Lyceum, 307.
Macrobius, 224, 243.
Megallanic Landmarks, 294.
Magini, Giovan Battista, 410.
Manius, Karl, 156.
Marinus Capella, 161, 301.
Melanchthon, Philipp, 343.
Mersenne the Roman, 317.
Mercurius, Gerhard, 301, 304-5.
Microcosmus, 300.
Mitterer, Daniel, 151.
moon, brightness of, 255-6.
circle of illumination of, see circle of illumination.
diameter of, 335, 360.
first appearance of, 268, 271.
Illumination of, 241-4, 263-8.
habitants of, 262.
shade of, 297, 3-3.
spots of, 279, 263.
Moses, 237-8.
motion, perception of, 335-345.
mountains, 19.
Mercury, 294.
Neugebauer, Otto, 340.
New Guinea, 294.
new stars, 250.

Novas, see new stars.

Novaya Zemlya, 153.

Oberhohenberg, see Hoebberg.

occultations, 314–8.

ordinate, 122.

Ovid, 337.

parabola, 90.

parallactic table, [in end pocket], use of, 330.

parallax, 319–33.

paralipomena, 91.

Pausanias, Spartan general, 302.

Peckham, John, 55–6.

Pedersen, Olaf, 340.

pellucid, distinguished from transparent, 46.

Pena, Ioannes, 142.

Pers. 294.


Pharmaccia, 237.

Pindar, 300.

pinholes, theory of, 56.

Placita philosophorum, 252.

Plato, 8.

Planer, Felix, 80, 171, 179, 222–3.

Pliny, 156, 160, 248, 258, 268, 277, 300, 302, 413.


Porta, J. B., 5, 54, 93–5, 216, 224.

Possidium, 260, 263.

Prague, 369, 376, 378, 381, 416, 425, 429.

Proclus, 158–61, 310–1, 315–6, 342, 354.

Hypotyposes, 310, 369.

Protagoras, 8.


Pythagoras, 342.

Pythagoreans, 243.

refraction, 93–169.

equation of, 126.

explanation of, 123–6.

refraction, atmospheric table, 138.

Regiomontanus, Ioannes, 5, 162, 164, 345, 374, 378.


Rhodes, 160.

Rhodius, Ambrosius, 360, 374.

Risner, Frideric, 145, 162.

Roeslin, Helianaeus, 151.

Rostock, 309.

Rothmann, Christoph, 124, 142–3, 148, 150, 231.

Rudolph II, Emperor, 5.

Sacrobosco, Ioannes, 297, 309.

Scaliger, Joseph Justus, 268.

Scaliger, Julius Caesar, 242.

Schebek, mountain, 262, 271.

Schoekel, see Schekel.

scripture, 337.

Seiffard, Matthias, 378.

Serenus, 357.

shadow, praise of, 15.

shadow of earth, dimensions of, 5, 289, 295.

Solomon Islands, 294.

Sosigenes, 301–11, 354, 369.

spider’s web, 219.

spirits, visual, 179, 219.

Stadios, Johannes, 285.


Stella, Tithonum, 398.

Steinschonar, 300.

Straussburg, 418.

Stynia, 5, 420, 423, 425, 429.

sun, diameter of, 350–4.


sunspots, thought to be planets, 316.

Tacitus, 315, 305.

Thales, 241.
Theon, 282, 300, 304, 354, 432.
Thucydides, 303.
Timocles, 304, 317.
Tomeau, Germain, 156, 315.
transparent, distinguished from pellucid, 496.
Tübingen, 406, 408, 418–9, 427.
latitude of, 491.
Tychonic observers, 427.
Urbannburg, 141, 292, 375, 398, 406, 408, 418.
uva, 80–3, 88.
Vienna, University of, 5.
vision, brownclar, 79–81, 321–3.
Vitruvius, 146.
lunar observation of, 411, 413.
Wandesburg, 375.
Wienhuber, 257.
Wolf, Hieronymus, 354.
Xerxes, 302.
Yacutan, 294.
Zembria, or Novaya Zemlya.
Zers, 416.